Proceedings of the 2012 International Conference on
Advanced Mechatronic Systems, Tokyo, Japan, September 18-21, 2012

# Online Hybrid Intelligent Tracking Control for Uncertain Nonlinear Dynamical Systems

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***Abstract*—A novel online hybrid direct/indirect adaptive Petri fuzzy neural network (PFNN) controller with stare observer for a class of multi-input multi-output (MIMO) uncertain nonlinear systems is developed in the paper. By using the Lyapunov synthesis approach, the online observer-based tracking control law and the weight-update law of the adaptive hybrid intelligent controller are derived. According to the importance and viability of plant knowledge and control knowledge, a weighting factor is utilized to sum together the direct and indirect adaptive PFNN controllers. In this paper, we prove that the proposed online observer-based hybrid PFNN controller can guarantee that all signals involved are bounded and that the system outputs of the closed-loop system can track asymptotically the desired output trajectories. An example including four cases is illustrated to show the effectiveness of this approach.**

## I. Introduction

In recent years, fuzzy neural network (FNN) has been developed into a powerful tool for modeling, analysis, and control of various engineering systems [1-3]. In [4-6], the authors investigated a T-S fuzzy neural approach for only considering the stabilization problem. Wang *et al.* [7, 8] developed an adaptive fuzzy-neural controller for SISO nonlinear systems and so is hardly practical in real applications. Although Hwang and Hu [9] proposed a robust fuzzy-neural learning controller for MIMO manipulators, the state feedback control scheme does not always hold in practical applications, because models of those systems are not always known. Furthermore, more inputs (linguistic terms) and membership functions of the FNN are required for higher-order complex systems [10]. Adjusting the vast numbers of parameters will aggravate the already heavy computational burden. To solve the problem of spending much computation time, an observer-based adaptive Petri fuzzy neural controller is developed for MIMO unknown nonlinear systems. In this case, using the Petri nets [11, 12] to alleviate the computation burden of parameter learning, the proposed robust tracking control scheme can deal with more kinds of unknown systems.

The conventional adaptive FNN control has direct and indirect FNN adaptive control categories [13]. The direct adaptive FNN control using fuzzy logic systems as controllers has been proposed in [8, 14]. Therefore, linguistic fuzzy control rules can be directly incorporated into the controller. Also, the indirect adaptive FNN control using fuzzy descriptions to model the plant has been developed in [7, 15]. Then, fuzzy IF–THEN rules describing the plant can be directly incorporated into the indirect FNN controller. Recently, it is an important issue [16] to choose suitably direct or indirect adaptive control for nonlinear systems. In this paper, a hybrid direct/indirect adaptive PFNN control scheme is constructed by using a weighting factor to sum together the direct adaptive PFNN controller and indirect adaptive PFNN controller. The weighting factor can be adjusted by the tradeoff between plant knowledge and control knowledge. Therefore, the free parameters can be flexibly tuned by the adaptive law.

In this paper, an observer-based adaptive tracking controller constructed by the PFNN is developed for a class of MIMO uncertain nonlinear systems. We replace the conventional fuzzy neural networks (FNNs) [7, 8, 10] with a novel PFNN and combine the direct adaptive controller with the indirect adaptive controller. The basic idea of the PFNN is that a system consists of a typical T-S fuzzy inference system constructed from a Petri neural network structure. Under the constraint that not all system states can be measured, the proposed output-feedback PFNN-based learning controller can guarantee that all signals involved are bounded and the outputs of the closed-loop system can track asymptotically the desired output trajectories, and the computation burden can be efficiently shortened.

The paper is organized as follows. Section II reviews the problem formulation. In Section III, a brief description of PFNN is presented. Section IV investigates the hybrid PFNN controller with observer. To demonstrate the performance of the control scheme, a simulation example is provided in Section V. Finally, Section VI concludes the paper.

