# Performance Evaluation of a Velocity Observer for Accurate Velocity Estimation of Servo Motor Drives

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***Abstract***- Because the position transducers commonly used in industry, i.e. encoders and resolvers, do not inherently produce an instantaneous velocity measurement, some signal processing technique is generally required to improve the accuracy of velocity estimation at each sample instant. This estimated signal is then used as the velocity feedback signal for the velocity loop control. The differential position technique commonly used in industry for velocity estimation suffers from the large quantization error, therefore limits the servo control loop bandwidth. This paper presents both the theoretical analysis and the experimental verification of a scheme that uses close loop observer for velocity estimation. The results have shown that the quantization error in the velocity feedback signal can be reduced dramatically when using a close loop observer for velocity estimation. The results also shown that at low speed, the velocity estimation could be improved with a simple compensation scheme.

## I. INTRODUCTION

All high performance servo drives require both the rotor position and the velocity feedback for feedback control, and incremental encoders are the most common positioning transducers used today in industry. Since a position transducer is already present in the drive, addition of a separate but directly coupled velocity transducer is both costly and mechanically difficult. Therefore numerical method is usually used to estimate the motor velocity from the position measurements. Most popular numerical method is by simply taking backward difference of the position measurements to approximate the differentiation of rotor position. The quantization of the encoder's position measurement results in significant noise in the differentiated signal, i.e. the estimated motor velocity. In order to limit the amount of quantization noise and prevent it from propagating to the motor current, it is necessary to filter the differentiated signal in the feedback controller. That is equivalent to reducing the bandwidth of the servo drive to compromise the magnitude of the noise. Another numerical approach consists in using the theory of observers to estimate the motor velocity from the position measurements. In this case, the position measurements and the motor current are used together with the motor dynamic model to obtain estimation of motor velocity.

Brown et al[1,2] have analyzed several velocity estimation algorithms from discrete position measurements. The techniques analyzed were essentially open loop, and required significant filtering to reduce the quantization noise. Sakai and Hori[3] have develop a scheme to improve the low speed performance of velocity estimation. The scheme required extra counters and software for implementation. Lorenz and Van Pattern[4] proposed an close loop observer for motor velocity estimation. Significant reduction in torque ripple was achieved compared to the use of backward difference for velocity estimation. However the scheme did not work well at low speed as will be discussed further in this paper. Lemkin et al[5] have developed a transition logic based scheme to improve the velocity estimation at low speed, and a multi-rate Luenberger observer for high speed velocity estimation. Bodson et al[6] have compared the performance of velocity estimation using the backward difference and the close loop observer.

This paper presents the theoretical analysis and experimental verification of a velocity estimation scheme that uses a close loop observer similar to the one used in [4]. Performance of the observer at both high and low speed is evaluated. A simple compensation technique is also proposed to improve the accuracy of the velocity estimation at low speed.

## II. PRINCIPLE OF THE VELOCITY OBSERVER

The most popular method to estimate velocity from position measurements in servo drives is by a backward difference,

$$\hat{\omega}(k)=\frac{\theta(kT)-\theta((k-1)T)}{T} \tag{1}$$

where $\omega$ and $\theta$ are motor velocity and position respectively, T is the sampling period, kT is the current sampling instant, and (k-1)T is the previous sampling instant. Although this method is simple, but it's usefulness is limited by the accuracy and quantization noise of the velocity estimation. Because the velocity loop is the innermost state loop, its performance must be generally better than the outer loops, therefore its gain is higher than the gains of the outer loops. But the higher gain requirement for the velocity loop will cause quantization noise to appear directly in the motor current command. This not only limits the achievable bandwidth of the feedback controller but also increase the power dissipation of the motor drive.

An alternative method for motor speed estimation is to use a close loop observer. Consider a dc motor model shown in Fig. 1, viscous damping is assumed negligible, Kt is the motor torque constant, J is the inertia, i is the motor current, and $T_d$ is the disturbance torque. The model can be expressed in the state variable form as follows,

$$\begin{bmatrix} \dot{\theta} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{Kt}{J} \end{bmatrix} \cdot i + \begin{bmatrix} 0 \\ -\frac{1}{J} \end{bmatrix} \cdot T_d \qquad (2)$$
$$= \overline{A}\,\overline{X} + \overline{B}\,\overline{U} + \overline{D} \cdot T_d$$

and since only the position is measured, the output equation is

$$\overline{Y} = \theta = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \omega \end{bmatrix} \qquad (3)$$
$$= \overline{C}\,\overline{X}$$

