THE VIBRATION REDUCTION ANALYSIS OF A ROTATING MECHANISM DECK SYSTEM

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ABSTRACT

In this paper, an optimized position of a mass-spring-damper vibration absorber is proposed for a rotating mechanism device (such as optical disk drive or rotary-wing and deck coupled system). A nonlinear 3-D theoretical model for a deck is established by Lagrange's equation. A 2-bladed rotor and deck (foundation) coupled aeroelastic system with vibration reduction device is presented and studied as well. The analytical solution is obtained by the Multiple-scales method for the case of no vibration absorber. The numerical results in time and frequency domain and with/no absorber are acquired. This research provides a theoretical background for the preliminary vibration reduction design for industries. It is found that the existing disk drives vibration can be reduced by simply adding the absorber at the end corner isolator of the deck, but without changing the main configurations. This will not only save costs but also increase testing efficiency, achieving the most cost-effective vibration reduction result.

Keywords: Vibration absorber, Rotating Fundation, Rotary-wing Blade.

1. INTRODUCTION

Vibration reduction of an optical disk drive or any of the rotating device deck such as rotary-wing coupled plate system by using a vibration absorber is presented in this paper. The traditional vibration reduction mechanism applied in normal optical disk is using the vibration isolator (also called damping washer) consisting of a resilient member (stiffness) and an energy dissipater (damping). Once the system of the disk drive is set, it is difficult to change the configuration, and so, the vibration reduction is limited for further modification. Furthermore, for the mini-scaled or micro-scaled unmanned rotary-wing aircraft, it is practically difficult to overcome the rotor shaft vibration by only the servo control. In this paper, a new vibration reduction device concept is presented for the optical disk drive or the mini-scaled unmanned rotary-wing aircraft.

For vibration-reduction, the development of all kinds of new dampers has allowed most kinds of vibration problems to be kept to a minimum. Developments in materials science have also resulted in various kinds of compound dampers in vibration-reduction solutions. In a published report on using damper attachments to moderate structural vibrations, Nakra [1] mentioned that compound structures made from metal and polymers can achieve the dampening effect while retaining strength and hardness. This has led to their use in a variety of machinery related industries such as aerospace, marine engineering, electronics and optical structures. For the optical disc drive, past vibration-reduction design focused on modifying the mass of the absorber so it achieved better vibration-reduction [2]. This however meant that different systems required different absorbers. A custom absorber must be built for each type of optical drive, significantly increasing costs.

S. Tada et al. [3,4] examined the correlation between path tracking error and directional vibrations along the principal axis to study the frequencies of vibration sources on the main spindle. At the same time they also looked at how axial vibration affected overall system performance. Additionally, Chung et al. [5-7] proposed motion analyses for the free vibration of a spinning platter, platter vibration for non-linear angular acceleration and how automatic balancers reduce optical drive vibration. The above studies all analyzed the vibration of the entire optical drive system, but Jin Wook Heo et al. [8] proposed the concept of using a single vibration absorber to reduce the vibration and noise in existing optical drives. However, the optimal position has not been studied in their research. To improve the rotational mechanism (such as for optical drives) vibration and do so in an economic manner, this study sought to discover if simply changing the absorber position could achieve vibration reduction in the rotational mechanism without changing the absorber material or modifying the main structure of the mechanism.

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The basic model of this study uses four degrees of freedom (1 translation moving of a deck \(Z_D\), 2 rotation of a deck \((\theta_X, \theta_Y)\) and 1 translation displacement of the absorber \(Z_A\) then derives the equations of motion of the deck-absorber coupled system using the Lagrange's equation. The \(\theta_X\) and \(\theta_Y\) are considered nonlinearly to allow large motions. The frequency domain response of deck amplitudes and the time domain phase plots of deck vibration velocities and displacements are studied. The numerical results are compared with the analytic solution to ensure its accuracy; from there, changes in the absorber position were observed to achieve vibration reduction. This study can be used in the vibration-reduction mechanism design of all rotation mechanisms such as hard disks, micro-scaled swashplate-less helicopters [9], and small rotor masts. This study is divided into two major parts. The first is the vibration-reduction analysis for standard optical drives or hard disks in order assess the effectiveness of the appliqué absorber. The second part is the vibration-reduction for an integrated system comprising a 2-bladed small main rotor and its foundation. This will hopefully open up new possibilities for appliqué absorbers.

2. THE EQUATIONS OF MOTION

See Fig. 1 for this deck's associated coordinates and definitions. \(M\) is the mass of the planar vibration deck. It is treated as a rigid body where \(m\) is the point mass of the absorber on which the deck is mounted. The point of origin for this system is the center of the deck, where \(C\) and \(d\) are the viscous damping coefficients; \(K\) and \(k\) are the Spring Constants; \(I_x\) and \(I_y\) are in the \(X\) and \(Y\) direction Moments of Inertia; \(X_i\) and \(Y_i\) \((i = 1 - 4)\) represent the direction at \(X_i\) and \(Y_i\) as well as the distance from the point of origin; \(X_T\) and \(Y_T\) represent the distance to the point of incidence; \(X_A\) and \(Y_A\) represent the relative position between the absorber and the point of origin. With horizontal translation displacements in the \(X\) and \(Y\) direction not taken into account with rotation displacements in the \(Z\) direction, then \(Z_D\) is the deck displacing in the \(Z\) direction, \(Z_A\) is the displacement of the absorber in the \(Z\) direction towards the axis, while \(\theta_x\) and \(\theta_y\) is the plane's rotational angle in the \(X\) and \(Y\) direction and considered as geometrical non-linearity. Next, we considered a rotor mechanism with a set of two blades as shown in Fig. 2. If the blade’s hinge with the mast is connected using a torsional spring, then the free body diagram for one single blade is as shown in Fig. 3. If the mast and vibration reduction deck are all rigid bodies, meaning that the lift generated by the rotor creates displacement in the \(Z\) direction that directly affects the deck, then the displacement caused by \(Z_D\) and the blade lift is considered no relative motion in this part. If the flapping angle \(\beta\) is an acute angle, then the kinetic energy \(T\) from the two blades is

\[
T = \frac{1}{2} I_B \Omega^2 \beta^2 + \frac{1}{2} I_B \dot{\beta}^2 + \frac{1}{2} M_B (\dot{Z}_D)^2 + \frac{1}{2} M_B (x_\theta \dot{\theta}_x)^2 + \frac{1}{2} M_B (y_\theta \dot{\theta}_y)^2 - M_B x_\theta \dot{Z}_D \dot{\theta}_x + M_B y_\theta \dot{Z}_D \dot{\theta}_y - M_B x_\theta y_\theta \ddot{\theta}_x \dot{\theta}_y. \tag{1}
\]

Here \(I_B = \int_0^L r^2 dm_B\), \(M_B\) and \(R\) represent the blades’ moment of inertia, mass and length while \(\Omega\) is the rotor’s speed of rotation. The potential energy \(V\) produced by the two blades is then

\[
V = 2 \left( \frac{1}{2} K_\beta \beta^2 \right). \tag{2}
\]

Of these, \(K_\beta\) is the mast’s torsional spring coefficient, while the work \((W_{\text{Lift}})\) done by the lift from the two blades is

\[
W_{\text{Lift}} = 2 \left\{ \frac{1}{2} \rho \alpha c \Omega^2 \int_0^R r^3 \left[ \sqrt{(x_\theta + (Z_D - x_\theta \dot{\theta}_x + y_\theta \dot{\theta}_y + r \beta)^2 + \dot{\theta}_x^2} \right] \right\}. \tag{3}
\]
where $\theta$ is the pitch angle of the blade, $c$ is the chord of the blades and we set the induced flow as $v_i = 0.3 R \Omega$.

