Modeling the convoy game for evaluation of tactical alternative

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ABSTRACT

The design of convoy formation is one of essential Anti-Submarine Warfare (ASW) issues. The objective of this paper is to develop a mathematic game model for optimizing the strategies on convoy operation in ASW scenario. The game model depends on the search theory for the convoy screens effectiveness and game theory for setting up a game matrix to analyze the tactical strategies on both commanders of convoy and submarine. With the given convoy design, there is saddle point existed in the game that leads us to calculate the optimal strategy for both sides. The result of the case study shows that the best strategy for blue is "double screens" and for red is the fast speed submarine. This game model can evaluate multiple screens convoy with different ASW assets but not for the stand-off missile attack from submarine scenario.

Keywords: ASW, convoy, game theory, payoff, search, strategy

應用賽局模型評估軍事決策之護航策略

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摘　要

護航作戰編隊之屏衛設計在反潛作戰中至為重要。本研究之目的在建立護航作戰之數學賽局模式，以之量化反潛護航策略並進行最佳化作業。賽局模式以搜索模式進行護航屏衛效益之計算，以賽局理論架構策略矩陣進行敵對雙方戰術策略之分析。研究案例之賽局中具鞍點，有單一報償值同為紅藍雙方之最佳策略；經分析，雙方最佳策略為：藍方採雙屏衛護航策略(第二策略)；紅方則採高速潛艦(第二策略)。本模式可對多屏衛、不同反潛兵力組合進行效益評估，限制則為無法用於具遠距飛彈攻擊能力潛艦的想定。

關鍵詞： 反潛作戰，護航，賽局理論，報償值，搜索，策略

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I. INTRODUCTION

Convoy is the most important national security alternatives, in particularly for island-wide countries, in maintaining the least national operations in wartime by the non-interrupted import via shipping while Sea Lines Of Communication (SLOC) is interfered by the enemy. Making sure the accessibility of SLOC, the fleet escorts play a key role that can be proved from the battle of the Atlantic in WWII.

Taiwan, for example, is an island-like country with insufficient natural resources; hence, it is unavoidable for the heavily dependence on international import of energies and bulk goods that there is over 99.25% of the energy, in average, is imported annually [1]. All of the energies rely on shipping, and the Middle East and South-East Asia are the major energies suppliers, which indicate the most of the energies, must be shipping from afar. In order to keep surviving or prosperity, the security of our shipping is one of the vital concerns of the nation security.

The most horrified attacker at sea, who is able to interfere the shipping with its invisible characteristics, is submarine.

By the advanced technology, submarine has made a great leap in its operational capability. Compared to WWII, the speed of diesel submarine is now about three times faster, the endurance is about 4.5 – 21 times longer, the submerged depth is about 2.5 - 4 times deeper, quieter (40 dB lesser), and diverse weapons on board may conduct long range attacks. Kilo submarine can be the representative of it.

The Anti-Submarine Warfare (ASW) capability has also improved in the meantime. The powerful ASW platform such as surface ship equipped with bot of active and passive sonar system as well as embarked ASW helicopter(s), Maritime Patrol Aircraft (MPA) carrying with about 100 sonobuoys, torpedoes, depth charges and Anti-Ship Missiles.

When these forces have some quite improvements in so many ways, we need to check if the convoy designed, one ASW screen, in WWII is still effective for today’s convoy scenario. The objective would be to evaluate the effectiveness of convoy design against the opponent’s strategies, which are different type of submarines by using gaming approach. In this study, the WWII convoy would be the basic alternative and make another plausible design for the purpose of contract.

Because of the complexity of ASW, we need to study some essential factors that affect the ASW performance such as ocean acoustic environment, shipping density and the concept of operation or tactics.

We adopt the theory of Two Person Zero Sum (TPZS) game as the major analytical method for evaluating the strategies presented by red force, submarine, and by blue force which is number of ASW screen. Other than that, the payoff in the game matrix is a model that will be developed by using the theory of search in ASW.

The anticipated contribution is to offer an analytical way for fleet commander in its latest convoy design issue with respect to the effectiveness of neutralizing modern submarine attack with the upgraded ASW taskforce.

