Grey Modeling and Control of a Thermal Barrel in Plastic Molding Processes

Kuen-Yih Shy — *Graduate student*
Yin-Tien Wang — *Associate Professor*
Robotics Laboratory, Department of Mechanical Engineering
Tamkang University
Tamsui, Taipei Hsien, TAIWAN CHINA 25137
http://mail.tku.edu.tw/ytwang

Abstract — **In this paper, we propose a self-tuning controller with a grey system for on-line process controls. The self-tuning mechanism is designed basing on a group of input-output data obtained from the process. The grey system in the tuning mechanism is utilized to reduce the random variation of the input-output data. The self-tuning mechanism is integrated into an internal model control, and the developed control system is applied to control the temperature distribution in a thermal barrel of plastic molding processes. From the experimental result, we conclude that the usage of the grey system can filter out the noise in the process and reduce the number of input-output data required in the tuning mechanism.**

I. INTRODUCTION

In process control, we need to establish an accurate mathematical model for the physical process, and design the process controller basing on the model. For a complex procedure, such as temperature control of plastic molding process, a controller with a linear model provides accurate results only in some application range and is not appropriate for precision process control. On the other hand, a controller with a nonlinear model costs too much time in calculation and is not applicable for on-line control.

Linear discrete models with self-tuning mechanism are simple and approximate methods, which can highly represent the properties of nonlinear systems and easily integrate a priori knowledge of data obtained from the processes. However, for a conventional self-tuning control, the tuning mechanism is suffered from the noise of the input-output data. In this paper, we propose a self-tuning controller integrated with a grey system for on-line process controls. The self-tuning mechanism is designed basing on a group of input-output data obtained from the process. The grey system in the controller is utilized to reduce the random variation of the input-output data.

We established two types of self-tuning mechanism for the purpose of comparison, one without any filter and the other with a filter based on the grey system theory. In order to compare the performance of these two controllers, we integrate the tuning mechanism into Internal Model Control (IMC) [Sousa *et al.* 1997] for temperature control of a thermal barrel in plastic molding processes. The systems are subjected to a step input and the responses will depict the dynamic performance of the controllers. The basic concept of IMC is to generate the command basing the inversion of the system model. If the mathematical model is accurate enough, the difference between the outputs of the mathematical model and the physical system will vanish, and the transfer function of the system is a unity function. Otherwise, if the model is slightly different from the physical system, a large difference will exist between the IMC output and the reference input. Therefore, the IMC is an appropriate tool that can be used to evaluate the accuracy of a mathematical model. We will show that the self-tuning model with a grey system presents a better control performance than that without any filter.

II. SELF-TUNING INTERNAL MODEL CONTROL

In plastic extrusion and injection molding processes, the particle polymer is fed into the thermal barrel and heated continuously. The mechanism forcing the polymer moving forward is a rotating screw in the thermal barrel. The configuration of a thermal barrel in plastic injection-molding machines is shown in Figure 1. Around the surface of the thermal barrel, several pairs of heaters are equipped to supply energy to the system. In the container of the barrel, the polymer is heated by the heaters in different zones. Meanwhile some melt polymer will mix with un-melt particle polymer.

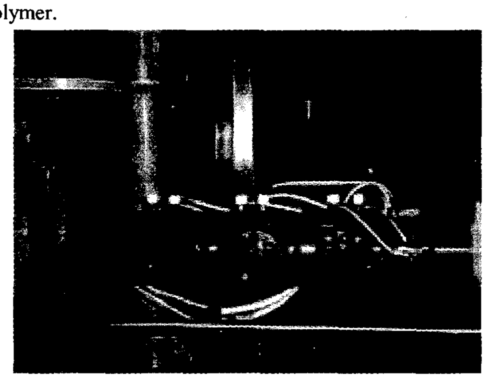

Figure 1 Thermal barrel of plastic product processes

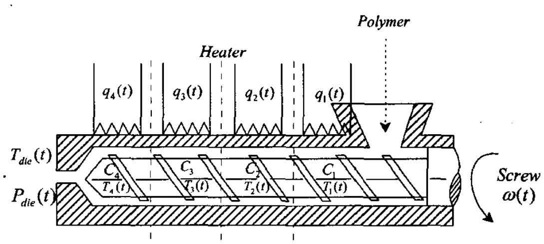

Figure 2 Model configuration of a thermal barrel

We divide the thermal barrel into four different temperature zones, as depicted in Figure 2, and analyze the thermal behavior in each zone separately. The thermal equation for each zone can be derived by the concept of thermal equilibrium of the system,