Once the two blades are coupled to the deck, we can then use the Lagrange’s equation to derive the collective blade mode equation as

$$I_0 \ddot{\beta}_C + \frac{1}{6} pacR^3 \Omega (\dot{Z}_D - X_D \dot{\theta}_X + Y_D \dot{\theta}_Y) = \frac{1}{8} pacR^4 \Omega \dot{\beta}_C \tag{4}$$

$$+ I_0 \Omega^2 \dot{\beta}_C + k_0 \dot{\beta}_C = \frac{1}{8} pacR^4 \Omega^2 \dot{Y}_A + 0.05 pacR^4 \Omega^2 \cdot$$

Here $\beta_C$ represents the flapping angle of the collective blade mode. While the differential blade mode equation for $\beta$ is

$$I_0 \ddot{\beta}_D + \frac{1}{8} pacR^4 \Omega \dot{\beta}_D + (J_\beta \Omega^2 + k_\beta) \dot{\beta}_D = 0 \cdot \tag{5}$$

Here $\beta_D$ represents the flapping angle of the differential blade mode.

After the two blades are coupled to the deck, the kinetic energy and potential energy of the two blades as well as the kinetic energy and potential energy of the deck are combined, so the equation of motion for four degrees of freedom of the deck are:

$Z_D$ equation of motion,

$$(M + 2M_D) \ddot{Z}_D + 2M_D Y_T \dot{\theta}_X + 2M_D X_T \dot{\theta}_Y$$

$$+ \left[ C + d + \frac{1}{2} pacR^2 \Omega \right] \dot{Z}_D + d \dot{Z}_D$$

$$+ \left[ \sum_{i=1}^n Y_i \right] C + X_D d + \frac{1}{2} pacR^2 \Omega Y_T \dot{\theta}_X$$

$$- \left[ \sum_{i=1}^n X_i \right] C + X_D d + \frac{1}{2} pacR^2 \Omega X_T \dot{\theta}_Y$$

$$+ \frac{1}{3} pacR^3 \Omega (\dot{\beta}_C + \dot{\beta}_D) + (4K + k) Z_D - kZ_D$$

$$+ \left[ \sum_{i=1}^n Y_i \right] K + X_D d + \left[ \sum_{i=1}^n X_i \right] K + X_D d \cdot$$

$$\frac{1}{3} \rho pacR^3 \Omega^2 \theta + \frac{0.3}{2} pacR^4 \Omega^2 \cdot \tag{6}$$

$Z_A$ equation of motion,

$$m \ddot{Z}_A + d \ddot{Z}_A - (Y_A d) \dot{\theta}_X + (X_A d) \dot{\theta}_Y - kZ_A + kZ_A$$

$$- (Y_A d) \left( \frac{0.3}{2} \rho \dot{\theta}_X \right) + (X_A d) \left( \frac{0.3}{2} \rho \dot{\theta}_Y \right) = 0 \cdot \tag{7}$$

To simplify the problem so it meets the conditions for the Duffing equation, we disregarded the cross multiplication of the coupled terms within the equation — that is, all the cross-multiplying items between $Z_D$, $Z_A$, $\theta_X$, and $\theta_Y$ were removed, so the equation for $\theta_X$ can be simplified to

$$2M_D Y_T \ddot{Z}_D + \left( I_X + 2M_D Y_T^2 \right) \dot{\theta}_X - 2M_D X_T \dot{\theta}_Y \dot{\theta}_X$$

$$+ \left[ \sum_{i=1}^n Y_i \right] C + X_A d + \frac{1}{2} pacR^2 \Omega Y_T \dot{\theta}_X$$

$$+ \left[ \sum_{i=1}^n Y_i \right] C + X_A d + \frac{1}{2} pacR^2 \Omega Y_T \dot{\theta}_X$$

$$- \left[ \sum_{i=1}^n X_i \right] C + X_A d + \frac{1}{2} pacR^2 \Omega X_T \dot{\theta}_Y$$

$$+ \frac{1}{3} pacR^3 \Omega Y_T (\dot{\beta}_C + \dot{\beta}_D) + \left[ \sum_{i=1}^n Y_i \right] K + X_A d \cdot$$

$$Z_D - (Y_A d) \dot{Z}_D + \left[ \sum_{i=1}^n Y_i \right] K + X_A d \cdot$$

$$\left( \theta_X + \frac{4}{3} \rho \dot{\theta}_X \right) \cdot$$

$$\frac{1}{3} \rho pacR^3 \Omega^2 \theta - \frac{0.3}{2} pacR^4 \Omega^2 Y_T \cdot \tag{8}$$

By the same principle then the equation for $\theta_Y$ can be simplified to:

$$-2M_D X_T \ddot{Z}_D - 2M_D X_T \dot{\theta}_X + (I_Y + 2M_D X_T^2) \dot{\theta}_Y$$

$$- \left[ \sum_{i=1}^n X_i \right] C + X_A d + \frac{1}{2} pacR^2 \Omega X_T \dot{\theta}_X$$

$$+ \left[ \sum_{i=1}^n X_i \right] C + X_A d + \frac{1}{2} pacR^2 \Omega Y_T \dot{\theta}_X$$

$$Z_D + (X_A d) \dot{Z}_D - \left[ \sum_{i=1}^n X_i \right] K + X_A d \cdot$$

$$\dot{\theta}_Y + \frac{1}{3} \rho \dot{\theta}_Y \cdot$$

$$\frac{1}{3} \rho pacR^3 \Omega X_T (\dot{\beta}_C + \dot{\beta}_D) - \left[ \sum_{i=1}^n X_i \right] K + X_A d \cdot$$

$$Z_D + (X_A d) Z_D - \left[ \sum_{i=1}^n X_i \right] K + X_A d \cdot$$

$$\left( \theta_Y + \frac{4}{3} \rho \dot{\theta}_Y \right) \cdot$$

$$- \frac{1}{3} \rho pacR^3 \Omega^2 \theta + \frac{0.3}{2} pacR^4 \Omega^2 X_T \cdot \tag{9}$$