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## II. PROBLEM FORMULATION

Consider the $n$th-order MIMO uncertain nonlinear systems of the form [17]

$$\dot{\boldsymbol{\chi}}_i = \mathbf{A}_i\boldsymbol{\chi}_i + \mathbf{B}_i\Big(f_i(\mathbf{x}) + \sum_{j=1}^{p} g_{ij}(\mathbf{x})u_j + d_{di}\Big) \qquad (1)$$

$$y_i = \mathbf{C}_i^T\boldsymbol{\chi}_i,\ i = 1, 2, \cdots, p$$

where

$$\mathbf{A}_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{r_i\times r_i},\ \mathbf{B}_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}_{r_i\times 1},\ \mathbf{C}_i = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{r_i\times 1} \qquad (2)$$

and $\boldsymbol{\chi}_1 = [x_1, x_2, \cdots, x_{r_1}]^T$, $\boldsymbol{\chi}_2 = [x_{(r_1+1)}, x_{(r_1+2)}, \cdots, x_{(r_1+r_2)}]^T, \cdots$, $\boldsymbol{\chi}_p = [x_{(n-r_p+1)}, x_{(n-r_p+2)}, \cdots, x_n]^T$ and $\mathbf{x} = [\boldsymbol{\chi}_1^T, \boldsymbol{\chi}_2^T, \cdots, \boldsymbol{\chi}_p^T]^T$ denote state vectors. $r_1 + r_2 + \cdots + r_p = n$ . $\mathbf{u} = [u_1, u_2, \cdots, u_p]^T$ and $\mathbf{y} = [y_1, y_2, \cdots, y_p]^T$ are vectors of control inputs and system outputs, respectively. $\mathbf{d}_d = [d_{d1}, d_{d2}, \cdots, d_{dp}]^T$ is a vector of external disturbances. $f_i$ and $g_{ij}$ are unknown smooth functions.

Define the reference vectors $\mathbf{y}_{mi} = [y_{mi}, \dot{y}_{mi}, \ddot{y}_{mi}, \cdots, y_{mi}^{(r_i-1)}]^T$, the tracking error vectors $\mathbf{e}_i = \mathbf{y}_{mi} - \boldsymbol{\chi}_i$ , and the estimated tracking error vectors $\hat{\mathbf{e}}_i = \mathbf{y}_{mi} - \hat{\boldsymbol{\chi}}_i$ where $\hat{\mathbf{e}}_i$ and $\hat{\boldsymbol{\chi}}_i$ denote the estimations of $\mathbf{e}_i$ and $\boldsymbol{\chi}_i$ , respectively. Based on the certainty equivalence approach, the control law is

$$\mathbf{u}^* = \frac{1}{\mathbf{G}(\mathbf{x})}\Big(-\mathbf{F}(\mathbf{x}) + [y_{m1}^{(r_1)}, y_{m2}^{(r_2)}, \cdots, y_{mp}^{(r_p)}]^T + [\mathbf{K}_{c1}^T\mathbf{e}_1, \mathbf{K}_{c2}^T\mathbf{e}_2, \cdots, \mathbf{K}_{cp}^T\mathbf{e}_p]^T\Big) \qquad (3)$$

where

$$\mathbf{F}(\mathbf{x}) = [f_1, f_2, \cdots, f_p]^T,\ \mathbf{G}(\mathbf{x}) = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1p} \\ g_{21} & g_{22} & \cdots & g_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ g_{p1} & g_{p2} & \cdots & g_{pp} \end{bmatrix}$$

and $\mathbf{K}_{ci} = [k_{r_i}^{ci}, k_{(r_i-1)}^{ci}, \cdots, k_1^{ci}]^T$ are the feedback gain vectors, chosen such that the characteristic polynomials of $\mathbf{A}_i - \mathbf{B}_i\mathbf{K}_{ci}^T$ are Hurwitz because ($\mathbf{A}_i, \mathbf{B}_i$) are controllable. However, $\mathbf{F}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ are unknown, the ideal controller (3) cannot be implemented, and not all system states can be measured. Therefore, we design an observer to estimate the state vector in the following context.

## III. THE PETRI FUZZY NEURAL NETWORK (PFNN) SYSTEMS

The basic configuration of the Petri fuzzy neural network (PFNN) consists of a typical T-S fuzzy inference system constructed from a Petri neural network structure. The fuzzy logic system can be divided into two parts: some fuzzy IF-THEN rules and a fuzzy inference engine. The fuzzy inference engine uses the fuzzy IF-THEN rules to perform a mapping from an input linguistic vector to an output linguistic variable. The $i$th fuzzy IF-THEN rule is written as

$$R^{(i)}: \text{If } z_1 \text{ is } F_1^i \text{ and} \ldots z_n \text{ is } F_n^i \text{ and } \cdots z_{n+p} \text{ is } F_{n+p}^i \qquad (4)$$
$$\text{Then } \bar{y}_l = p_{l1}^i z_1 + p_{l2}^i z_2 + \ldots + p_{l(n+p)}^i z_{n+p}$$

where $\mathbf{z} = [z_1, z_2, \cdots, z_{n+p}]^T \in \Re^{n+p}$ is a vector of linguistic variables, $\bar{y}$ represents the output of the fuzzy-neural network, $F_j^i\ (i = 1, 2, \cdots, h,\ j = 1, 2, \cdots, p)$ are fuzzy sets, and $p_{lk}^i\ (l = 1, 2, \cdots, n,\ k = 1, 2, \cdots, (n+p))$ are adjustable parameters which are called the weighting factors.