Based on the work of Luenberger[7], a full state observer for motor position can be formulated as:

$$\dot{\hat{X}} = \overline{A}\hat{X} + \overline{B}\,\overline{U} + \overline{K}\left(\overline{Y} - \overline{C}\hat{X}\right) \qquad (4)$$

where $\overline{K}$ is the gain matrix of the observer, and the symbols with '^' represent estimated variables. Note the observer shown in Eq.(4) can be simplified and expressed in the block diagram shown in Fig. 2. As shown in this block diagram, the inputs to the observer include measured motor position and motor current, the feedback loops are consisted of a derivative, a proportional, and an integral gain. Since the summing point for the derivative gain has been moved to behind the $1/\hat{J}s$ block, the derivative 's' was cancelled. Note also that there is no integral gain in the gain matrix of the original observer, it was added to reduce the steady state tracking error of the observer. The feedback loops force the error between the measured position and the estimated position to zero by manipulating the input to a model of the physical system.

The estimated velocity can be derived and expressed as a function of the observer gains and motor parameters as follows,

$$\hat{\omega} = \frac{\left(\frac{J}{\hat{J}}\frac{\hat{Kt}}{Kt}s^3 + K_1 s^2 + \frac{K_2}{\hat{J}}s + \frac{K_3}{\hat{J}}\right)\omega + \frac{1}{\hat{J}}\frac{\hat{Kt}}{Kt}s^2\,T_d}{s^3 + K_1 s^2 + \frac{K_2}{\hat{J}}s + \frac{K_3}{\hat{J}}} \qquad (5)$$

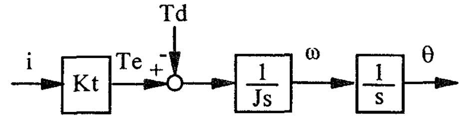



Fig. 1 Block Diagram of a dc Motor Model

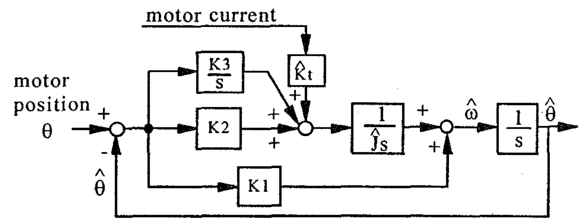
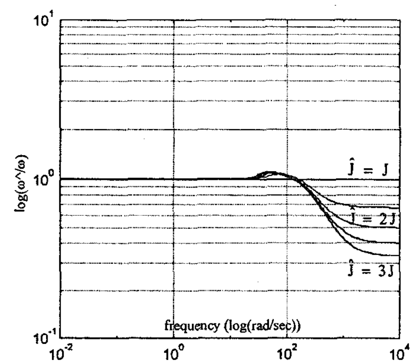
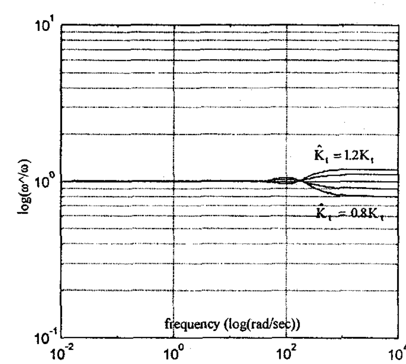



Fig. 2 Block Diagram of the Velocity Observer

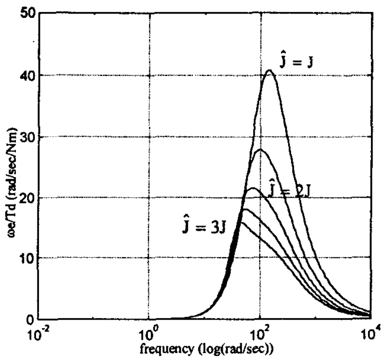



a) $\hat{J}$ Varied 1, 1.5, 2, 2.5 and 3 times of J

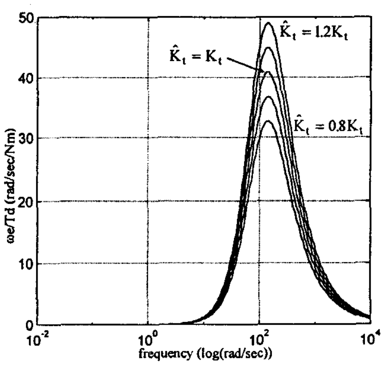



b) $\hat{Kt}$ Varied 0.8, 0.9, 1.0, 1.1 and 1.2 times of Kt

Fig. 3 Parameter Sensitivity of $\hat{\omega}/\omega$ When a) J Varied, b) Kt Varied