It is noted that if no rotor is coupled, the equations of motion for the deck can be reduced to a four degrees of freedom system from Eqs. (6) ~ (9). The $Z_D$ direction can be expressed in the following manner:
\[ M \dddot{Z}_D + (4C + d) \dddot{Z}_D - d\dddot{Z}_A + \left( \sum_{i=1}^{4} Y_i \right) C + Y_A d \right] \dot{\theta}_x \]

\begin{align*}
&- \left( \sum_{i=1}^{4} X_i \right) C + X_A d \right] \ddot{\theta}_x \right) + (4K + k) Z_D - kZ_A \right) \ddot{Z}_A \\
&+ \left[ \sum_{i=1}^{4} Y_i \right] K + Y_A k \right] \left( \theta_x + \frac{1}{3} \theta_x^2 \right) \\
&- \left[ \sum_{i=1}^{4} X_i \right] K + X_A k \right] \left( \theta_y + \frac{1}{3} \theta_y^2 \right) \right) = F(t) \quad \text{(10)}
\end{align*}

\[ Z_A \text{ directional equation of motion can be written as:} \]

\[ m \dddot{Z}_A - d\dddot{Z}_A - (Y_A d) \dot{\theta}_x + (X_A d) \dot{\theta}_y - kZ_D + kZ_A \ddot{Z}_A \\
+ kZ_A - (Y_A d) \left( \theta_x + \frac{1}{3} \theta_x^2 \right) + (X_A d) \left( \theta_y + \frac{1}{3} \theta_y^2 \right) \right) = 0 \quad \text{(11)}
\]

The equation of motion for the \(\theta_x\) direction can be simplified to:

\[ I_x \ddot{\theta}_x + \left[ \sum_{i=1}^{4} Y_i \right] C + Y_A d \right] \dddot{Z}_D - (Y_A d) \dot{Z}_D \dot{\theta}_x \\
+ \left[ \sum_{i=1}^{4} Y_i^2 \right] C + Y_A d \right] \ddot{\theta}_x - \left[ \sum_{i=1}^{4} X_i Y_i \right] C + X_A Y_A d \right] \ddot{\theta}_x \\
+ \left[ \sum_{i=1}^{4} Y_i^2 \right] K + X_A d \right] Z_D - (Y_A d) \dot{Z}_D \dot{\theta}_x \\
+ \left[ \sum_{i=1}^{4} Y_i^2 \right] K + X_A d \right] \left( \theta_x + \frac{4}{3} \theta_x^3 \right) \\
- \left[ \sum_{i=1}^{4} X_i Y_i \right] K + X_A d \right] \left( \theta_y + \frac{4}{3} \theta_y^3 \right) \right) = F(t) \dot{\theta}_x \quad \text{(12)}
\]

Similarly, the equation of motion for the \(\theta_y\) direction can be simplified to:

\[ I_y \ddot{\theta}_y - \left[ \sum_{i=1}^{4} X_i \right] C + X_A d \right] \dddot{Z}_A + (X_A d) \dot{Z}_A \dot{\theta}_y \\
- \left[ \sum_{i=1}^{4} X_i Y_i \right] C + X_A d \right] \ddot{\theta}_y - \left[ \sum_{i=1}^{4} X_i^2 \right] K + X_A d \right] \dddot{Z}_A + (X_A d) \dot{\theta}_y \dot{\theta}_x \\
- \left[ \sum_{i=1}^{4} X_i Y_i \right] K + X_A d \right] \left( \theta_x + \frac{4}{3} \theta_x^3 \right) \\
+ \left[ \sum_{i=1}^{4} X_i^2 \right] K + X_A d \right] \left( \theta_y + \frac{4}{3} \theta_y^3 \right) \right) = -F(t) \dot{\theta}_y \quad \text{(13)}
\]

where \(F(t)\) is the external load of this deck system.

At this point we have derived the equations of motion for the deck and the rotor-deck coupled model. We can use the 4th order Runge-Kutta method to solve this directly and find out the effect of the vibration absorber on this mechanism. For Eqs. (4) ~ (9) and (10) ~ (13) we ignored the \(\theta_x\) and \(\theta_y\) terms with an order greater than 3 to meet the classical form of Dufing equation. Additionally, we also had \(Z_D = Z_{DC} \cos \omega t + Z_{DS} \sin \omega t, Z_A = Z_{AC} \cos \omega t + Z_{AS} \sin \omega t, \theta_x = \theta_{XC} \cos \omega t + \theta_{XS} \sin \omega t, \theta_y = \theta_{YC} \cos \omega t + \theta_{YS} \sin \omega t, \) and \(F(t) = F \sin \omega t\). Each equation was then separated into a cos \(\omega t\) and sin \(\omega t\) harmonic where the amplitude in the \(Z_D\) direction was \(Z_{DC}^2 + Z_{DS}^2\), in the \(Z_A\) direction was \(Z_{AC}^2 + Z_{AS}^2\), in the \(\theta_x\) direction was \(\theta_{XC}^2 + \theta_{XS}^2\), in the \(\theta_y\) direction was \(\theta_{YC}^2 + \theta_{YS}^2\), in the \(\beta_C\) direction was \(\beta_{CC}^2 + \beta_{CS}^2\) and in the \(\beta_D\) direction was \(\beta_{DC}^2 + \beta_{DS}^2\). By substituting in different \(\omega\), we can acquire the frequency response for each degree of freedom.

3. THE ANALYTIC STUDY OF THE NON-LINEAR MODEL

3.1 The Analytic Study of the Deck Model

In this section, we shall use the Multiple-scales method [10] to solve this non-linear equation of motion for the deck model in order to verify the correctness of the numerical results. Due to the complexity of this non-linear equation of motion, we have left out the effect of the vibration absorber to make the solution easier to derive. First the \(Z_i\) related items in Eqs. (10), (12) and (13) are deleted with the remaining coefficients substituted by \(A_h B_t\) and \(C_i\) (where \(i = 1 \to 8\)). Among these \(\Xi = F(t)\) simplifies each expression making them easy to develop, so Eqs. (10), (12) and (13) can be rewritten as:

\[ A_h \dddot{Z}_D + A_2 \dddot{Z}_D + A_3 \dot{Z}_D + A_4 \ddot{Z}_D + A_5 \dot{Z}_D + A_6 Z_D \]

\[ + A_h \left( \theta_x + \frac{1}{3} \theta_x^2 \right) + A_2 \left( \theta_y + \frac{1}{3} \theta_y^2 \right) = A_h \Xi, \quad \text{(14)}
\]

\[ B_1 \dddot{Z}_D + B_2 \dddot{Z}_D + B_3 \dot{Z}_D + B_4 \ddot{Z}_D + B_5 \dot{Z}_D + B_6 Z_D \]

\[ + B_1 \left( \theta_x + \frac{4}{3} \theta_x^3 \right) + B_2 \left( \theta_y + \frac{4}{3} \theta_y^3 \right) = B_1 \Xi, \quad \text{(15)}
\]

\[ C_1 \dddot{Z}_D + C_2 \dddot{Z}_D + C_3 \dot{Z}_D + C_4 \ddot{Z}_D + C_5 \dot{Z}_D + C_6 Z_D \]

\[ + C_1 \left( \theta_x + \frac{4}{3} \theta_x^3 \right) + C_2 \left( \theta_y + \frac{4}{3} \theta_y^3 \right) = C_1 \Xi. \quad \text{(16)}
\]

where \(A_h, B,\) and \(C_i\) are the coefficients in Eqs. (10), (12) and (13), respectively.