Fig. 1 shows the configuration of the Petri fuzzy neural function approximator. It has a total of five layers. Nodes at input layer are input nodes (linguistic nodes) that represent input linguistic variables. Nodes at membership layer are term nodes which act as membership functions to represent the terms of the respective linguistic variables. The Petri layer of the PFNN in this paper for producing tokens makes use of competition laws as follows to select suitable fired nodes:

$$t_j^i = \begin{cases} 1, & \mu_{F_j^i}(z_j) \geq d_{th} \\ 0, & \mu_{F_j^i}(z_j) < d_{th} \end{cases} \qquad (5)$$

where $t_j^i$ is the transition and $d_{th}$ is a dynamic threshold value varied with the corresponding tracking error to be tuned by the following equation:

$$d_{th} = \frac{k_a \exp(-k_b\|\mathbf{e}\|)}{1 + \exp(-k_b\|\mathbf{e}\|)} \qquad (6)$$

where $k_a$ and $k_b$ are positive constants. $\mathbf{e}$ represents a tracking error vector. It means that if tracking errors become large, the threshold values will be decreased in order to fire more control rules for the present situation. Each node at rule layer is a fuzzy rule. Nodes at output layer are output nodes.

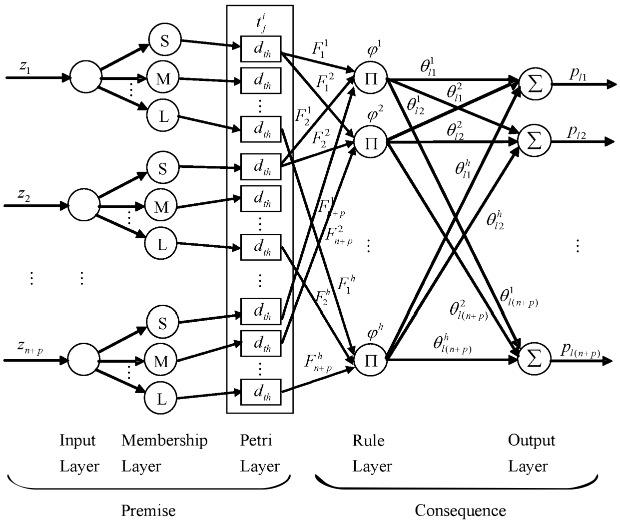



Fig. 1. Configuration of the Petri fuzzy neural approximator.

From Fig. 1, the coefficients, $p_{lk}\ (l = 1, 2, \cdots, p,\ k = 1, 2, \cdots, (n+p))$, of the Petri fuzzy neural model are

$$p_{lk} = \frac{\sum_{i=1}^{h} \theta_{lk}^{i} (\prod_{j=1}^{n+p} \mu_{F_j^i}(z_j))}{\sum_{i=1}^{h} (\prod_{j=1}^{n+p} \mu_{F_j^i}(z_j))} = \boldsymbol{\theta}_{lk}^{T} \boldsymbol{\psi}(\mathbf{z}) \tag{7}$$

where $\mu_{F_j^i}(z_j)$ are the membership function values of the fuzzy variable $z_j$, $h$ is the number of the total IF-THEN rules, $\boldsymbol{\theta}_{lk} = [\theta_{lk}^1, \theta_{lk}^2, \cdots, \theta_{lk}^h]^T$ is a adjustable parameter vector, and a fuzzy basis function vector $\boldsymbol{\psi} = [\psi^1, \psi^2, \cdots, \psi^h]^T$ is defined as

$$\psi^i \equiv \frac{\prod_{j=1}^{n+p} \mu_{F_j^i}(z_j)}{\sum_{i=1}^{h} (\prod_{j=1}^{n+p} \mu_{F_j^i}(z_j))}, \quad i = 1, 2, \cdots, h. \tag{8}$$