a) $\hat{J}$ Varied 1, 1.5, 2, 2.5 and 3 times of J




b) $\hat{K}t$ Varied 0.8, 0.9, 1.0, 1.1 and 1.2 times of Kt

Fig. 4 $\omega_e/T_d$ Responses When a) J Varied, b) Kt Varied

It can be seen from Eq. (5) that the estimated velocity equals the actual motor velocity when all the motor parameters are correctly estimated. To investigate the parameter sensitivity the velocity observer, the frequency response of $\hat{\omega}/\omega$ was calculated and the results were plotted in Fig. 3. All the roots of the characteristic equation of Eq.(5) were set to 100 rad/sec in the calculations for convenience. As can be seen from Fig. 3, the frequency responses varied only slightly for frequencies near and below the observer bandwidth. This implies that the tracking performance of $\hat{\omega}$ is not sensitive to motor parameters within the observer's operating frequency.

The frequency response between the velocity estimation error, i.e. $\omega_e = \omega - \hat{\omega}$, and $T_d$ when $\hat{J}$ and $\hat{K}t$ varied was also calculated and plotted in Fig.4. Notice there is no steady state velocity estimation error for step and ramp disturbance. However, it can be seen that the response of $\hat{\omega}$ to load disturbance is sensitive to motor parameters within the bandwidth of the observer. In fact, significant error could be introduced due to the external disturbance if $\hat{J}$ and $\hat{K}t$ are not correctly estimated.

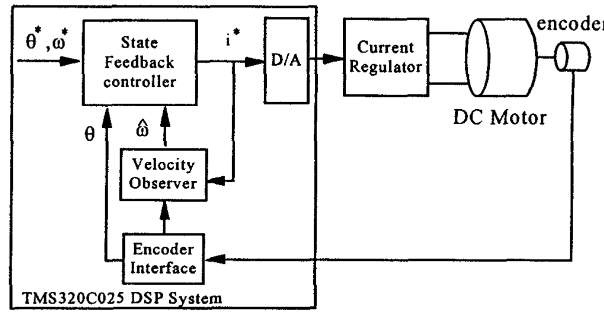



Fig. 5 Experimental Setup

## III. Performance of the Velocity Observer at Normal Speed

Several experiments were performed to evaluate the performance of the velocity observer described in the previous section. The observer was implemented with a TMS320C25 DSP based development system. A 200 Watts DC motor was used in the experimental setup, a 1000 pulse/rev encoder was mounted on the motor for position measurement. The motor was controlled with a state feedback controller. The sampling frequency is set to 1000 Hz. Since the bandwidth of the current controller is in the order of 500Hz, therefore it can be assumed ideal. Block diagram of the experimental setup is shown in Fig. 5.

Fig. 6-7 compared the responses of the feedback controller when using the measured velocity (backward difference) and the estimated velocity as the feedback signal for the velocity control loop. The motor was cycling from zero to 900 rpm, and the loop gains were kept the same for both experiments. It can be seen in Fig. 6 that the current command has a significant noise on it due to the quantization of the motor position measurements. But the noise was reduced dramatically when using the estimated velocity for the velocity feedback as shown in Fig.7. Note this result implies that the bandwidth of the state feedback controller can be set to a higher value when the estimated velocity is used as the velocity feedback signal.

The third signal shown in Fig. 7 was the velocity estimation error $\omega_e$. The error was within +-1 encoder pulse most of the time. The noise on the error signal was due to the quantization noise of the position measurements. Also note that there was very little following error in the estimated velocity, this is because the current command was input to the observer, this input is similar to a feedforward signal which help to reduce the tracking error.

Fig. 8 compared the measured motor velocity and the estimated velocity as the motor reversed its speed from 0 rpm to 60 rpm. The estimated velocity was used as the velocity feedback in this experiment. Fig. 9 shows the measured and the estimated motor velocity when the motor was running at 900 rpm and subjected to a step load disturbance. It is clear from the above results that the observer was able to track the motor velocity with relatively high accuracy when the motor was running at normal speeds.