II. LITERATURE REVIEW

The definitions of convoy are almost the same. Steven Schwinghamer defined convoy as: 「A convoy is a group of merchant ships traveling together. This group would either be equipped and arranged for self-defense or be escorted by warships. 」[2] The Admiralty has the definition as: 「A convoy is a group of merchant ships traveling together. This group would either be equipped and arranged for self-defense or be escorted by warships」[3] These definitions indicate the defense capability is required for safety cause. The war at sea in modern history, the merchant ships confronted multiple threats, which were air-attackers in air, submarines under sea and warships at sea. In WWII, German submarine, U-boat, was the major killer of the merchant ship until convoy was underway. The loss rates per month at sea in WWII in terms of the effectiveness of convoy, Ralph V. Buck indicated, 20% for independent sailing merchant ships and 4 % for convoyed merchant ships. There were 18% lesser shipping losses while the merchant ships were organized in convoy at sea. [4]

The convoy design in WWII is similar to Fig.1 with main body, 42 ships, and the High Value Unit (HVU) is in the center escorted by warships in the front and flanks that is mainly because U-Boat commanders preferred to lay in wait ahead of the convoy if possible. The escort screen, with six destroyers, is 5,000 yards from main body. The typical convoy, low speed, has about forty ships with the formation of ten columns and with four ships in each. [5]

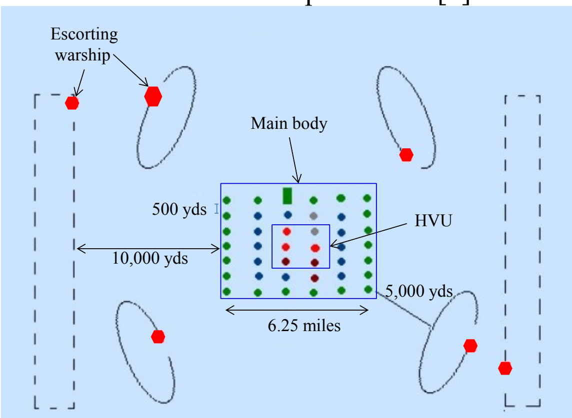

Fig. 1. Convoy formation design in WWII. (Source: WWII Anti-Submarine Warfare Tactics, the U.S. Navy Destroyer Operations in World War II.)

NATO developed concept for protect its convoy from submarines attack in 1990 with exercises for testing. The basic convoy design is as Fig.2.

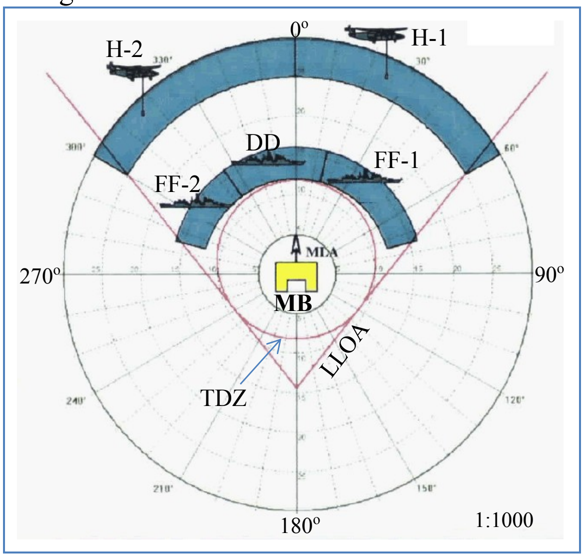

Fig. 2. NATO convoy formation designs in 1990. [7]

There are two screens ahead of main body, which has higher speed, Main Line of Advance (MLA), than the one in WWII. The outer screen deploys two dipping sonar capable helicopters; the inner screen has two frigates (FF) and one destroyer (DD) for active sonar search. The circle with two tangent lines indicates the Torpedo Danger Zone (TDZ) where the submarine must enter to be within effective conventional torpedo range. The tangent lines are the Limiting Lines Of Approach (LLOA), which only appear if the submarine speed is less than the speed of advance of the convoy. [6] As the main body speed increases, the LLOA will tend to narrow, which means the submarine has less chance to take an attack position of the circle. [7]

In fact, one of the interest areas in this study is in the screen effectiveness that relies on search theory for a plausible calculation. Submarine would always keep stealthy across the entire engagement process that causes trouble for ASW asset in detection. Most of analyses treat this issue by using random search, also called exponential detection function. The formula of random search, derived by Koopman:

$$POD = 1 - e^{-C}$$

where, POD = Probability Of Detection; C= effective Coverage. Converge = Area effective swept/ Physical size of the area where sweeping was done. [8]