## IV. OBSERVER-BASED HYBRID DIRECT/INDIRECT TRACKING CONTROL SCHEME

In this section, we develop the observer-based hybrid direct/indirect PFNN controller. The overall control law is constructed as

$$\mathbf{u} = \alpha \mathbf{u}_I(\hat{\mathbf{x}} \mid \boldsymbol{\theta}_{fi}) + (1-\alpha)\mathbf{u}_D(\hat{\mathbf{x}} \mid \boldsymbol{\theta}_{gij}) + \mathbf{u}_s(\hat{\mathbf{x}} \mid \boldsymbol{\theta}_{Di}) \tag{9}$$

where $\mathbf{u}_I$ and $\mathbf{u}_D$ are indirect PFNN controller and direct PFNN controller, respectively. $\mathbf{u}_s$ is the compensated control input vector. The indirect control law is described as

$$\mathbf{u}_I = \frac{1}{\hat{\mathbf{G}}(\hat{\mathbf{x}})}(-\hat{\mathbf{F}}(\hat{\mathbf{x}}) + [y_{m1}^{(r_1)}, y_{m2}^{(r_2)}, \cdots, y_{mp}^{(r_p)}]^T$$
$$+ [\mathbf{K}_{c1}^T \hat{\mathbf{e}}_1, \mathbf{K}_{c2}^T \hat{\mathbf{e}}_2, \cdots, \mathbf{K}_{cp}^T \hat{\mathbf{e}}_p]^T). \tag{10}$$

Applying (9) and (10) to (1), we can obtain the error dynamic equation as

$$\dot{\mathbf{e}}_i = \mathbf{A}_i \mathbf{e}_i - \mathbf{B}_i \mathbf{K}_{ci}^T \hat{\mathbf{e}}_i + \mathbf{B}_i(\alpha(\hat{f}_i(\hat{\mathbf{x}}) - f_i(\mathbf{x}) + \sum_{j=1}^{p}(\hat{g}_{ij}(\hat{\mathbf{x}}) - g_{ij}(\mathbf{x}))u_{Ij})$$
$$+ (1-\alpha)\sum_{j=1}^{p} g_{ij}(\mathbf{x})(u^* - u_{Dj}) - \sum_{j=1}^{p} g_{ij}(\mathbf{x})u_{sj} - d_{di})$$
$$e_{oi} = \mathbf{C}_i^T \mathbf{e}_i \tag{11}$$

where $e_{oi} = y_{mi} - y_i$ denote the output tracking errors. The observation errors are defined as: $\tilde{\mathbf{e}}_i = \mathbf{e}_i - \hat{\mathbf{e}}_i$ and $\tilde{e}_{oi} = e_{oi} - \hat{e}_{oi}$. Then the error dynamics are

$$\dot{\tilde{\mathbf{e}}}_i = (\mathbf{A}_i - \mathbf{K}_{oi}\mathbf{C}_i^T)\tilde{\mathbf{e}}_i + \mathbf{B}_i(\alpha(\hat{f}_i(\hat{\mathbf{x}}) - f_i(\mathbf{x}) + \sum_{j=1}^{p}(\hat{g}_{ij}(\hat{\mathbf{x}}) - g_{ij}(\mathbf{x}))u_{Ij})$$
$$+ (1-\alpha)\sum_{j=1}^{p} g_{ij}(\mathbf{x})(u^* - u_{Dj})) - \mathbf{B}_i \sum_{j=1}^{p} g_{ij}(\mathbf{x})u_{sj} - \mathbf{B}_i d_{di}$$
$$\tilde{e}_{oi} = \mathbf{C}_i^T \tilde{\mathbf{e}}_i. \tag{12}$$

where $\mathbf{K}_{oi} = [k_1^{oi}, k_2^{oi}, \cdots, k_{r_i}^{oi}]^T$ are the observer gain vectors, chosen such that the characteristic polynomials of $\mathbf{A}_i - \mathbf{K}_{oi}\mathbf{C}_i^T$ are strictly Hurwitz because ( $\mathbf{C}_i$ , $\mathbf{A}_i$ ) are observable.

By using $\hat{f}_i(\hat{\mathbf{x}} \mid \boldsymbol{\theta}_{fi}) = \boldsymbol{\theta}_{fi}^T \boldsymbol{\psi}(\hat{\mathbf{x}})$, $\hat{g}_{ij}(\hat{\mathbf{x}} \mid \boldsymbol{\theta}_{gij}) = \boldsymbol{\theta}_{gij}^T \boldsymbol{\psi}(\hat{\mathbf{x}})$, and $u_{Dj}(\hat{\mathbf{x}} \mid \boldsymbol{\theta}_{Di}) = \boldsymbol{\theta}_{Di}^T \boldsymbol{\varphi}(\hat{\mathbf{x}})$, (12) can be rewritten as