We assumed a parameter \(\varepsilon\), and this \(\varepsilon\) is used to identify the time scale. By setting \(T_0 = t\) (fast rate of change of time), and \(T_1 = \varepsilon t\) (slow rate of change of time)
then we can make the following assumption about the form of the desired solution:

\[
Z_D(t, \varepsilon) = Z_{D0}(T_0, T_1, T_2, \ldots) + \varepsilon Z_{D1}(T_0, T_1, T_2, \ldots),
\]

(17)

\[
\theta_1(t, \varepsilon) = \theta_{10}(T_0, T_1, T_2, \ldots) + \varepsilon \theta_{11}(T_0, T_1, T_2, \ldots),
\]

(18)

\[
\theta_2(t, \varepsilon) = \theta_{20}(T_0, T_1, T_2, \ldots) + \varepsilon \theta_{21}(T_0, T_1, T_2, \ldots).
\]

(19)

The differential for the time of each equation can thus be written as:

\[
\frac{\partial}{\partial t} = \frac{\partial}{\partial t_0} + \varepsilon \frac{\partial}{\partial t_1} + \ldots = \frac{\partial^2}{\partial t_0^2} + 2\varepsilon \frac{\partial^2}{\partial t_0 \partial t_1} + \ldots.
\]

(20)

Using the above assumption and ignoring the \( \varepsilon \) with an order over two, Eqs. (14) to (16) can then be written in the following manner:

\[
A_k \frac{\partial^2}{\partial t_0^2} Z_{D0} + \varepsilon A_k \frac{\partial^2}{\partial t_1^2} Z_{D1} + 2\varepsilon A_k \frac{\partial^2}{\partial t_0 \partial t_1} Z_{D0} + A_k Z_{D0} + \varepsilon A_k Z_{D1}
\]

\[+ \varepsilon A_k \theta_{10} = - \left[ A_2 \frac{\partial}{\partial t_0} Z_{D0} + A_2 \frac{\partial}{\partial t_1} \theta_{10} \right] + A_4 \frac{\partial}{\partial t_0} \theta_{10},
\]

\[+ A_0 \left( \theta_{10} + \frac{1}{3} \theta_{10}^3 \right) + A_1 \left( \theta_{10} + \frac{1}{3} \theta_{10}^3 \right) - A_k \Xi \right].
\]

(21)

\[
B_k \frac{\partial^2}{\partial t_0^2} \theta_{10} + \varepsilon B_k \frac{\partial^2}{\partial t_1^2} \theta_{10} + 2\varepsilon B_k \frac{\partial^2}{\partial t_0 \partial t_1} \theta_{10} + B_k \theta_{10} + \varepsilon B_k \theta_{21}
\]

\[= - \left[ B_2 \frac{\partial}{\partial t_0} Z_{D0} + B_2 \frac{\partial}{\partial t_1} \theta_{10} + B_4 \frac{\partial}{\partial t_0} \theta_{10} \right] + B_5 Z_{D0} + \frac{4}{3} B_6 \theta_{10}^3 + B_1 \left( \theta_{10} + \frac{1}{3} \theta_{10}^3 \right) - B_k \Xi \right].
\]

(22)

\[
C_k \frac{\partial^2}{\partial t_0^2} \theta_{10} + \varepsilon C_k \frac{\partial^2}{\partial t_1^2} \theta_{10} + 2\varepsilon C_k \frac{\partial^2}{\partial t_0 \partial t_1} \theta_{10} + C_k \theta_{10} + \varepsilon C_k \theta_{21}
\]

\[= - \left[ C_2 \frac{\partial}{\partial t_0} Z_{D0} + C_2 \frac{\partial}{\partial t_1} \theta_{10} + C_4 \frac{\partial}{\partial t_0} \theta_{10} + C_5 Z_{D0} \right] + C_1 \left( \theta_{10} + \frac{1}{3} \theta_{10}^3 \right) + C_3 \theta_{10}^3 - C_k \Xi \right].
\]

(23)

From Eq. (27), we can see that the modal frequency of \( Z_{D0} \) is \( \sqrt{A_k/A_k} \). To prevent the occurrence of resonance, all coefficients with a frequency of \( \sqrt{A_k/A_k} \) on the right-hand-side of Eq. (27) must be vanished. Therefore the following two equations are zero:

\[
2 \sqrt{A_k/A_k} \frac{\partial}{\partial t_0} a_0 = 0,
\]

(28)

\[
2 \sqrt{A_k/A_k} \frac{\partial}{\partial t_0} b_0 = 0.
\]

(29)

By the same principle, the equation \( \theta_{21} \) can be arranged as below:

\[
\frac{\partial^2}{\partial t_0^2} \theta_{21} + \frac{B_k}{B_k} \theta_{21} = 2 \left[ \frac{B_k}{B_k} \frac{\partial}{\partial t_0} a_0 + \frac{B_k}{B_k} \frac{\partial}{\partial t_1} a_0 \right] \sin \left( \frac{B_k}{B_k} T_0 + b_2 \right)
\]

\[+ \frac{B_k}{B_k} \left( \frac{B_k}{B_k} \frac{\partial}{\partial t_0} b_0 + \frac{B_k}{B_k} \frac{\partial}{\partial t_1} b_0 \right) \cos \left( \frac{B_k}{B_k} T_0 + b_2 \right) - \frac{B_k}{B_k} a_0 \cos \left( \frac{B_k}{B_k} T_0 + b_2 \right)
\]

\[+ \frac{B_k}{B_k} \frac{\partial}{\partial t_0} a_0 \sin \left( \frac{B_k}{B_k} T_0 + b_2 \right) + \frac{B_k}{B_k} \frac{\partial}{\partial t_0} b_0 \right) \sin \left( \frac{B_k}{B_k} T_0 + b_2 \right)
\]

\[+ \frac{B_k}{B_k} a_0 \cos \left( \frac{B_k}{B_k} T_0 + b_2 \right) - \frac{B_k}{B_k} \frac{\partial}{\partial t_0} a_0 \sin \left( \frac{B_k}{B_k} T_0 + b_2 \right) + \frac{B_k}{B_k} \frac{\partial}{\partial t_0} b_0 \right) \sin \left( \frac{B_k}{B_k} T_0 + b_2 \right)
\]

(30)

