Ho and Hosotani Reply: Hagen and Sudarshan (HS) claim that Eq. (6) of Ref. [1] is incorrect and that their new solution (2) of Ref. [2] implies the nondynamical nature of nonintegrable phase θ_j's. We show that the argument in Ref. [1] is correct and consistent, and that HS's solution has inconsistencies, leading to nonvanishing commutators of $[P^1, P^2]$ and $[P^j, H]$ even in physical states.

In Ref. [1] Chern-Simons theory is formulated by first eliminating unphysical degrees of freedom. Dynamical variables are $\psi(x)$ and θ_j's. There are four ingredients: (i) the Hamiltonian given by (5) and (6), (ii) equal time commutation relations among ψ, $\psi^\dagger$, and θ_j's, (iii) physical state condition (9), and (iv) boundary conditions (BC) on a torus (3).

The theory thus formulated is equivalent to the original theory described by (1)–(3). Our Eq. (6) contains the field equation $(\kappa/4\pi)\varepsilon^{\mu\nu\rho}f_{\nu\rho} = j^\mu$, except for three relations. Two of them are equations for θ_j's, which in this formulation follows from $i\dot{\theta}_j = [\theta_j, H]$. The third one is the relation between Q and Φ, which does not follow from (i) and (ii), but is imposed as a physical state condition. The last point has not been fully appreciated in the earlier Letter [3].

Although HS state that there are "errors" in Ref. [1], there is no error and the argument in Ref. [1] is perfectly consistent. What HS do is to propose their "new solution" with different BC, whose validity and consistency we shall now check.

HS's solution (2) of Ref. [2] is explicitly

$$\begin{aligned} a_0(x) &= \sum_{k=1}^{2} \frac{x_k}{L_k}\left\{\dot{\theta}_k + \alpha\epsilon^{kl}\frac{2\pi}{\kappa L_l}J^l\right\} \\ &\quad + \frac{2\pi}{\kappa}\int d\mathbf{y}\, \epsilon^{kl}\nabla_k^x D(\mathbf{x}-\mathbf{y})j^l(y), \\ a_k(x) &= \frac{\theta_k}{L_k} + \frac{\epsilon^{kl}x_l}{2L_1L_2}\frac{2\pi Q}{\kappa} \\ &\quad + \frac{2\pi}{\kappa}\int d\mathbf{y}\, \epsilon^{kl}\nabla_l^x D(\mathbf{x}-\mathbf{y})j^0(y), \end{aligned} \tag{A1}$$

where $J^k = \int d\mathbf{x}\, j^k(x)$. We have introduced a parameter α in the expression for a_0. HS's solution gives $\alpha = \frac{1}{2}$. Insertion of (A1) to f_{0k} yields $(\kappa/2\pi)\epsilon^{kl}f_{0l}(x) = j^k(x) + (\alpha-1)J^k/L_1L_2$, so that the equations are not satisfied unless $\alpha = 1$. Note that $\Delta D(\mathbf{x}) = \delta(\mathbf{x}) - (L_1L_2)^{-1}$.

The comparison of (A1) with (6) of Ref. [1] shows two differences. (a) (A1) has an additional term $\sum(x_k/L_k)\{\cdots\}$ in the expression for a_0, which vanishes on shell for $\alpha = 1$. (b) In the second term in the expression for a_k, (A1) has an operator $2\pi Q/\kappa$, whereas ours has a c-number $-\Phi$. [Note that $\int d\mathbf{y}\,\nabla_l D(\mathbf{x}-\mathbf{y}) = 0$.]

HS's solution satisfies different BC. In Eq. (3) of Ref. [1], $\beta_j = \beta_j^{\rm HS} = \epsilon^{jk}x_k\pi Q/L_k\kappa$, for $\alpha = 1$ on shell. For $\alpha = \frac{1}{2}$, $\beta_j^{\rm HS}$ contains an additional term. In either case $\beta_j^{\rm HS}$ is an operator satisfying $[\beta_j^{\rm HS}, \psi] \neq 0$. This BC must be respected in order that physical gauge invariant operators be single valued on a torus.

The most serious problem in HS's solution arises in commutators of P^j and H. We have evaluated the commutators in the nonrelativistic theory by adopting Eq. (5) of Ref. [1] with HS's solution for $a_k(x)$ in (A1) substituted. HS also claim that θ_j's are not dynamical. So we have evaluated the commutators in two ways, by taking $[\theta_1, \theta_2] = 2\pi i/\kappa$ (as in Ref. [1]) or $[\theta_1, \theta_2] = 0$.

The evaluation is straightforward, but requires extra care on the ordering of operators. The result is

$$\begin{aligned} [P^j, P^k] &= \epsilon^{jk}\frac{2\pi i}{\kappa L_1 L_2}[Q \text{ or } Q(1-Q)], \\ [P^j, H] &= \epsilon^{jk}\frac{2\pi i}{\kappa L_1 L_2}[J^k \text{ or } J^k(1-Q)]. \end{aligned} \tag{A2}$$

This contradicts HS's statement that all commutators vanish. (A2) casues a serious problem. P^j's and H do not commute with each other even in physical states except for $Q = 0 = J^k$, or $Q = 1$.

The correct commutators (11) and dynamical nature of θ_j's are important in establishing the connection to anyon quantum mechanics [1]. With (A2) such connection cannot be achieved. Hence the argument in Ref. [1] is correct, whereas HS's solution leads to inconsistency.

This research was supported in part by Republic of China Grant No. NSC 83-0208-M032-017 (C.-L.H.) and by the U.S. Department of Energy under Contract No. DE-AC02-83ER-40105 (Y.H.).

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Received 28 March 1994
PACS numbers: 11.15.–q, 05.30.–d

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0031-9007/95/74(6)/1033(1)$06.00