$$\dot{\tilde{\mathbf{e}}}_i = \boldsymbol{\Lambda}_i \tilde{\mathbf{e}}_i + \mathbf{B}_i(\alpha(\tilde{\boldsymbol{\theta}}_{fi}^T \boldsymbol{\psi}(\hat{\mathbf{x}}) + \sum_{j=1}^{p} \tilde{\boldsymbol{\theta}}_{gij}^T \boldsymbol{\psi}(\hat{\mathbf{x}}) u_{Ij})$$
$$-(1-\alpha)\sum_{j=1}^{p} g_{ij}(\mathbf{x}) \tilde{\boldsymbol{\theta}}_{Di}^T \boldsymbol{\varphi}(\hat{\mathbf{x}})) - \mathbf{B}_i v_i + \mathbf{B}_i w_{mi}$$
$$\tilde{e}_{oi} = \mathbf{C}_i^T \tilde{\mathbf{e}}_i \tag{13}$$

where $\tilde{\boldsymbol{\theta}}_{fi} = \boldsymbol{\theta}_{fi} - \boldsymbol{\theta}_{fi}^*$ , $\tilde{\boldsymbol{\theta}}_{gij} = \boldsymbol{\theta}_{gij} - \boldsymbol{\theta}_{gij}^*$ , $\tilde{\boldsymbol{\theta}}_{Di} = \boldsymbol{\theta}_{Di} - \boldsymbol{\theta}_{Di}^*$ , $v_i = \sum_{j=1}^{p} g_{ij}(\mathbf{x}) u_{sj}$ , $\boldsymbol{\Lambda}_i = \mathbf{A}_i - \mathbf{K}_{oi}\mathbf{C}_i^T$ , and

$$w_{mi} = \alpha(\hat{f}_i(\hat{\mathbf{x}} \mid \boldsymbol{\theta}_{fi}^*) - f_i(\mathbf{x}) + \sum_{j=1}^{p}(\hat{g}_{ij}(\hat{\mathbf{x}} \mid \boldsymbol{\theta}_{gij}^*) - g_{ij}(\mathbf{x}))u_{Ij})$$
$$+(1-\alpha)\sum_{j=1}^{p} g_{ij}(\mathbf{x})(u^* - u_{Dj}(\hat{\mathbf{x}} \mid \boldsymbol{\theta}_{Di}^*)) - d_{di} \tag{14}$$

By using the strictly-positive-real (SPR) Lyapunov design approach to analyze the stability of (13) and generate the adaptive output feedback update laws for $\boldsymbol{\theta}_f$ , $\boldsymbol{\theta}_{gij}$ , and $\boldsymbol{\theta}_{Di}$ , (13) can be rewritten as

$$\tilde{e}_{oi} = H_i(s)(\alpha(\tilde{\boldsymbol{\theta}}_{fi}^T \boldsymbol{\psi}(\hat{\mathbf{x}}) + \sum_{j=1}^{p} \tilde{\boldsymbol{\theta}}_{gij}^T \boldsymbol{\psi}(\hat{\mathbf{x}}) u_{Ij}) - (1-\alpha)\sum_{j=1}^{p} g_{ij}(\mathbf{x}) \tilde{\boldsymbol{\theta}}_{Di}^T \boldsymbol{\varphi}(\hat{\mathbf{x}})$$
$$-v_i + w_{mi}) \tag{15}$$

where

$$H_i(s) = \mathbf{C}_i^T (s\mathbf{I} - (\mathbf{A}_i - \mathbf{K}_{oi}\mathbf{C}_i^T))^{-1} \mathbf{B}_i = \frac{1}{s^{r_i} + k_1^{oi} s^{(r_i-1)} + \cdots + k_{r_i}^{oi}}. \tag{16}$$

The transfer functions $H_i(s)$ are known stable transfer functions. In order to be able to use the SPR-Lyapunov design approach, (15) can be rewritten as

$$\tilde{e}_{oi} = H_i(s) L_i(s)(\alpha(\tilde{\boldsymbol{\theta}}_{fi}^T \boldsymbol{\psi}(\hat{\mathbf{x}}) + \sum_{j=1}^{p} \tilde{\boldsymbol{\theta}}_{gij}^T \boldsymbol{\psi}(\hat{\mathbf{x}}) u_{Ij}) - (1-\alpha)\tilde{\boldsymbol{\theta}}_{Di}^T \boldsymbol{\varphi}(\hat{\mathbf{x}})$$
$$-v_{fi} + w_{fi}) \tag{17}$$