With the two following conditions,
By the same principle, the equation $\theta_{11}$ can be arranged as below:

$$\frac{\partial^2}{\partial t^2} \theta_{11} + \frac{C_v}{C_1} \theta_{11} = \left( 2 \frac{C_v}{C_1} \frac{\partial a_3}{\partial t} + \frac{C_v}{C_1} \frac{\partial a_3}{\partial t} \right) \sin \left( \frac{C_v}{C_1} T_0 + b_1 \right)$$

$$\left( \frac{2}{C_1} \frac{C_v}{C_1} \frac{\partial a_3}{\partial t} \frac{\partial a_3}{\partial t} \right) \cos \left( \frac{C_v}{C_1} T_0 + b_1 \right)$$

$$+ \frac{C_v}{C_1} \frac{B_k}{B_1} a_3 \sin \left( \frac{B_k}{B_1} T_0 + b_1 \right) - \frac{C_v}{C_1} \frac{B_k}{B_1} a_3 \cos \left( \frac{B_k}{B_1} T_0 + b_1 \right)$$

$$- \frac{C_v}{C_1} \frac{B_k}{B_1} a_3 \cos \left( \frac{B_k}{B_1} T_0 + b_1 \right) + \frac{C_v}{C_1} \frac{B_k}{B_1} \Xi .$$

(33)

By the same principle, with the following condition:

$$\frac{\partial^2}{\partial t^2} a_3 = \left( \frac{2}{C_1} \frac{C_v}{C_1} \frac{\partial a_3}{\partial t} + \frac{C_v}{C_1} \frac{\partial a_3}{\partial t} \right) \sin \left( \frac{C_v}{C_1} T_0 + b_1 \right)$$

$$\left( \frac{2}{C_1} \frac{C_v}{C_1} \frac{\partial a_3}{\partial t} \frac{\partial a_3}{\partial t} \right) \cos \left( \frac{C_v}{C_1} T_0 + b_1 \right)$$

$$+ \frac{C_v}{C_1} \frac{B_k}{B_1} a_1 \sin \left( \frac{B_k}{B_1} T_0 + b_1 \right) - \frac{C_v}{C_1} \frac{B_k}{B_1} a_1 \cos \left( \frac{B_k}{B_1} T_0 + b_1 \right)$$

$$- \frac{C_v}{C_1} \frac{B_k}{B_1} a_1 \cos \left( \frac{B_k}{B_1} T_0 + b_1 \right) + \frac{C_v}{C_1} \frac{B_k}{B_1} \Xi .$$

(34)

(35)

By solving Eqs. (28), (29), (31), (32), (34) and (35) with initial conditions, we can get the solutions for $Z_{d1}$, $\theta_{12}$, and $\theta_{11}$ as:

$$Z_{d1} = \frac{1}{A_3 - B_1} \left[ - \frac{A_6}{A_4} \left( a_2 + \frac{a_3^2}{4} \right) \cos \left( \frac{B_k}{B_1} T_0 + b_2 \right) \right]$$

$$+ \frac{A_6}{A_4} \frac{B_k}{B_1} a_2 \sin \left( \frac{B_k}{B_1} T_0 + b_2 \right)$$

$$- \frac{1}{A_3 - B_1} \frac{A_6}{A_4} \frac{A_3^2}{4} \cos \left( \frac{B_k}{B_1} T_0 + 3b_2 \right)$$

$$+ \frac{1}{A_3 - B_1} \frac{A_6}{A_4} \frac{A_3^2}{4} \cos \left( \frac{C_v}{C_1} T_0 + b_3 \right)$$

$$+ \frac{A_6}{A_4} \frac{C_v}{C_1} a_3 \sin \left( \frac{C_v}{C_1} T_0 + b_3 \right)$$

$$- \frac{1}{A_3 - B_1} \frac{A_6}{A_4} \frac{A_3^2}{4} \cos \left( \frac{C_v}{C_1} T_0 + 3b_3 \right) + \frac{A_8}{A_3 - A_4 \omega^2} \Xi .$$

(36)

The solutions for $Z_{d1}$, $\theta_{12}$ and $\theta_{11}$ can now be easily acquired. By differentiating the time for the solutions of $Z_{d1}$, $\theta_{12}$ and $\theta_{11}$ it is also possible to derive their individual velocity expressions though these are omitted here in the interest of brevity.

### 3.2 Analytic Study of the Rotor and Deck Coupled Equation

In order to verify the numerical results, the Multiple-scales method was used to find the analytic solution. As for the simple deck analysis previously conducted, we have omitted the effect of the vibration absorber to make getting an analytic solution easier. We removed the $Z_{d}$ related items in Eqs. (6), (8) and (9) then the re-
main coefficients were substituted using $A_i, B_i$, and $C_i$ (where $i = 1 \sim 12$). It is noted that the coefficients ($A_i, B_i$, and $C_i$) are different from Eqs. (14) ~ (16), although we use the same variables for simplicity. The coefficients in Eq. (4) were replaced with $D_j$ ($j = 1 \sim 6$) and for Eq. (5) the coefficients were substituted with $E_k$ ($k = 1 \sim 3$). This simplified the expression making it easier to solve, so Eqs. (4), (5), (6), (8) and (9) can be rewritten as:

\[
D_1 \ddot{\theta}_x + D_2 (\dddot{\theta}_x - X_T \dot{\theta}_x + T_T \dddot{\theta}_x) + D_3 \dot{\theta}_x + D_4 \dot{\theta}_x = D_0 \theta_0 + D_1 \theta_1 ,
\]

(39)

\[
E_1 \ddot{\theta}_z + E_2 \dot{\theta}_z + E_3 B_0 = 0 ,
\]

(40)

\[
A_1 \dddot{\theta}_x + A_2 \ddot{\theta}_x + A_3 \dot{\theta}_x + A_4 \dddot{\theta}_x + A_5 \theta_0 + A_6 \dddot{\theta}_x + A_7 \theta_0 + A_8 \dddot{\theta}_x + A_9 \theta_0 + A_{10} \theta_0 - A_{11} \theta_0 + A_{12} .
\]

(41)

\[
B_1 \dddot{\theta}_x + B_2 \ddot{\theta}_x + B_3 \dot{\theta}_x + B_4 \dddot{\theta}_x + B_5 \theta_0 + B_6 \dddot{\theta}_x + B_7 \theta_0 + B_8 \dddot{\theta}_x + B_9 \theta_0 + B_{10} \theta_0 = B_{11} \theta_0 + B_{12} ,
\]

(42)

\[
C_1 \dddot{\theta}_x + C_2 \ddot{\theta}_x + C_3 \dot{\theta}_x + C_4 \dddot{\theta}_x + C_5 \dot{\theta}_x + C_6 \theta_0 + C_7 \dddot{\theta}_x + C_8 \dot{\theta}_x + C_9 \theta_0 + C_{10} \theta_0 - C_{11} \theta_0 + C_{12} .
\]

(43)