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$$C_i \frac{dT_i(t)}{dt} = q_i(t) - q_o(t) \tag{1}$$

where C_i is the thermal capacitance of zone i; $T_i(t)$ is the temperature of zone i in centigrade (°C); $q_i(t)$ and $q_o(t)$ represent the input and output heat rate, respectively. In general, Equation (1) is a nonlinear function that can be expressed as

$$\dot{x} = f(x, u, t) \tag{2}$$

where x and u are the state and input variables of the system, respectively; and f is an nonlinear function of the input and state variables. A linear approximation of Equation (2) can be written as

$$\dot{x} = A_c x + B_c u \tag{3}$$

$$y = C_c x + D_c u \tag{4}$$

where y is the output of the system; A_c, B_c, C_c, and D_c are constant matrices. We can derive a discrete model as follows for Equations (3) and (4)[Phillips and Nagle 1995],

$$x(k) = Ax(k-1) + Bu(k-1) \tag{5}$$

$$y(k) = Cx(k-1) + Du(k-1)$$

In these equations, the coefficient matrices are determined by

$$A = Q(t_s) = e^{A_c t_s}$$

$$B = \int_0^{t_s} Q(t_s - \tau) d\tau\, B_c$$

$$C = C_c$$

$$D = D_c$$

where t_s is the sampling time of the discrete system. The coefficients of the discrete model are estimated basing on a group of input and output data. Rearrange Equation (5) to be a homogenous equation

$$x = Wh \tag{6}$$

where x is a vector contains the state variables at $t=k$; W is a matrix contains the input and state variables at $t=k-1$; h is a vector of undetermined coefficients in matrices A and B. These vectors and matrix can be expressed as the follows.

$$x = [x_1(k) \quad x_2(k) \quad \cdots \quad x_n(k)]^T$$

$$W = \begin{bmatrix} x_1(k-1) \cdots x_n(k-1) & 0 & 0 & u_1(k-1) \cdots u_m(k-1) & 0 & 0 \\ 0 & x_1(k-1) \cdots x_n(k-1) & 0 & 0 & u_1(k-1) \cdots u_m(k-1) & 0 \\ \vdots & 0 & 0 & \vdots & 0 & 0 \\ 0 & 0 & x_1(k-1) \cdots x_n(k-1) & 0 & 0 & u_1(k-1) \cdots u_m(k-1) \end{bmatrix}$$

$$h = [a_{11} \ \cdots \ a_{1n};\ a_{21} \ \cdots \ a_{2n};\ \cdots;\ a_{n1} \ \cdots \ a_{nn};$$
$$b_{11} \ \cdots \ b_{1m};\ b_{21} \ \cdots \ b_{2m};\ \cdots;\ b_{n1} \ \cdots \ b_{nn}]$$

The coefficient vector, h, in Equation (6) can be determined using a pseudo-inverse matrix method

$$\hat{h} = \left(W^T W\right)^{-1} W^T x \tag{7}$$

The caret denotes the estimated valued of h. We can see that the model coefficients are tuned basing on a group of input and output data. The estimated coefficients $\hat{a}_{ij}(k)$ and $\hat{b}_{ij}(k)$ can be substituted into Equation (5) to get a linear approximate model for a nonlinear physical system.

In order to evaluate the control performance of the developed discrete model, we construct an internal model controller (IMC) [Sousa *et al.* 1997] as shown in Figure 3. In the block diagram, the wiggle and caret represent the reference input and the estimated output, respectively. We calculate the output difference, Δy, between the output measured from the real system and that estimated from model. This output error represents the inaccuracy of the discrete model. This error value is fed back to the controller to compensate the command. In the controller, we use an inverse model to generate the control command, $u(k)$, and the command is sent into the real system and the discrete mathematical model simultaneously. A new output can be obtained at the outlets of the system and model. If the difference between the system and the model vanishes, i.e. the mathematical model matches the real system exactly, there is no feedback signal, and the IMC become an open-loop feed-forward controller. On the other hand, if the model and the system are slightly different, the output difference, Δy, will not disappear. Therefore, according to the characteristic of IMC, the output, $y(k+1)$, will not follow the reference input exactly.