Alan Washburn defines game theoretic models as prescriptive model in combat modeling spectrum with the characteristics of optimal orientation. [9] The military issues are mostly two sided games, in particular in the field of ASW with the players of submarine and the ASW asset. Ingolf Norman Kiland, Jr. and Jerry Allen present their analysis on the convoy-ASW issue by using game theoretic analysis. Concentration on the payoff function development, they succeed have it derived from search theory, random search in particular. The probability of successful kill to submarine (PS) is expresses by a chain of probability:

$PS = P_w P_l P_d$, where w is a weapon launched effectively; l is a Blue attacking unit getting into an attack position and launching a weapon given that a Red submarine is detected by a Blue attacking unit; d represents a Red submarine is detected by a Blue attacking unit, given the submarine is in the area being searched.

By using the "formula for random search" as the probability of detection, the submarine killed probability is

$$PS = P_w P_l \left[1 - \exp \frac{-\omega X T}{AS} \right]$$

where, ω = the relative speed of the Blue attacking unit and Red submarine; X = the effective sweep width of the Blue attacking unit; AS = the Blue attacking units are deployed in an area which is called the Area of Search. [10]

The battlefield situation is always ambiguous, called as "fog of war", with insufficient information for commanders making their decisions hard. Von Neumann proved finite TPZS game has a value as long as randomized strategies are permitted. TPZS game with its matrix where the row player tries to maximize the payoff while column player tries to minimize it that is beneficial for decision-making process in tactical scenario. In gaming, the saddle point is a way to check if the following process is pure or mixed strategies. Let a_{ij} be the payoff for row i and column j, and let $v_I = \max_i(\min_j a_{ij})$, $v_{II} = mix_j(\max_i a_{ij})$. If $v_I = v_{II} \equiv v$, then the game has a saddle point, v is the value of the game, and the optimal strategies for Players I and II are the row and column that guarantee v. If there is without a saddle point, then this game has only one logical choice for the opponent and uses mixed strategies concept to find out the optimal strategy instead of choosing a strategy randomly. [11]

III. MODELING

In modeling, we divide its process into the following steps:

Step 1. Scenario and concept of convoy operation
Step 2. Gaming structure and strategies analysis
Step 3. Payoff analysis
Step 4. Optimization process

3.1 Scenario and Concept of Convoy Operation

There are four points for us to concern as follows.

(1) Scenario and deployment. Red force establishes zones at sea for deploying its submarines executing the plan of sea blockade by a way of ambush. Its operational objective is to attack the High Value Unit (HVU) in convoy. With sufficient submarines, including nuclear-powered sub and diesel sub that they all are capable of ambush mission, red side considers the deployment of which type of submarine. The decision factors would be the effectiveness in attack and the penalty as submarine killed given the different performance such as speed, stealth or noise level, endurance, and firepower.

In order to ensure the security of the shipping-in product from the sea, blue force send out its ASW force to perform escort mission. With the available ASW ships and the considerations of convoy speed, size, weather condition, ocean acoustic characteristics, ASW ships' sonar performance as well as the red submarines' capability, the force combination for the build-up of escort screens attached to the assembled merchant ships needs to evaluate before the convoy plan being forged.

(2) The constraints. One of the constraints for submarine is the weapon used to do the attack will not be sub-launched missile because of the extremely difficult target identification process at sea where the shipping density stays high and the unique solo character of submarine. The left choice will be the torpedo that the submarine is subject to high risk because of the necessity of approaching target with the action of penetrating the blue's convoy screens where the ASW assets are performing their search on top. Diesel submarine, another constraint, has to concern its battery-capacity during the engagement phase because speed and endurance are very much dependent on the batteries.

(3) The submarine operations. When in the phase of waiting, submarine maintains under ultra-quiet operation mode with the submerge speed at about 4 knots, which causes very limited noise that may keep it as stealthy as possible.[12] In approaching phase, submarine would do the target analysis for selecting several targets of the convoy for attacks. The actions in this phase are the approaching speed, depth, heading and weapon selection as well as the penetrating screens, etc. The phase after attacks, avoid retaliation from ASW assets at sea or in air is the major concern for submarine that it may take an evasive action

for maximizing its survivability, in other words, minimizing the probability of being killed . The major concern for this study is on the second phase: approaching phase.