where $v_{fi} = L_i^{-1}(s) v_i$, $w_{fi} = L_i^{-1}(s) w_i$, $w_i = w_{mi} + \varepsilon_i$, $\varepsilon_i = \alpha(\tilde{\boldsymbol{\theta}}_{fi}^T \boldsymbol{\psi}(\hat{\mathbf{x}}) + \sum_{j=1}^{p} \tilde{\boldsymbol{\theta}}_{gij}^T \boldsymbol{\psi}(\hat{\mathbf{x}}) u_{Ij}) - (1-\alpha)\sum_{j=1}^{p} g_{ij}(\mathbf{x}) \tilde{\boldsymbol{\theta}}_{Di}^T \boldsymbol{\varphi}(\hat{\mathbf{x}}) - L_i(s)(\alpha(\tilde{\boldsymbol{\theta}}_{fi}^T \boldsymbol{\psi}(\hat{\mathbf{x}}) + \sum_{j=1}^{p} \tilde{\boldsymbol{\theta}}_{gij}^T \boldsymbol{\psi}(\hat{\mathbf{x}}) u_{Ij}) - (1-\alpha)\tilde{\boldsymbol{\theta}}_{Di}^T \boldsymbol{\varphi}(\hat{\mathbf{x}}))$ and $L_i(s)$ are chosen so that $L_i^{-1}(s)$ are proper stable transfer functions and $H_i(s) L_i(s)$ are proper SPR transfer functions. Suppose that $L_i(s) = s^{(r_i-1)} + b_1 s^{(r_i-2)} + b_2 s^{(r_i-3)} + \cdots + b_{(r_i-1)}$, such that $H_i(s) L_i(s)$ are proper SPR transfer functions. Then the state-space realization of (17) can be rewritten as

$$\dot{\tilde{\mathbf{e}}}_i = \mathbf{A}_{ci} \tilde{\mathbf{e}}_i + \mathbf{B}_{ci}(\alpha(\tilde{\boldsymbol{\theta}}_{fi}^T \boldsymbol{\psi}(\hat{\mathbf{x}}) + \sum_{j=1}^{p} \tilde{\boldsymbol{\theta}}_{gij}^T \boldsymbol{\psi}(\hat{\mathbf{x}}) u_{Ij}) - (1-\alpha)\tilde{\boldsymbol{\theta}}_{Di}^T \boldsymbol{\varphi}(\hat{\mathbf{x}})$$
$$-v_{fi} + w_{fi})$$
$$\tilde{e}_{oi} = \mathbf{C}_{ci}^T \tilde{\mathbf{e}}_i \tag{18}$$

where $\mathbf{A}_{ci} = \mathbf{A}_i - \mathbf{K}_{oi}\mathbf{C}_i^T \in \Re^{r_i \times r_i}$ , $\mathbf{B}_{ci} = [1, b_1, b_2, \cdots, b_{(r_i-1)}]^T \in \Re^{r_i}$ , and $\mathbf{C}_{ci} = [1, 0, \cdots, 0]^T \in \Re^{r_i}$ .

*Theorem 1:* Consider the nonlinear systems (1) with the following adaptive laws

$$\dot{\boldsymbol{\theta}}_{fi} = -\gamma_{1i}\tilde{e}_{oi}\boldsymbol{\psi}(\hat{\mathbf{x}}) \tag{19}$$

$$\dot{\boldsymbol{\theta}}_{gij} = -\gamma_{2i}\tilde{e}_{oi}\boldsymbol{\psi}(\hat{\mathbf{x}})u_{Ij} \tag{20}$$

$$\dot{\boldsymbol{\theta}}_{Di} = \gamma_{3i}\tilde{e}_{oi}\boldsymbol{\varphi}(\hat{\mathbf{x}}) \tag{21}$$

where $\gamma_{1i}, \gamma_{2i}$, and $\gamma_{3i}$ are learning rates. Suppose that the compensated control inputs are chosen as

$$u_{si} = \begin{cases} \rho_i, & \text{if } \tilde{e}_{oi} \geq 0 \text{ and } |\tilde{e}_{oi}| > \varpi_i, \\ -\rho_i, & \text{if } \tilde{e}_{oi} < 0 \text{ and } |\tilde{e}_{oi}| > \varpi_i, \\ \rho_i\tilde{e}_{oi}/\varpi_i, & \text{if } |\tilde{e}_{oi}| < \varpi_i, i = 1, 2, \cdots, p \end{cases} \tag{22}$$

where $\varpi_i$ are positive constants. The control law is chosen as (9). Then $\tilde{e}_{oi}$ and $e_{oi}$ converge to zero as $t \to \infty$.