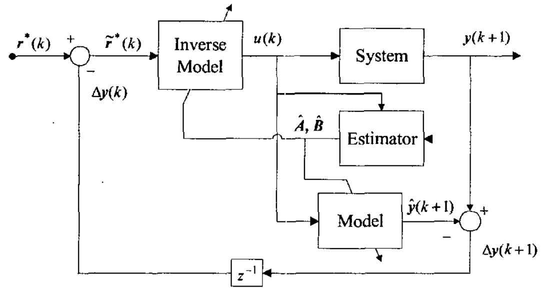

Figure 3 Self-tuning Internal Model Control

III. SELF-TUNING WITH A GREY FILTER

In this section, we apply Grey System theory [Deng 1982, Wong *et al.* 2001] to predict the output of the real system and filter out the random variation of the output. Instead of using linear approximation, we imitate the relationship between the input and output by a so-called grey differential equation based on the grey system theory [Wong *et al.* 2001]. According to the grey system theory [Deng 1982, Wong *et al.* 2001], we define two data-generating operations, namely, Accumulated Generating Operation (AGO) and mean operation (MEAN). Let $y^{(0)}$ be a non-negative original data sequence

$$y^{(0)} = \{y^{(0)}(1), y^{(0)}(2), \ldots, y^{(0)}(n)\}, \quad n \geq 4 \tag{8}$$

The number "0" in the parentheses on the superscript indicates the original data sequence. Define the AGO on $y^{(0)}$ by the operation

$$y^{(1)}(k) = AGO \circ y^{(0)} = \sum_{m=1}^{k} y^{(0)}(m), \quad k = 1,2,\ldots,n. \tag{9}$$

The number "1" in the parentheses on the superscript denotes the first-order AGO. Then we define the MEAN operation on $y^{(1)}$ as

$$z^{(1)}(k) = MEAN \circ y^{(1)} = 0.5y^{(1)}(k) + 0.5y^{(1)}(k-1), \quad k = 2,3,\ldots,n. \tag{10}$$

We can see that the AGO operation will filter out the random variation of the raw data and generate a regular sequence $y^{(1)}$. We now can begin to create a grey system model. In the grey system theory [Deng 1989], a dynamic grey model is denoted by $GM(i,j)$ which contains a group of differential equations, where i and j indicate the order and the number of variables of the differential equation, respectively. A grey model $GM(1,N)$ is represented as the following equation

$$y^{(0)}(k) + az^{(1)}(k) = \sum_{i=2}^{N} b_i x_i^{(1)}(k) \tag{11}$$

where $y^{(0)}(k)$ is the output of the system; a is called the development coefficient; b_i are the input coefficients; $x_i^{(1)}(k)$ are (N-1) grey inputs in first-order AGO form which is expressed as the following equation

$$x_i^{(1)}(k) = AGO \circ x_i^{(0)} = \sum_{m=1}^{k} x_i^{(0)}(m), \quad k = 1,2,\dots,n.$$

The (N-1) input variables are

$$x_i^{(0)} = \{x_i^{(0)}(1), x_i^{(0)}(2), \dots, x_i^{(0)}(n)\}, \quad n \geq 4, \quad i = 2,3,\dots,N$$

Equation (11) can be expressed in homogenous form

$$\boldsymbol{y} = \boldsymbol{W}\boldsymbol{h} \tag{12}$$

The vectors and matrices in Equation (12) are denoted as follows:

$$\boldsymbol{y} = \left[y^{(0)}(2) \quad y^{(0)}(3) \quad \cdots \quad y^{(0)}(n)\right]^T$$

$$\boldsymbol{W} = \begin{bmatrix} -z^{(1)}(2) & u_2^{(1)}(2) & \cdots & u_N^{(1)}(2) \\ -z^{(1)}(3) & u_2^{(1)}(3) & \cdots & u_N^{(1)}(3) \\ \vdots & \vdots & \ddots & \vdots \\ -z^{(1)}(n) & u_2^{(1)}(n) & \cdots & u_N^{(1)}(n) \end{bmatrix}$$

$$\boldsymbol{h} = \left[a \quad b_2 \quad \cdots \quad b_N\right]^T$$

The coefficients a and b_i can be determined according to the input and output data. They can be solved by means of the pseudo-inverse matrix method as in Equation (7). The first-order ordinary differential equation

$$\frac{dy^{(1)}}{dt} + ay^{(1)} = \sum_{i=2}^{N-1} b_i x_i^{(1)} \tag{13}$$

is called the whiten equation corresponding to the grey differential equation (11). A general solution of the whiten equation is