(4) The convoy operations. The main body, number of assembled ships, is in the center with several escort screens in its front. The concept of convoy formation is as fig. 3. The apple shape zone is TDZ and the inner screen, screen 1, locates over the edge of TDZ, range $D_1$from main body, for searching the submarine who maybe intense while in the attacking procedure. The next screen, screen 2, would be at range D_2 from main body. We design the ASW force in both screens with same medium frequency active sonar in the screen for maximizing the sonar performance and the awe effect to submarine. There can be outer screen, screen 3, for air-ASW asset which is not included here. In Fig.3, L and W denote the length and width of searched area for each ASW asset.

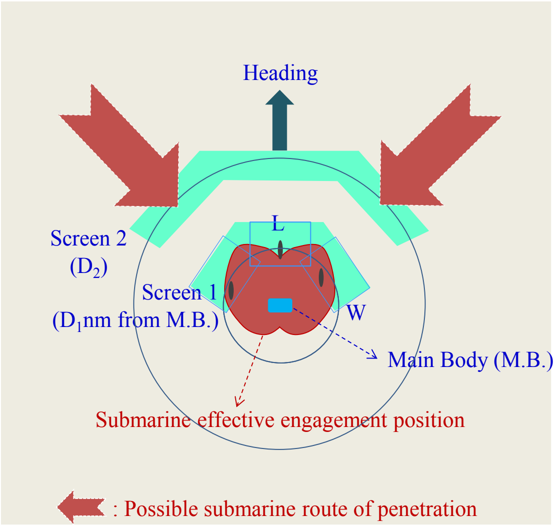

Fig. 3. Basic concept of convoy ASW design. (Source: United States Fleet Anti-Submarine Instructions)

3.2 Game structure and strategies analysis

The convoy against submarine is a confrontation problem with two players in the game, blue (convoy) and red (submarine). Same as tick-tack-toe, chess and tennis, Game theorists refer to these games as Two-Person Zero Sum (TPZS) game. The screen of convoy can be regarded as the variant ASW barrier, the searcher has the option of lower speed for hearing well or faster for covering much more area; the penetrator has the options of slower for being quiet as possible or fast for minimize the time in searcher's area. [13]

However, in this TPZS game, red side has two strategies in sending submarine that includes low speed diesel submarine, R1, and high speed nuclear submarine, R2. Blue side has not so much choice in slow speed for the main body is consisted of high-speed nowadays-merchant ships. The strategy of blue will be two force combinations that they are one screen, b_1, and two screens, b_2, protection to the convoy. The TPZS game matrix sets up as Fig. 4, where v_{ij} denotes the payoff for both sides to select the relative strategy.

		Red strategy	
		r_1	r_2
Blue strategy	b_1	v_{11}	v_{12}
	b_2	v_{21}	v_{22}

Fig. 4. Convoy game matrix.(Made by author)

The payoff is the outcome after engagement. Measure Of Effectiveness, MOE, in this game should be submarine's survivability with the advantage in evaluating the effectiveness of convoy protection by escorts. However, game theory used to put the opponent strategies on the column of matrix in where the strategy tends to be a way of minimizing the loss. Using survivability will contradict the essence of game theory design for changing the position of strategies of blue and red, or seeking for the maximized payoff in column instead of minimized. Hence, in order to avoid the conflict in analysis, we use the probability of killing submarine, i.e. blue screen-effectiveness in the engagement, instead of survivability, which means the payoff is the probability of kill, P_k, to the submarine. Let S be a function of the blue's strategy b and the red's strategy r. The blue wants to choose b to maximize S(b,r) and red wants to choose r to minimize S(b,r). Payoff is the chain probability, which is

$$P_k = P_d P_h \tag{1}$$

where,
P_d= the probability of detection
P_h= the probability of torpedo hit

3.3 Payoff analysis

There are three factors need to be concerned for developing the payoff: Measure

Of Effectiveness (MOE), screen search effectiveness, and the aggregated convoy effectiveness.

(1) MOE. The insight of secured convoy closely relates to the chance of the submarine taking its torpedo firing position, the search and kill effectiveness of ASW assets on top of submarine, the hit main body ship(s) by submarine. Base on submarine's progress in the engagement sequence, MOE can be the probability of taking attack position, the probability of submarine being killed (screen-effectiveness), survivability, or the expected numbers of main body ships are hit by submarine. Nevertheless, as to measure the effectiveness of convoy screen, we need to focus on how much chance of the screen can catch the penetrating submarine. The "catch" is regarded as successful search and hit the submarine, which means we need to figure out the probability of detection, p_d, and probability of hit, p_h prior to the probability of kill, p_k. In fact, product of these two, $p_d \cdot p_h$, the result is the probability of kill, p_k. The effectiveness of the designed convoy would allow the analyst and decision maker understand how much difference between the strategies for downgrading submarine's attack effect. Therefore, the screen-effectiveness is the best fit for this study for checking on the blue force effectiveness in the convoy against different red submarine attack.