As described above, we assume a parameter $\varepsilon$. This parameter $\varepsilon$ is used to identify the time scale, with the assumption that

\[
\beta_C(x, \varepsilon) = \beta_{C0}(T_0, T_1, T_2) + \varepsilon \beta_{C1}(T_0, T_1, T_2) .
\]

(44)

\[
\beta_B(x, \varepsilon) = \beta_{B0}(T_0, T_1, T_2) + \varepsilon \beta_{B1}(T_0, T_1, T_2) .
\]

(45)

\[
\beta_D(x, \varepsilon) = \beta_{D0}(T_0, T_1, T_2) + \varepsilon \beta_{D1}(T_0, T_1, T_2) .
\]

(46)

\[
\beta_{T}(x, \varepsilon) = \beta_{T0}(T_0, T_1, T_2, \ldots) + \varepsilon \beta_{T1}(T_0, T_1, T_2, \ldots) ,
\]

(47)

\[
\beta_{D}(x, \varepsilon) = \beta_{D0}(T_0, T_1, T_2, \ldots) + \varepsilon \beta_{D1}(T_0, T_1, T_2, \ldots) .
\]

(48)

It is noted that we omit those $\varepsilon$ with orders greater than 2, we can further sort these into two equations $\varepsilon^0$ and $\varepsilon^1$. By following the previous step the solution for $\varepsilon^0$ part is as written below:

\[
\beta_{C0} = a_1(T_1, T_2, \ldots) \cos \left[ \frac{\sqrt{D_{T_0}}}{A_1} T_0 + b_0(T_1, T_2, \ldots) \right]
\]

(49)

\[
\beta_{D0} = 0 .
\]

(50)

\[
Z_{D0} = a_1(T_1, T_2, \ldots) \cos \left[ \frac{\sqrt{A_1}}{A_1} T_0 + b_1(T_1, T_2, \ldots) \right] ,
\]

(51)

\[
\theta_{D0} = a_2(T_1, T_2, \ldots) \cos \left[ \frac{\sqrt{B_2}}{B_2} T_0 + b_2(T_1, T_2, \ldots) \right] ,
\]

(52)

\[
\theta_{T0} = a_3(T_1, T_2, \ldots) \cos \left[ \frac{C_{10}}{C_3} T_0 + b_3(T_1, T_2, \ldots) \right] .
\]

(53)

The explanation for the $\varepsilon^1$ part is then as follows:

\[
\beta_{C1} = \frac{1}{D_1 - A_1} \frac{D_2 + D_3}{A_1} a_1 \sin \left( \frac{A_1}{A_1} T_0 + b_1 \right)
\]

(54)

\[
\beta_{D1} = 0 .
\]

(55)

\[
\beta_{T1} = \frac{1}{A_1} \left[ \left( \frac{A_1 B_2}{A_1 B_2} \right) a_2 - \left( \frac{A_1 A_2}{A_1} \right) \right] \cos \left( \frac{B_2}{B_2} T_0 + b_2 \right)
\]

(56)
Its velocity can also be easily derived by differentiating the time.
Table 1 Properties of the deck-absorber system

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Deck mass (kg)</td>
<td>0.164</td>
</tr>
<tr>
<td>Absorber mass (kg)</td>
<td>$m$</td>
</tr>
<tr>
<td>Deck damper Coeff. (kg/sec)</td>
<td>$C$</td>
</tr>
<tr>
<td>Absorber damper Coeff. (kg/sec)</td>
<td>$d$</td>
</tr>
<tr>
<td>Deck spring Const. (kg/sec$^2$)</td>
<td>$K$</td>
</tr>
<tr>
<td>Absorber spring Const. (kg/sec$^2$)</td>
<td>$k$</td>
</tr>
<tr>
<td>Moment of Inertia (x-dir.) (kg-mm$^2$)</td>
<td>$I_x$</td>
</tr>
<tr>
<td>Moment of Inertia (y-dir.) (kg-mm$^2$)</td>
<td>$I_y$</td>
</tr>
<tr>
<td>External force coordinate (x-dir) mm</td>
<td>$X_T$</td>
</tr>
<tr>
<td>External force coordinate (y-dir) mm</td>
<td>$Y_T$</td>
</tr>
<tr>
<td>Coordinate of deck’s 4 corners (X1 mm)</td>
<td>$X_1$</td>
</tr>
<tr>
<td>Coordinate of deck’s 4 corners (X2 mm)</td>
<td>$X_2$</td>
</tr>
<tr>
<td>Coordinate of deck’s 4 Corners (X3 mm)</td>
<td>$X_3$</td>
</tr>
<tr>
<td>Coordinate of deck’s 4 Corners (X4 mm)</td>
<td>$X_4$</td>
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<tr>
<td>Coordinate of deck’s 4 Corners (Y1 mm)</td>
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<td>Coordinate of deck’s 4 Corners (Y2 mm)</td>
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<td>Coordinate of deck’s 4 Corners (Y3 mm)</td>
<td>$Y_3$</td>
</tr>
<tr>
<td>Coordinate of deck’s 4 Corners (Y4 mm)</td>
<td>$Y_4$</td>
</tr>
</tbody>
</table>

Fig. 4 Phase plot of $Z_{D}$ (numerical method)

Fig. 5 Phase plot of $Z_{D}$ (analytical method)

Fig. 6 Diagonal locations of the absorber
### Table 2  The effect of absorber on $|Z_D|$  

| Absorber position (mm) | $|Z_D|_{\text{max}}$ (10^{-4}mm) | Linear Result | Resonant frequency (rad/sec) |
|------------------------|-----------------------------------|---------------|-----------------------------|
| $X_a = 0, Y_a = 0$     | 1.3040799                         | 1.2388758     | 329.0                       |
| $X_a = 57, Y_a = 87.75$| 1.3280159                         | 1.2616151     | 329.0                       |
| $X_a = 76, Y_a = 117$  | 1.3360969                         | 1.2692920     | 329.0                       |
| $X_a = -57, Y_a = 87.75$| 1.3090641                        | 1.2436109     | 329.0                       |
| $X_a = -76, Y_a = 117$ | 1.3105442                         | 1.2450170     | 329.0                       |
| $X_a = -57, Y_a = -87.75$| 1.2793457                         | 1.2153785     | 328.0                       |
| $X_a = -76, Y_a = -117$| 1.2706181                         | 1.2070872     | 328.0                       |
| $X_a = 57, Y_a = -87.75$| 1.2981096                         | 1.2320309     | 329.0                       |
| $X_a = 76, Y_a = -117$  | 1.2958012                         | 1.2310111     | 329.0                       |