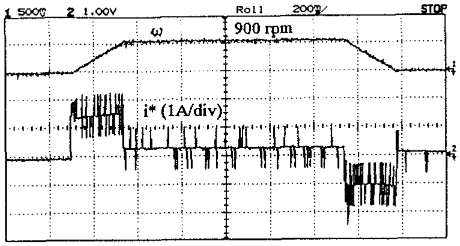



Fig. 6 Measured Velocity and Current Command When Using the Measured Velocity as Speed Feedback Signal, the Motor was Cycling from Zero to 900 rpm

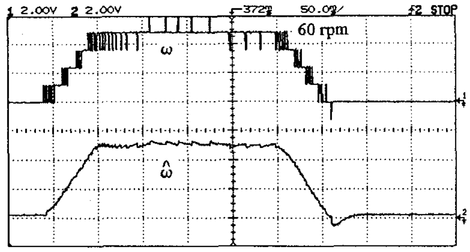



Fig. 8 Measured Velocity and Estimated Velocity When the Motor was Cycling from 0 to 60 rpm

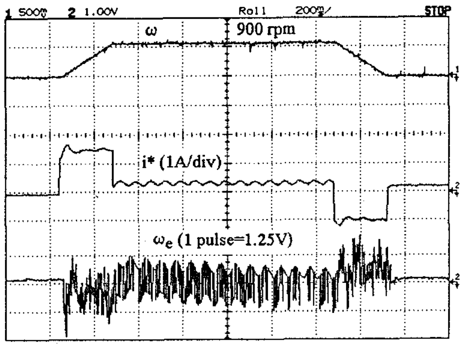



Fig. 7 Measured Velocity, Current Command, and Velocity Estimation Error When Using the Estimated Velocity as Speed Feedback Signal

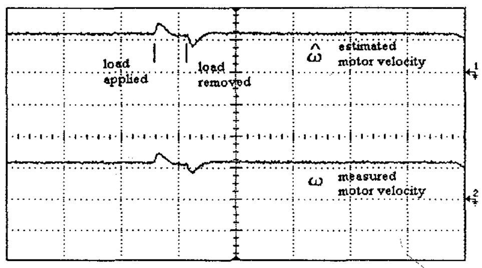



Fig. 9 Measured Velocity and Estimated Velocity When the Motor was Subjected to a Step Load Disturbance

## IV. LOW SPEED PERFORMANCE AND COMPENSATION

This section presents the experimental results of velocity observer when the motor was running at speeds below 15 rpm. Note that 15 rpm corresponds to 1 pulse/T in the experimental system. The performance of the velocity observer at steady state motor speed was evaluated first. Fig. 10 shows the measured velocity and the estimated velocity when the motor was running at 0.5, 0.25 and 0.125 pulse/T respectively. Notice the measured encoder pulses were discontinuous at these speeds. The velocity estimation became oscillatory due to the discontinuous position measurement input to the observer. The observer output rose up quickly when an incremental position pulse was detected, and decayed exponentially when no pulse was detected in the following sampling instants. Fig. 11 shows the measured motor velocity and the estimated velocity when the motor was a) accelerating to 0.5 pulse/T, and b) decelerating down to zero. The results are similar to the ones obtained in Fig. 10, the estimated speed has ripple on it due to the discontinuous position pulse measurements. This is undesirable since the ripple will cause oscillation in the actual motor speed and torque.

It is possible to reduce the magnitude of the ripple on the estimated velocity by reducing the bandwidth of the observer at low speed. However, it would be difficult to tune the observer correctly without good speed estimation. Sakai[3] attempted to improve the low speed performance by adding a static fiction compensation to the observer. This solution was not effective since the friction is generally non-linear and difficult to identify.

A simple method to improve the low speed velocity estimation without adding extra hardware to the system is by predicting the present velocity from the previous position measurements. The scheme can be illustrated with a typical low position pulse diagram shown in Fig. 12. The time duration between the position pulses can be tracked in the control program via counting of the sampling instants. Then the input position measurement to the observer is compensated with the average pulse per sample from the previous measured average speed, and the maximum compensation is one pulse. For example in the situation shown in Fig. 12, the average speed between the two position pulses is 0.25 pulse/T. Therefore 0.125 pulse is added to the position measurement at the sampling instants after the second pulse was detected. But the compensation stopped after a total of 1 pulse is added to the position measurement. The compensation also stopped when the measured position pulse becomes continuous, or when there are more than one pulse measured at any sampling instant.