(2) Screen search effectiveness. Detection is the first concern for the surface ASW assets with torpedo hit in succession for developing the payoff.

There are several means for detection the submerged submarine such as sonar systems or Magnetic Anomaly Device (MAD). In this study, we use the active sonar system as the detection device that includes medium and low frequency sonar with different range of detection, noted as r_1 and r_2 respectively, and deployed in two screens separately. Search theory, by Koopman, is the method we apply to calculate the probability of detection. Two points we need to evaluate, the lateral range curve for sonar system and the search pattern we adopt for calculating the probability of detection, note p_d, in the engagement scenario.

Lateral range curve. The lateral range, say r, is the distance of the closest approach (CPA) to the target from sensor. A lateral range curve is the plotting of the integrated encounter probability against CPA as in Fig.5 in which the number of missed detections (B) inside the effective sweep width equals the number of detections (A) that occur outside the sweep width, which is 2r. [14]

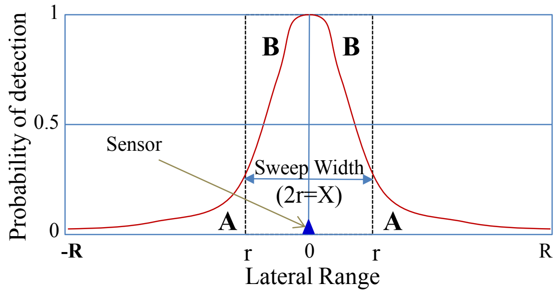

Fig.5. Lateral range curve.
(Source: Principle of search theory)

The sensors' detection performance is not a constant when against the environment including weather, ambient noise, depth and type of sea floor that would directly affect the acoustic character. Therefore, we may assume the sensors are imperfect when in the detection process that allows using cookie cutter technique to cut a clean swath reasonably in favor of analytical search modeling process. [15] With the highly interacted operational situation, the fate of the penetrating submarine would largely depend on blue ASW assets searched effort. The random search, by Koopman, can be the best representation of the action the blue assets in the assigned area of screen performing their search. In fact, Washburn indicates that random search is not a goal, but rather a skeptical prediction of the ultimate effect of trying to cover an area uniformly.

Let $P_d(t)$ be the detection probability by time t, the random search formula is

$$P_d(t) = P(T < t) = 1 - \exp^{(-\lambda t)} \qquad (2)$$

where, $\lambda = \dfrac{VX}{A}$

λ, the search effort
V, the speed of ASW asset (searcher)
X, the sweep width of searcher's sonar
A, the assigned searched area of searcher
t, the time submarine staying at A while penetrating.

The given scenario, submarine needs to

penetrate blue convoy escort screens in order to take the torpedo attack position. The speed of submarine will determine the time it staying in blue searched area (screen). In a sense of the time staying in an area where is under searching, the probability would be higher for a longer stay than the short time pass. Moreover, it is easy to prove it by random search formula. As matter of fact, the length of time the submarine staying in the screen is up to its speed that implies the power on board for propulsion system such as the speed of diesel submarine is lower than the nuclear-powered submarine.

The time of the effective search by escort is the time the submarine stay inside the area of screen that is t. The time span is dependent on the angle of submarine entering the area of screen, say θ and with the condition of $0 < \theta \le 90$, which determines the distance, d_i, the submarine passes. Let U be the speed of submarine and W be the width of assigned area of escort. Therefore,

$$\mathrm{t} = \frac{d_i}{\mathrm{U}} = \frac{\frac{W_i}{\sin\theta}}{\mathrm{U}} = \frac{W_i}{U\sin\theta} \tag{3}$$

The best choice for submarine is to minimize t, which the entering angle against the screen would be $\theta = 90^o$.