### Table 3  The effect of absorber on $\theta_X$  

| Absorber position (mm) | $|\theta_X|_{\text{max}}$ (10^{-10}rad) | Linear Result | Resonant frequency (rad/sec) |
|------------------------|----------------------------------------|---------------|-----------------------------|
| $X_a = 0, Y_a = 0$     | 4.2988017                              | 4.0838617     | 600.0                       |
| $X_a = 57, Y_a = 87.75$| 4.3291590                              | 4.1127009     | 605.0                       |
| $X_a = 76, Y_a = 117$  | 4.3886143                              | 4.1691834     | 599.0                       |
| $X_a = -57, Y_a = 87.75$| 4.2198158                             | 4.0088247     | 600.0                       |
| $X_a = -76, Y_a = 117$ | 4.1954218                              | 3.9856505     | 593.0                       |
| $X_a = -57, Y_a = -87.75$| 4.5304898                            | 4.3039654     | 588.0                       |
| $X_a = -76, Y_a = -117$| 4.6670581                              | 4.4337051     | 579.0                       |
| $X_a = 57, Y_a = -87.75$| 4.4239283                             | 4.2027321     | 577.0                       |
| $X_a = 76, Y_a = -117$  | 4.4797184                              | 4.2557323     | 563.0                       |

### Table 4  The effect of absorber on $\theta_Y$  

| Absorber position (mm) | $|\theta_Y|_{\text{max}}$ (10^{-10}rad) | Linear Result | Resonant frequency (rad/sec) |
|------------------------|----------------------------------------|---------------|-----------------------------|
| $X_a = 0, Y_a = 0$     | 3.9674291                              | 3.7690577     | 273.0                       |
| $X_a = 57, Y_a = 87.75$| 4.1409996                              | 3.9339494     | 295.0                       |
| $X_a = 76, Y_a = 117$  | 4.2383206                              | 4.0260474     | 298.0                       |
| $X_a = -57, Y_a = 87.75$| 4.1064844                             | 3.9011260     | 413.0                       |
| $X_a = -76, Y_a = 117$ | 4.1527278                              | 3.9450913     | 413.0                       |
| $X_a = -57, Y_a = -87.75$| 4.2705710                             | 4.0570427     | 456.0                       |
| $X_a = -76, Y_a = -117$| 4.4579537                              | 4.2350561     | 491.0                       |
| $X_a = 57, Y_a = -87.75$| 4.0475447                             | 3.8451674     | 283.0                       |
| $X_a = 76, Y_a = -117$  | 4.0680393                              | 3.8646374     | 283.0                       |
conflicts with the principle of cost-effectiveness and keeping weight down. To simplify the problem, we therefore chose not to factor in the effect of change of its moment of inertia $I_x$ and $I_y$. We can use the largest amplitude found in the frequency response to find out its corresponding region for comparison. The primary goal of this study is to observe the effect on vibration-reduction from moving the vibration absorber. The $Z_d$ degrees of freedom represent the vibration absorber’s own vibration amplitude. We will not discuss this aspect here and only observe the three degrees of freedom for $Z_d$, $\theta_X$, and $\theta_Y$. From the data in Tables 2 to 4, it can be projected that the effectiveness of the vibration absorber will vary in accordance with its distance from the origin and with changes in moment arm distance. The linear model results are included in the tables for comparison. It was found that for this deck if the absorber is moved to the furthest point of incidence (in this example, the corner of the 3rd quadrant) it can effectively suppress transverse vibrations though its ability to suppress vibrations in the rotational direction ($\theta_x$, $\theta_y$) is impaired. Both linear and nonlinear models discovered this effect. It is noted that the nonlinear model shows larger amplitudes than the linear model, we believe that the nonlinear terms still have contributions to this system (although are limited). However, for an optical disk or the micro devices, a tiny vibration may cause reading data errors or other unpredicted damages. Furthermore, when we examined Tables 2 to 4 closely it was found that the maximum transverse $Z_d$ amplitude was much greater than the rotation amplitudes ($\theta_x$, $\theta_y$), so the actual effect should still be based on $Z_d$. The vibration in this direction may need more attention on the preliminary design of this system.

4.2 Results from Vibration-Reduction Mechanism of Coupled Rotor and Deck

The reference rotor data required in this study is listed in Table 5 and the pitch angle selected for the rotor blade was $10^\circ = 0.1745\text{rad}$. First an analytical approach is taken to define a deck structure with no absorber; its data are shown in Table 1. As the rotor blade transmits far more force, a higher viscous damping coefficient of 45kg/sec was selected first so the graph can be more easily observed. Using the numerical method (Runge-Kutta method) with its equation of motion, the phase diagram at the time domain when its $Z_d$ degrees of freedom achieve convergence was acquired and was plotted with the convergent result from analytical method shown in Fig. 7. We found that when convergence was achieved, both vibrated in the same zone so the results correlate. The results of this study are therefore reliable.

Next, we study the cases of the absorber presented. Equations (4) to (9) are considered both in Runge-Kutta method or frequency response. By using the frequency response analysis perspective to validate this mode, we tested the vibration absorber at the position (19, 29.25) to see the amplitude relationship between $Z_d$, $Z_a$, $\theta_x$, and $\theta_y$ as well as how it relates to the frequency. First we observed that when the vibration absorber was located (19, 29.25), the displacement of $Z_d$ can be seen in Fig. 8, when $\omega = 9.3\text{Hz}$ the amplitude is around 1.2747. The maximum displacement from time dependent result is 1.2715 so there’s an error of 0.3%. Next, from Fig. 9 we can see that when $\omega = 9.3\text{Hz}$, the vibration amplitude of $Z_d$ is 1.7533. Compared to the maximum displacement from time dependent of 1.7531, the error is approximately 0.01%. The vibration amplitude of $\theta_x$ can be seen in Fig. 10. When $\omega = 9.3\text{Hz}$, the vibration amplitude is approximately 9.6197E-03. Compared to the maximum displacement from time dependent of 9.5953E-03, the error is approximately 0.3%. The vibration amplitude of $\theta_y$ can be seen in Fig. 11. When $\omega = 9.3\text{Hz}$, the vibration amplitude is approximately 9.6661E-03. Compared to the maximum displacement from time dependent of 9.5953E-03, the error is approximately 0.3%. The vibration amplitude of $\theta_y$ is approximately 0.01%. The vibration frequency of $\theta_y$ is approximately 9.6661E-03. The results are proved to be reliable.

During this we observed that when $\omega = 6.7\text{Hz}$, the four degrees of freedom in $Z_d$, $Z_a$, $\theta_x$, and $\theta_y$ decrease rapidly. This is because this frequency is the resonant frequency of the flapping angle so it causes rapid fluctuations in the deck vibration amplitude. For a better view of this phenomenon, the magnified resonant area of $Z_d$ plot (Fig. 8) is shown in Fig. 12. Consider an undamped and decoupled blade-flapping mode from Eq. (4), we can obtain the natural frequency of $\beta_C$ mode.