*Proof:* To be omitted for matching the requirement of length.

## V. Simulation Results

This section presents the simulation results of the proposed observer-based hybrid Petri fuzzy neural network control for unknown nonlinear dynamical systems to illustrate that the tracking error of the closed-loop system can be made arbitrarily small. In addition, the simulation results confirm that the effect of all the estimation errors and external disturbances on the tracking error is attenuated efficiently by the proposed controller.

*Example 1:* Consider the problem of balancing of an inverted pendulum on a cart shown in Fig. 2. Let $x_1$ be the angle of the pendulum with respect to the vertical line. The dynamic equations of the inverted pendulum system [16] are

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}(f + gu + d_d) \tag{23}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

where

$$f = \frac{g_v \sin(x_1) - \dfrac{mlx_2^2 \cos(x_1)\sin(x_1)}{M+m}}{l\left(\dfrac{4}{3} - \dfrac{m\cos^2(x_1)}{M+m}\right)}; \; g = \frac{\dfrac{\cos(x_1)}{M+m}}{l\left(\dfrac{4}{3} - \dfrac{m\cos^2(x_1)}{M+m}\right)}$$

and $M$ is the mass of the cart, $m$ is the mass of the rod, $g_v = 9.8 \, {}^{m}\!/\!_{\sec^2}$ is the acceleration due to gravity, $l$ is the half length of the rod, $u$ is the control input, and $d_d$ is the external disturbance which is assumed to be a square-wave with amplitude $\pm 0.03$ and period $2\pi$. In this example, we assume that $M$=1$kg$, $m$=0.1$kg$, and $l$=0.5$m$, and four different cases for the initial states $\mathbf{x}(0)$ and $\hat{\mathbf{x}}(0)$ are simulated. The four cases are shown in Table I.

TABLE I
Four Cases of Initial States for Example 1

| Cases | Initial states |
|---|---|
| Case 1 | $\mathbf{x}(0) = [0.1,\ 0]^T$, $\hat{\mathbf{x}}(0) = [-0.15,\ 0]^T$ |
| Case 2 | $\mathbf{x}(0) = [0.25,\ 0.25]^T$, $\hat{\mathbf{x}}(0) = [0.1,\ 0.1]^T$ |
| Case 3 | $\mathbf{x}(0) = [-0.1,\ 0]^T$, $\hat{\mathbf{x}}(0) = [0.15,\ 0]^T$ |
| Case 4 | $\mathbf{x}(0) = [-0.15,\ -0.15]^T$, $\hat{\mathbf{x}}(0) = [0.15,\ 0.15]^T$ |

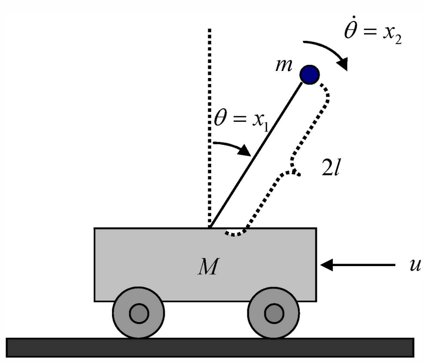
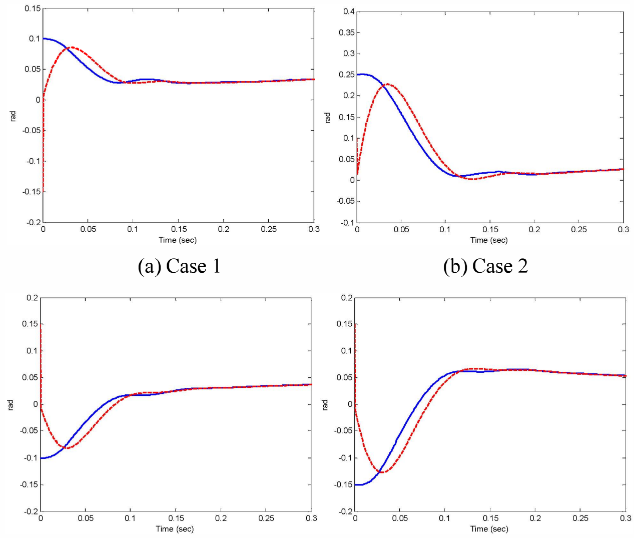



Fig. 2. Inverted pendulum system.