$Let\ \mathrm{C} = \frac{VX}{A}\frac{W_i}{U\sin\theta}$, $P_d(t)$ would be

$$p_d(t) = 1 - \exp^{-C} \tag{4}$$

For each ASW asset area of screen, we may re-write equation (1) as

$$P_{k_i} = \left[1 - \exp^{-C}\right]P_h \tag{5}$$

P_h is for one torpedo performance. With the number of two or salvo, n, attacks to the submarine, the outcome for at least one torpedo hit is to accumulate all of the torpedoes hitting result as

$$Acc.P_h = 1 - (1 - P_h)^n \tag{6}$$

Taking (5) into account, (4) turns out to be

$$P_{k_i} = \left[1 - \exp^{-C}\right]\left[1 - (1 - p_h)^n\right] \tag{7}$$

For multiple screens convoy, say *m* screens, with surface ASW escort, the accumulative effectiveness of screens, convoy effectiveness, is

$$P_k = \begin{cases} 1 - (1 - P_{k_1})^m \ , \ if P_{k_1} = P_{k_2} \cdots = P_{k_m} \\ 1 - \prod_{I=1}^{m}(1 - P_{k_i}), \ otherwise \end{cases} \tag{8}$$

In this analysis, we have the convoy design with at most two identical screens that each of the screens generates the same screen effectiveness. In light of this point, we may use $P_k = 1 - (1 - P_{k_1})^m$ for the outcome of payoff. For b_1 and b_2, we may have the following expression

$$P_k = \begin{cases} P_k \text{, in terms of } \mathrm{b}_1 \\ 1 - (1 - P_{k_1})^2 \text{ , in terms of } \mathrm{b}_2 \end{cases} \tag{9}$$

Substitute (9) with (7), and the consideration of submarine's two strategies (various speed), denoted as *j*, we can re-formulate the payoff generally as:

$$v_{1j} = \left[1 - \exp^{-C_j}\right]\left[1 - (1 - p_h)^n\right] \tag{10}$$

$$v_{2j} = 1 - \left\{1 - \left[1 - \exp^{-C_j}\right]\left[1 - (1 - p_h)^n\right]\right\}^2 \tag{11}$$

where, (10) represents the payoff of first blue strategy against red's strategies; (11) is the payoff of second blue strategy against red's strategies.

3.4 Optimization process

The assumed strategy in ASW convoy scenario for both blue and red are expressed as set. Blue set of strategy is $B = (b_1, b_2)$; red set of strategy is $R = (r_1, r_2)$. The criterion of the game for blue is maximin strategy and for red is mimmax.

One check must do in the beginning of optimizing process that is to exam if saddle point exists for determining it is a pure strategy or mixed strategy. If it does, then it is a pure strategy and there is payoff, say V_G that is the value of the game, and the optimal strategies for blue and red are the row and column containing V_G; otherwise, it is a mix strategy, which is with a probability assigned to each pure strategy. The saddle point is the situation when Maximin equals to Minimax, i.e., refer to Fig.4,

$$\max_b \min_r M(b,r) = \min_r \max_b M(b,r) \qquad (12)$$

where, *M(b,r)* is the matrix of Fig.4.

The following analysis is to find out the result of both sides of (12) for seeing if V_G exists.

Firstly, check on L.H.S. of (12) for $\max_b \min_r M(b,r)$:

In terms of red strategies, the variable in (10) is U_j, which is the submarine speed, where $U_1 < U_2$. The value of exponential functions, $1\text{-}\exp^{-C}$, is proportional to value C that indicates $v_{11} > v_{12}$ under the condition of $U_1 < U_2$. Likewise, equation (11) has the same consequence as (10) for $v_{21} > v_{22}$.

In terms of blue strategies, the variable is the screen number that we need to find out if (11)-(10)>0, if it is then $v_{2j} > v_{1j}$. Let us consider (10) and (11) for processing their subtraction,

$$\begin{aligned}
&(11)-(10) = v_{2j} - v_{1j} \\
&= 1 - \left\{1 - \left[1 - \exp^{-C_j}\right]\left[1-(1-p_h)^n\right]\right\}^2 - v_{1j} \\
&= 1 - (1 - v_{1j})^2 - v_{1j} = (1 - v_{1j})v_{1j}
\end{aligned}$$

$$where\ 1 > v_{1j} > 0$$

$$\therefore v_{2j} - v_{1j} > 0 \rightarrow v_{2j} > v_{1j}$$

Thus, L.H.S. of (12) can be as

$$\begin{aligned}
&\max_b \min_r M(b,r) \\
&= \max[\min(v_{11}, v_{12}), \min(v_{21}, v_{22})] \\
&= \max[v_{12}, v_{22}] \\
&= v_{22}
\end{aligned} \qquad (13)$$

Secondly, check on R.H.S. of (12) for $\min_r \max_b M(b,r)$:

From previous analysis, we may obtain

$$\begin{aligned}
&\min_r \max_b M(b,r) \\
&= \min[\max(v_{11}, v_{21}), \max(v_{12}, v_{22})] \\
&= \min[v_{21}, v_{22}] \\
&= v_{22}
\end{aligned} \qquad (14)$$

Thirdly, check equality of (13) and (14)

$$\max_b \min_r M(b,r) = v_{22} = \min_r \max_b M(b,r)$$

that we obtain v_{22} is the game value and there is saddle point exists. We say M(b,r) is a pure game.