![Fig. 7 The convergent cycle of the ZD phase plot (numerical and analytical method)](image)

<table>
<thead>
<tr>
<th>Table 5 Properties of the rotor system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of the rotor blade (kg)</td>
</tr>
<tr>
<td>Moment of inertia of the blade (kg-mm²)</td>
</tr>
<tr>
<td>Torsional spring Constant (N-mm/rad)</td>
</tr>
<tr>
<td>Blade length (span) (mm)</td>
</tr>
<tr>
<td>Blade chord length (mm)</td>
</tr>
<tr>
<td>Rotor speed (rad/sec)</td>
</tr>
<tr>
<td>Blade lift slope</td>
</tr>
<tr>
<td>Air density (kg-mm³)</td>
</tr>
</tbody>
</table>

Using the data from Table 5, one can acquire the natural frequency as 6.71Hz for $\beta_C$. The plot of the collective flap mode is shown in Fig. 13. It shows that the spike appears at 6.71Hz, and this becomes a significant result in the system and should be concerned in the future design.

In this study, we coupled the rotor blade with the deck with the external force that had been applied to the deck replaced by the lift generated from the rotor blades. The vibration reduction analysis was still based on the absorber’s position however to see how the positioning of the absorber affects the magnitude of vibration amplitude. See Fig. 6 for the absorber’s positioning. The main goal of this study is to observe the effect of changing the absorber’s positioning on vibration reduction for the deck. Only the three degrees of freedom $\theta_x$, $\theta_y$ and $Z_D$ were therefore observed. From the data in Tables 6 to 8, it can be projected that the effectiveness of the vibration absorber will vary in accordance with its distance from the origin and with changes in moment arm distance. The linear model results are again included in the tables for comparison. It was found that for this coupled rotor and deck system, just like for the deck only system, if the vibration absorber is moved to the furthest point of incidence (in this example, the corner of the 3rd quadrant) it can effectively suppress transverse vibrations, though its ability to suppress vibrations in the rotational direction is impaired. However, the transverse $Z_D$ amplitude was much greater than the rotation amplitudes ($\theta_x$, $\theta_y$), so the actual effect should still be based on $Z_D$. 

Fig. 8 The $|Z_D|$ frequency response (absorber location (19, 29.25))

Fig. 9 The $|Z_A|$ frequency response (absorber location (19, 29.25))

Fig. 10 The $|\theta_X|$ frequency response (absorber location (19, 29.25))

Fig. 11 The $|\theta_Y|$ frequency response (absorber location (19, 29.25))

Fig. 12 A magnified resonant area from Fig. 8

Fig. 13 The collective blade mode ($\beta_C$) frequency response
Table 6  The effect of absorber on $|Z_D|$ (rotor-deck system)

<table>
<thead>
<tr>
<th>Absorber position (mm)</th>
<th>Frequency Response</th>
<th>Runge-Kutta Method</th>
<th>Linear Result</th>
<th>Resonant frequency (rad/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Xa = 0, Ya = 0$</td>
<td>1.274638</td>
<td>1.271582</td>
<td>1.195469</td>
<td>58.6</td>
</tr>
<tr>
<td>$Xa = 57, Ya = 87.75$</td>
<td>1.274766</td>
<td>1.271133</td>
<td>1.195579</td>
<td>58.6</td>
</tr>
<tr>
<td>$Xa = 76, Ya = 117$</td>
<td>1.274629</td>
<td>1.270692</td>
<td>1.195446</td>
<td>58.6</td>
</tr>
<tr>
<td>$Xa = -57, Ya = 87.75$</td>
<td>1.274717</td>
<td>1.271608</td>
<td>1.195541</td>
<td>58.6</td>
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<tr>
<td>$Xa = -76, Ya = 117$</td>
<td>1.274738</td>
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<td>1.195340</td>
<td>58.6</td>
</tr>
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</table>

Table 7  The effect of absorber on $|\theta_X|$ (rotor-deck system)

<table>
<thead>
<tr>
<th>Absorber position (mm)</th>
<th>Frequency Response</th>
<th>Runge-Kutta Method</th>
<th>Linear Result</th>
<th>Resonant frequency (rad/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Xa = 0, Ya = 0$</td>
<td>9.6188141</td>
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<td>9.5956493</td>
<td>9.0264529</td>
<td>58.6</td>
</tr>
<tr>
<td>$Xa = 76, Ya = 117$</td>
<td>9.6258838</td>
<td>9.5958328</td>
<td>9.0287365</td>
<td>58.6</td>
</tr>
<tr>
<td>$Xa = -57, Ya = 87.75$</td>
<td>9.6238907</td>
<td>9.6000591</td>
<td>9.0269623</td>
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</tr>
<tr>
<td>$Xa = -76, Ya = 117$</td>
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<td>9.0239234</td>
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<tr>
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<tr>
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<tr>
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<td>9.5923441</td>
<td>9.0187872</td>
<td>58.6</td>
</tr>
</tbody>
</table>

Table 8  The effect of absorber on $|\theta_Y|$ (rotor-deck system)

<table>
<thead>
<tr>
<th>Absorber position (mm)</th>
<th>Frequency Response</th>
<th>Runge-Kutta Method</th>
<th>Linear Result</th>
<th>Resonant frequency (rad/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Xa = 0, Ya = 0$</td>
<td>9.6641043</td>
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<tr>
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<td>9.8491516</td>
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</tr>
<tr>
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</tr>
<tr>
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<tr>
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</table>
5. CONCLUSIONS

This study successfully used several methods including the Runge-Kutta method, the frequency response, and the Multiple-scales method to prove the correctness of the non-linear equations for the motion constructed. It also successfully discovered the best vibration absorber positioning to minimize a rotating mechanism’s deck vibration is to set it parallelly with the four corners’ isolator of the deck, depending on the positioning of the external force. Additionally, this result’s process and results are summarized as follow:

(1) Deck Only Structure
1. The four degrees of freedom $Z_D$, $Z_A$, and nonlinear $\theta_X$ and $\theta_Y$ are mutually dependent, and this study was able to successfully emulate this combination.
2. It was found that the maximum transverse $Z_D$ amplitude was much greater than the rotation amplitudes $(\theta_x, \theta_y)$, so the actual effect should still be based on $Z_D$. The vibration in this direction may need more attention on the preliminary design of this system.
3. The best position for the vibration absorber to minimize the vibration in the deck is at the end corner of the deck of the diagonal to the point of incidence.

(2) Coupled Rotor and Deck Structure
1. When the blade’s pitch angle ($\theta$) frequency is the resonant frequency of their flapping angle, the three degrees of freedom $Z_D$, $\theta_X$ and $\theta_Y$ fluctuate greatly.
2. The best position for the vibration absorber to minimize the coupled vibration system is at the end corner of the deck of the diagonal to the point of incidence.
3. This research has successfully proven that the vibration absorber’s position can be changed to achieve the vibration reduction effect. It is therefore unnecessary to change the vibration absorber material or modify the greater part of the deck. To achieve vibration reduction, simply lock the vibration absorber to different locations. Although this can be predicted based on basic vibration theory, no other research works regarding this subject are presented. This study provides a theoretical background for the preliminary vibration reduction design for industries. This will not only save costs but also increase testing efficiency, achieving the most cost-effective vibration reduction result.

REFERENCES


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