Fig. 13 shows the measured velocity and the estimated velocity with the compensation described previously, the motor was running at 0.5, 0.25 and 0.125 pulse/T respectively. $\hat{\omega}'$ represents the estimated velocity with compensation. Fig. 14 shows the measured velocity and the compensated velocity when the motor was accelerating to 0.5 pulse/T, and decelerating down to zero. It can be seen from these results that the velocity estimations were smoother with compensation, and the ripple became less significant when comparing to the results obtained in Fig. 10-11. The velocity ripple diminished completely when the motor was running at constant speed. These results have demonstrated the effectiveness of the compensation in improving the velocity estimation at low speed.

At last, the speed response when the motor was cycling from zero to 15 rpm is shown in Fig. 15. As can be seen from this figure that the motor oscillated during the acceleration period. This was primarily due to the time delay of the observer at low speed. As described earlier, the average speed of the previous encoder pulses is used to predict and compensate for the current velocity estimation. Although this scheme is very effective when at constant speed, but it inevitably introduces delay to the velocity estimation when motor speed is changing.

## V. CONCLUSIONS

In this paper, a scheme for accurate velocity estimation for servo motor drives was presented. The performance of the proposed control system was investigated both by analysis and by experiments, and at normal and very low speed operations. The experimental results have shown that the quantization error in the velocity feedback signal can be reduced dramatically when using a close loop observer instead of a backward difference to estimate motor velocity. It was also shown that at low speed, the velocity estimation could be improved with a simple compensation scheme.

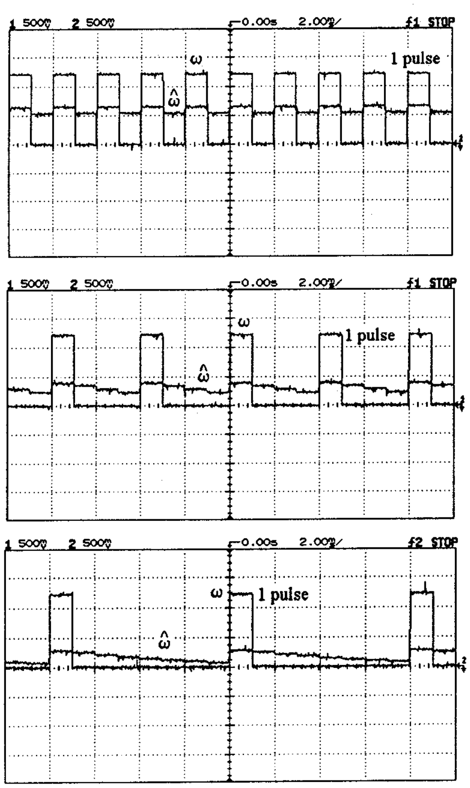
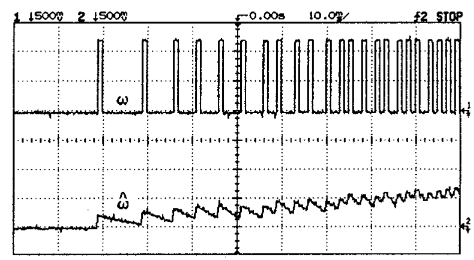
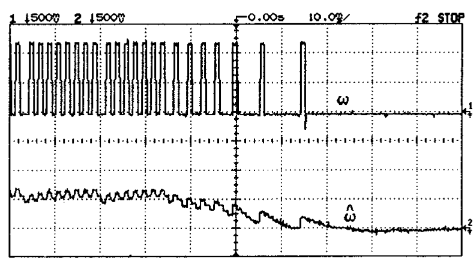



Fig. 10 Measured Velocity and Estimated Velocity When the Motor was Running at 0.5, 0.25, and 0.125 Pulse/T Separately




a) Motor Accelerating from Zero to 0.5 Pulse/T




b) Motor Decelerating from 0.5 Pulse/T to Zero

Fig. 11 Measured Velocity and Estimated Velocity When the Motor was a) Accelerating, b) Decelerating

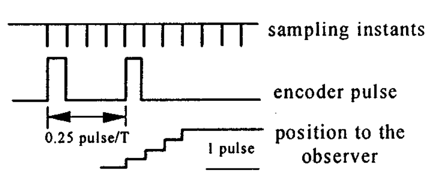



Fig. 12 Timing Diagram Showing the Compensated Position Measurement to the Observer

