

## Frequency Dependence on Image Reconstruction for a Buried Conductor

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**Abstract** - Frequency dependence on image reconstruction for a buried imperfectly conducting cylinder is investigated. A conducting cylinder of unknown shape and conductivity is buried in one half-space and scatters the incident wave from another half-space. By using measured scattered field, the image problem is reformulated into an optimization problem and solved by the genetic algorithm. Frequency dependence on image reconstruction is investigated and numerical results show that the reconstruction is quite good in the resonant frequency range. On the contrary, if the frequency is too high or too low, the reconstruction becomes bad. It is worth noting that the present work provides not only comparative information but quantitative information.

**Keywords** - Imperfectly conducting cylinder, Image reconstruction, Frequency dependence, Genetic algorithm

### I. INTRODUCTION

The electromagnetic inverse scattering problem of conductors has been a subject of considerable importance in remote sensing and noninvasive measurement. In the past twenty years, many rigorous methods have been developed to solve the exact equation. However, inverse problem of this type are difficult to solve because they are illposed and nonlinear. As a result, many inverse problems are reformulated as optimization problems. General speaking, two main kinds of approaches have been developed. The first is based on gradient search approach such as the Newton-Kantorovitch method [1]-[3], the Levenberg-Marguarat algorithm [4]-[5]. Since this first approach applies the gradient search method to find extreme values of the cost function. It is highly dependent on the initial guess and tends to get trapped in a local minima and maxima. In contrast, a second approach is based on the genetic algorithm [6]-[8]. The genetic algorithm is a well-known algorithm that uses the stochastic random choice to search through a coding of a

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parameter space. Compared to gradient search optimization techniques, the genetic algorithm is less prone to convergence to a local minimum, which in turn renders it an ideal candidate for global optimization. It usually converges to the global extreme of the problem, no matter what the initial estimate is [9].

In this paper, frequency dependence on image reconstruct for a buried imperfectly conducting cylinder is presented. The genetic algorithm is used to reconstruct the shape and conductivity of a buried scatterer. In section II, the theoretical formulation is briefly presented. Numerical results for different frequencies of incident waves are given in section III. Finally, conclusions are drawn in section IV.

### II. THEORETICAL FORMULATION

Let us consider a buried imperfectly conducting cylinder with cross section described in polar coordinates in the  $xv$  plane by the equation  $\rho = F(\theta)$  located in free space. A plane wave whose electric field vector is parallel to the  $Z$ -axis is incident from one half-space to the metallic cylinder buried in another half-space. By using the induced current concept, the scattered field can be expressed as the integral of the two-dimensional Green's function multiplies by the induced surface current density, which is proportional to the normal derivative of the electric field on the conductor surface [3], [6]. For an imperfectly conducting scatterer with finite conductivity, the boundary condition can be approximated by assuming that the total tangential electric field on the scatterer surface is related to surface current density through a surface impedance [3], [10]. As a result, for the direct problem, given the shape and the conductivity of the object, we can use the boundary condition to solve the surface current density, then calculate the scattered field by using the two dimensional half-space Green's function. For numerical calculation of the direct problem, the contour of the object is first divided into sufficient small segments so that the induced surface current density can be consider constant over each segment. Then the moment method is used to solve the equations with a pulse basis function for expanding and Dirac delta function for testing.

Let us consider the following inverse problem: given the scattered field, determine the shape  $F(\theta)$ . Here the shape  $F(\theta)$  is assumed to be star-like. In other words,  $F(\theta)$  can be expanded as:

$$F(\theta) = \sum_{n=0}^{N/2} B_n \cos(n\theta) + \sum_{n=1}^{N/2} C_n \sin(n\theta)$$

where  $B_n$  and  $C_n$  are real coefficient to be determined, and  $N$  is the number of unknowns for shape function of the object. In the inversion procedure, the genetic algorithm is used to minimize the root mean square error of the measured scattered field and the calculated scattered field, through three genetic operators: reproduction, crossover and mutation. When the root mean square error changes by less than 1% in two successive generations, the genetic algorithm is terminated and a solution is then obtained. Note that the regularization term can be added to avoid illposed problems. Please refer the reference [3], [6] for detail.