$$y^{(1)}(t) = e^{-at} \int \sum_{i=2}^{N-1} b_i x_i^{(1)} e^{at} dt + Ce^{-at} \tag{14}$$

where C is a coefficient to be determined by substituting the initial condition into Equation (14). Assuming known quantity of inputs x_i, the solution will become

$$y^{(1)}(t) = \left(y^{(0)}(1) - \sum_{i=2}^{N-1} \frac{b_i x_i^{(1)}}{a} \right) e^{-at} + \sum_{i=2}^{N-1} \frac{b_i x_i^{(1)}}{a} \tag{15}$$

It is proved that the solution of the corresponding grey differential equation can be expressed by

$$\hat{y}^{(1)}(n+p) = \left(y^{(0)}(1) - \sum_{i=2}^{N-1} \frac{b_i x_i^{(1)}}{a} \right) e^{-a(n+p-1)} + \sum_{i=2}^{N-1} \frac{b_i x_i^{(1)}}{a}, \quad n \geq 4 \tag{16}$$

where the parameter p is the forecasting step-size and the caret indicates that the value $\hat{y}$ is a predicting value of y. To get the prediction output at time t=n+p, we take the inverse AGO (IAGO) on $\hat{y}^{(1)}$. The corresponding IAGO is defined by

$$\hat{y}^{(0)}(k) = IAGO \circ y^{(1)} = \hat{y}^{(1)}(k) - \hat{y}^{(1)}(k-1), \quad k = 2,3,\dots,n.$$

Then we can obtain the prediction output at t=n+p, expressed as the follows:

$$\hat{y}^{(0)}(n+p) = \left(y^{(0)}(1) - \sum_{i=2}^{N-1} \frac{b_i x_i^{(1)}}{a} \right) e^{-a(n+p-1)} \left(1 - e^{a}\right), \quad n \geq 4 \tag{17}$$

We conclude that the grey system composed of the operations AGO, IAGO and GM(i,j) can be constructed by

$$\hat{y}^{(0)} = IAGO \circ GM(i,j) \circ AGO \circ y^{(0)}$$

This grey system can be utilized to estimate the output of a real system.

We propose a modified self-tuning IMC basing on the input-output data obtained by the grey system. The control block diagram is shown in Figure 4. We apply a grey system to reduce the random variation of input-output data of the system, and a estimator to tuning the coefficient matrices, $\boldsymbol{A}$ and $\boldsymbol{B}$, basing on the filtered data.

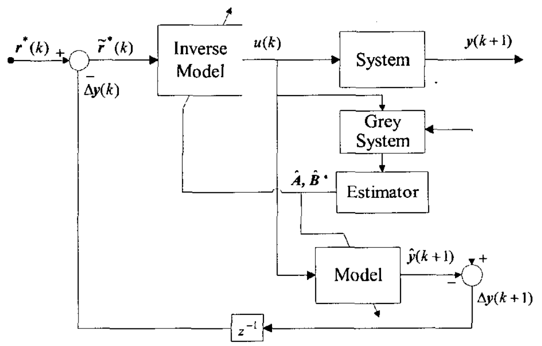

Figure 4 Internal Model Control with a grey system

IV. THERMAL CONTROL IN PLASTIC MOLDING PROCESSES

We construct an experimental setup for temperature control of the thermal barrel in the plastic molding processes. In the setup, four thermocouples are used to measure temperature at each temperature zone of the thermal barrel. Then the measured signal is converted to digital signal by an ADC interface, PCL-818HG [Advantech 1994], and fed back to the PC-based controller. We can employ a temperature control algorithm to calculate the heat rate command basing on the feedback temperature. In order to increase the efficiency of the heating device, the heat rate command is sent to the heater using PWM format through a PWM interface PCL-836 [Advantech 1994].