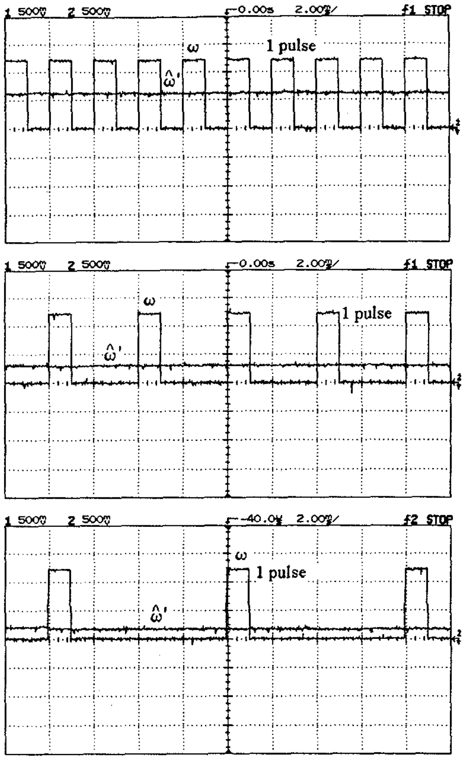
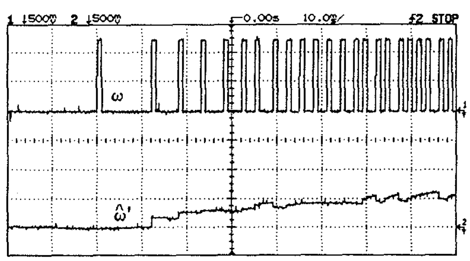
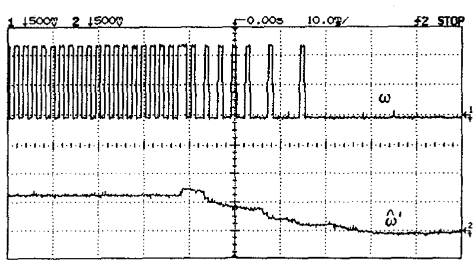



Fig. 13 Measured Velocity and Compensated Estimated Velocity When the Motor was Running at 0.5, 0.25, and 0.125 Pulse/T Separately



a) Motor Accelerating from Zero to 0.5 Pulse/T



b) Motor Decelerating from 0.5 Pulse/T to Zero

Fig. 14 Measured Velocity and Compensated Estimated Velocity When the Motor was a) Accelerating, b) Decelerating

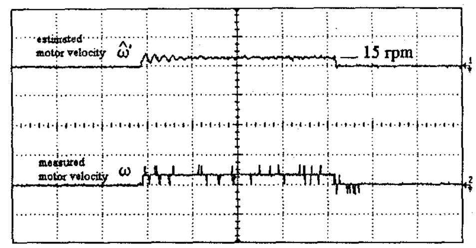



Fig. 15 Measured Velocity and the Compensated Velocity When the Motor Cycled from Zero to 15 rpm

## References


[1] R. Brown, S. Schneider, and M. Mulligan, "Analysis of algorithms for velocity estimation from discrete position versus time data", IEEE Trans. on Industrial Electronics, Vol.39, No.1, pp. 11-19, Feb. 1992.

[2] R. Brown and S. Schneider, "Velocity observations from discrete position encoder", Proceedings of the 1987 IECON, pp. 1111-1118.

[3] S. Sakai and Y. Hori, "Ultra-low speed control of Servomotor using low resolution rotary encoder", Proceedings of the 1995 IECON, pp. 615-620.

[4] R. Lorenz and K. Van Patten, "High-resolution velocity estimation for all digital, ac servo drives", IEEE Trans. on Industry Applications, Vol.27, No.4, July/Aug. 1991.

[5] M. Lemkin, P. Yang, A. Huang, J. Jones, and D. Auslander, "Velocity estimation from widely spaced encoder pulses", Proceedings of the 1995 American Control Conference, pp. 998-1002.

[6] M Bodson, J. Chiasson, and R. Novotnak, "Nonlinear speed observer for high-performance induction motor control", IEEE Trans. on Industrial Electronics, Vol.44, No.4, pp. 337-343, Aug. 1995.

[7] D. G. Luenberger, "An introduction to observers", IEEE Trans. On Automatic Control, Vol. 16, No. 6, Dec. 1971, pp. 596-602.