The optimal strategy for blue is b_2 and for red is r_2.

IV. CASE STUDY

In this section, we propose a convoy case for evaluating both sides' optimal strategy with previous process.

4.1 Strategy

Assume a blue convoy passing through the South China Sea bounds for Taiwan with speed 13 knots and screened by Perry class frigates (FFG). The FFG is assigned to the ASW area in screen with maximum sonar effective speed 15 knots for its random search. Between the LLOA, shown in Fig. 2, there are three FFGs form the screen.

The available force size to support convoy is always a major concern in the wartime. There are two strategies for blue can offer, which are screen number established for the protection of main body, i.e. b_1 for one screen only and b_2 for two screens. The red side would concern what type of the force should be used to against blue convoy. Red force has two types submarine, the diesel submarine and nuclear powered submarine, with different performance in attacking convoy. In penetration blue's screen, diesel submarine would take a slower speed for reserving its battery capacity in terms of after attacks evasive consideration. Nuclear powered submarine has no such concern and it can take a higher speed for taking the torpedo attack position. Assume, diesel submarine has 10 knots and nuclear powered submarine uses 20 knots in this penetration period.

4.2 Tactical assumptions

All assets' parameters and performance are as follows:

The sonar predicted range of FFG is 2 nautical miles (nm) which make 4 nm of the sweep width, X. The speed of FFG, V, is 15 knots. The angle of heading for both of FFG and submarine, θ, is 90^o. The searched area of FFG is 50 nm^2, which means 5 nm width, W, and 10 nm length, L, of the area. The ASW torpedo has P_h =0.6 and the doctrine of torpedo firing is two.

4.3 Analysis

Let b_1 be the first strategy for analysis, which is only one screen scenario. Use of random search, the P_d of FFG against submarine's two strategies is as Fig.6 and Fig.7.

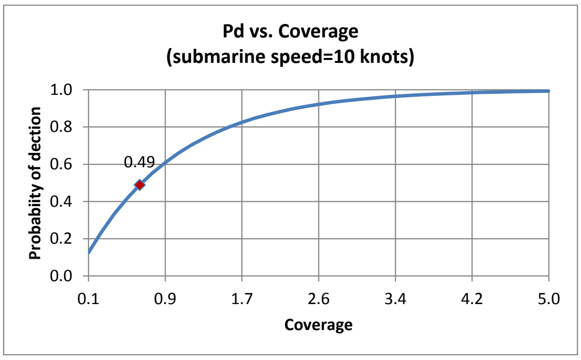

Fig.6. FFG's probability of detection as submarine speed=10 knots.

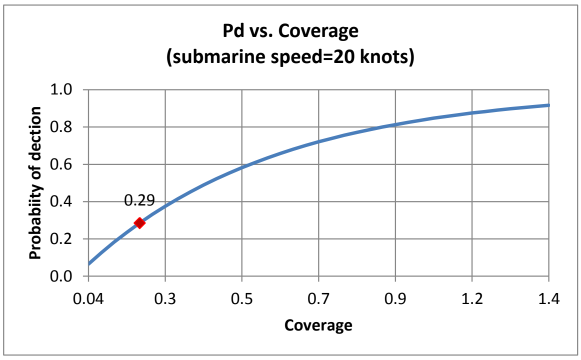

Fig.7. FFG's probability of detection as submarine speed=20 knots.

The time of penetrations, for two different speeds of submarines, are main factor to affect searched coverage of FFG. The faster speed of the submarine, the shorter the time staying in FFG's searching area, and it leads to smaller coverage of the searcher. In these two figures, with 10 knots and 20 knots submarine speed, display the coverage with 0.67 and 0.19 respectively that would generate the P_d for 0.49 and 0.29.