We use the linear discrete model in Equation (5) to estimate the temperature output. For the discrete model, the coefficients $a_{ij}(k)$ and $b_{ij}(k)$ in matrices A and B can be determined by a parameter estimation method. We can see that the model coefficients are estimated basing on a group of input and output data. The estimated coefficients $\hat{a}_{ij}(k)$ and $\hat{b}_{ij}(k)$ can be substituted into Equation (5) to get a linear approximate model for a nonlinear physical system. The resultant model is determined [Wang *et al.* 1999] as

$$\begin{bmatrix} T_1(k) \\ T_2(k) \\ T_3(k) \\ T_4(k) \end{bmatrix} = \begin{bmatrix} 1.0718 & -0.0625 & 0 & 0 \\ -0.0862 & 0.9463 & 0.1485 & 0 \\ 0 & -0.0649 & 1.0355 & 0.0503 \\ 0 & 0 & -0.0014 & 1.0059 \end{bmatrix} \begin{bmatrix} T_1(k-1) \\ T_2(k-1) \\ T_3(k-1) \\ T_4(k-1) \end{bmatrix} + \begin{bmatrix} 0.06430 & 0 & 0 & 0 \\ 0 & 0.08838 & 0 & 0 \\ 0 & 0 & 0.04079 & 0 \\ 0 & 0 & 0 & 0.09842 \end{bmatrix} \begin{bmatrix} u_1(k-d) \\ u_2(k-d) \\ u_3(k-d) \\ u_4(k-d) \end{bmatrix}$$
$$+ \begin{bmatrix} 0.1005 & 0 & 0 & 0 \\ 0 & 0.0089 & 0 & 0 \\ 0 & 0 & 0.0201 & 0 \\ 0 & 0 & 0 & 0.0184 \end{bmatrix} \begin{bmatrix} u_1(k-1-d) \\ u_2(k-1-d) \\ u_3(k-1-d) \\ u_4(k-1-d) \end{bmatrix} + \begin{bmatrix} e_1(k) \\ e_2(k) \\ e_3(k) \\ e_4(k) \end{bmatrix} \quad (19)$$

Where the heat-transfer delay time is measured to be eight sampled periods (d=8).

The proposed self-tuning IMC can be utilized to control the temperature distribution in the thermal barrel of plastic molding processes. We compare the control performance of the self-tuning IMC in Figure 3 and the self-tuning IMC with a grey system in Figure 4. The control results for temperature Zone 3 and 4 are plotted in Figures 5 and 6. We can see that the controllers without any filter involve with steady-state error, while the controllers with grey system have better performance.

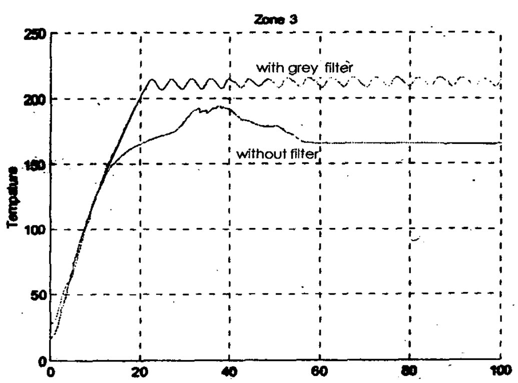

Figure 5 Self-tuning IMC for Zone 3

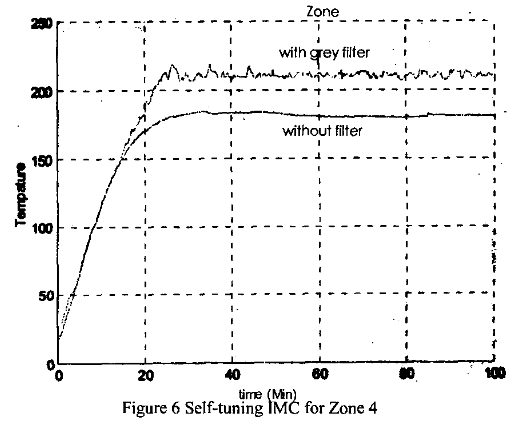

Figure 6 Self-tuning IMC for Zone 4

V. CONCLUSION REMARK

In this research, we establish a self-tuning mechanism with a grey system and apply to control the temperature distribution in a thermal barrel of plastic molding processes. In conventional self-tuning control, the tuning mechanism is suffered from the noise of the input-output data. The usage of the grey system can reduce the random variation of the input-output data, and the tuning mechanism can estimated the system parameters basing on a least number of data obtained from the process.

In order to compare the control performance, we integrate the self-tuning mechanism into an internal model controller and apply to thermal control in plastic molding processes. The systems are subjected to a step input and the responses depict the different dynamic performance of the controllers. The IMC with linear discrete model has large steady-state errors, while the control with grey system has better performance, even though it has oscillation in Zone 3 and 4 at steady state.

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