(a) Case 1 (b) Case 2

(c) Case 3 (d) Case 4

Fig. 3. Transient trajectories (0-0.3 sec) of the state $x$ (solid line) and $\hat{x}_1$ (dashed line) of four cases.

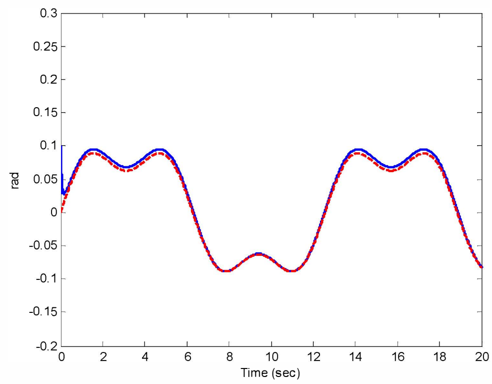



Fig. 4. Trajectories of output $y$ (solid line) of Case 1 and reference $y_m$ (dashed line) with $\alpha = 0.5$ (time=0-20 s).

The design parameters are selected as $\mathbf{K}_c = [144,\ 24]^T$, $\mathbf{K}_o = [60,\ 900]^T$, $k_a = 0.4$, $k_b = 300$, $\gamma_1 = \gamma_2 = \gamma_3 = 5$, and $\rho = 20$. We use the proposed control law in (9) to control the state $x_1$ of the system to track the reference signal $y_m(t) = 0.03\pi\sin(0.5t) + 0.01\pi\sin(1.5t)$. Fig. 3 illustrates that the curves of the states $x_1$ and $\hat{x}_1$ of four cases if $\alpha = 0.5$ is chosen. The trajectories of system outputs $y$ of four cases and reference signal $y_m$ with $\alpha = 0.5$ are shown in Figs. 4 and 5. The response of control input $u$ of Case 1 with $\alpha = 0.5$ is shown in Fig. 6. The simulation results indicate that the estimation state $\hat{x}_1$ takes very short time to catch up to the system state $x_1$. Moreover, the tracking performances of four cases are also very good.

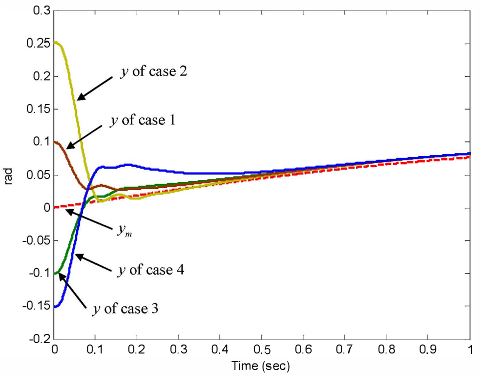



Fig. 5. Trajectories of outputs $y$ of four cases and reference $y_m$ with $\alpha = 0.5$ (time=0-1 s).

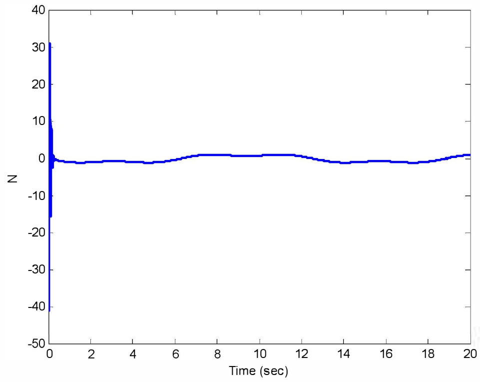



Fig. 6. The response of control input $u$ of Case 1 with $\alpha = 0.5$ (time=0-20 s).

## VI. CONCLUSION

For a class of MIMO uncertain nonlinear systems under the constraint that not all system states can be measured, a novel design of adaptive observer-based hybrid intelligent tracking controller using a weighting factor to combine direct PFNN controller with indirect PFNN controller is proposed in this paper. The free parameters of the adaptive hybrid direct/indirect intelligent controller can be tuned on-line by the observer-based robust control law and the adaptive update law. The proposed control scheme guarantees that all the tracking error of the closed-loop system can be made arbitrarily small. The computer simulation results for the inverted pendulum system show that the proposed observer-based hybrid adaptive PFNN controller can achieve the successful control and the desired tracking performance.

## ACKNOWLEDGMENT


This work was partially supported by the National Science Council (NSC), Taiwan, under Grant NSC 100-2221-E-003-007 and 100-2221-E-003-008.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