For screen effectiveness, according to (10), v_{11} and v_{12} are

$$v_{1j} = \left[1 - \exp^{-C_j}\right]\left[1 - (1 - p_h)^n\right] \rightarrow$$

$$v_{11} = (0.49)\left[1\text{-}(1\text{-}0.6)^2\right] = 0.41, \text{ and}$$

$$v_{12} = (0.29)\left[1\text{-}(1\text{-}0.6)^2\right] = 0.24$$

As for the strategy of b_2, taking advantage of (11), we may obtain the payoff of v_{21} and v_{22} as 0.65 and 0.42 respectively.

With all of the payoffs calculated, we may renew Fig.4 with numbers as Fig.8.

		Red strategy	
		r_1	r_2
Blue strategy	b_1	0.41	0.24
	b_2	0.65	0.42

Fig. 8. Convoy game matrix with numbers

The optimization doctrine for Fig.8 is to have mimmax in column and maximin in row, such an operation is as in Fig.9.

		Red strategy		Doctrine	
		r_1	r_2	min	max
Blue strategy	b_1	0.41	0.24	0.24	
	b_2	0.65	0.42	0.42	0.42
Doctrine	max	0.65	0.42		
	min		0.42		

Fig. 9. Optimization of the Convoy game matrix

For blue strategy, in sequence, the first step of the doctrine is to filter out the minimum of b_1 and b_2 row that the set is (0.24, 0.42) as in the column of "min". Next step is to find out the maximum number in "min" column and that is 0.42.

For red strategy, the first step is to get the maximum number from each column forming the set of (0.65, 0.42) as the row "max". The next step is to have the minimum number the set, which is 0.42.

0.42, i.e. v_{22}, is the game value for both of the optimal strategy, having the same payoff that is a pure game. We may conclude that the b_2 and r_2 is the optimal strategy for blue and red force.

From this case study, we also validate the process used for optimizing the game in 3.4.

V. CONCLUSIONS

Military decision may never leave the threat's thinking alone. It is the reason why the game theory is one of the beneficial optimal method used in military decision process. In this study, the objective is to find out the best strategy for two gamers based on the convoy design and various threats characteristics by game theory. Search theory is applied to calculate the payoffs in game matrix and with the game doctrine for optimizing process.

We have this case as a pure game with the game value v_{22} that keeps game process simple. However, some other convoy issues may not be a pure game but mixed game if some of the ASW assets use passive sonar system for submarine detection, then the faster speed of submarine would create more self-noise that is easy detected by "listening" device at longer distance offering the convoy an early warning for its defense or evasive action. We also prove that game theory is an effective tool for having the insight of strategies interaction and getting the optimal solution.

We find that the main factors would affect the convoy design are as follows:

(1) Force size and formation
More screens on convoy would increase the ASW effectiveness

(2) The coverage of searcher
More time for submarine stay in the screen, bigger the coverage area for ASW asset.

(3) The type of sonar system
Active and passive sonar systems used as mixed detection device with different detection range against various ocean conditions that would affect lateral range in search model. Submarine speed could not be the significant advantage for its self-noise.

Convoy is a serious issue for island-like countries when most of trades and energies are heavily dependent upon the international. The traditional convoy tactics are for U boat-like submarine with very limited performance in all aspects. The submarine nowadays is much more capable than the old days' submarine is such as less noisy, stealthier, faster speed, deeper, longer endurance underwater and with powerful weapon systems. The way to cope with this formidable threat from underwater, we need to consider the way to enhance the convoy effectiveness. In this study, we provide an analytical gaming method for finding the optimal strategy for both sides.

Since military decision has large part of tactical concerns that are hard to be ignored, game theory is the one of very delicate analytical tool for optimizing the proposed strategies or alternative in regard of force requirement and deployment, tactics as well as respond actions. There are some of the works are worthwhile for further study:

(1) Composite ASW forces convoy: More convoy designs are concerned in military decision for considering the composite ASW forces and multiple screens such as the use of various types of sonar systems or the mixed force with air and surface ASW assets on ASW task.

(2) Multiple-threats scenario: the long distance convoy could face Anti Air Warfare (AAW) problem while in ASW task on convoy. The convoy design should be deliberate on AAW performance with robust C^4ISR capability.

(3) Variable testing on convoy design: Convoy speed is the main factor affecting the probability of submarine taking its attack position. The faster speed of the convoy is the smaller angle (θ) in between two LLOAs and the chance for submarine to do the attacks is smaller. It could be another blue's strategies in game to do the analysis.

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