### III. NUMERICAL RESULTS

Let us consider an imperfectly conducting cylinder buried in a lossless half-space. The permittivity in region 1 and region 2 is characterized by  $\epsilon_0$  and  $2.56\epsilon_0$ , respectively. A TM polarization plane wave of unit amplitude is incident from region 1. The frequency of the incident wave is chosen to be 1GHz, 2GHz, 3GHz, 4GHz and 5GHz. The object is buried at a depth 0.1m and the scattered field is measured on a probing line along the interface between region 1 and region 2. Our purpose is to reconstruct the shape of the object by using the scattered field at different incident angles. To reconstruct the shape of the object, the object is illuminated by incident waves from three different directions and 20 measurement points at equal spacing are used along the interface for each incident angle. There are 60 measurement points in each simulation. The measurement is taken from  $x=0$  to 0.2m for incident angle  $-\pi/3$ , from  $x=-0.1$  to 0.1m for incident angle 0, and from  $x=-0.2$  to 0m for incident angle  $\pi/3$ . To save computing time, the number of unknowns is set to be 10 (i.e.  $N+2=10$ ). The population size is chosen as 300. The binary string length of the unknown coefficient,  $B_n$  and  $C_n$ , is set to be 16 bits. The binary string length of conductivity is also set to be 16 bits. In other words, the bit number of a chromosome is 160. The search range for the unknown coefficient of the shape function is chosen to be from 0 to 0.1. The search range for the unknown conductivity is chosen to be from  $3 \times 10^7$  to  $7 \times 10^7$ . The extreme value of the coefficient of the shape function and conductivity can be determined by the prior knowledge of the objects. The crossover probability and mutation probability are set to be 0.8 and 0.1 respectively. In the first example, the shape function is chosen to be  $F(\theta) = (0.026 + 0.009 \sin 2\theta)$  m with aluminum material (i.e.,  $\sigma = 3.54 \times 10^7$  s/m). The reconstructed relative root mean square errors for the shape and conductivity (ES and EC) versus different frequencies are plotted in Fig. 1. It is found that the reconstruction for 1GHz and 5GHz are poor, which the reconstruction for 2GHz, 3GHz and 4GHz are fair.

Roughly speaking, the reconstruction for 3GHz is fairly satisfactory. Physically this can be explained by the fact that the information available from the shadow region decreases if the wave number increases. On the other hand, the scattering pattern becomes isotropic at very low frequencies and thus insensitive to the variation of shape and conductivity resulting in large errors. In addition, we also see that the EC is always larger than ES. This is due to the fact that the shape function makes a stronger contribution to the scattered field than the conductivity does. In other words, the reconstruction of the shape function has a higher priority than the reconstruction of the conductivity.

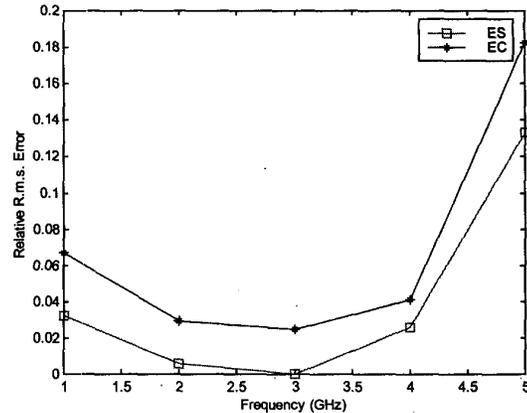


Fig. 1 Shape function error (ES) and conductivity error (EC) versus different frequencies in each generation.

### IV. CONCLUSIONS

We have presented the effect of the incident frequency on the shape and conductivity reconstruction for a buried imperfectly conducting cylinder. It is found that the information available from the shadow region decreases if the wave number increases. On the other hand, the scattering pattern becomes isotropic at low frequencies and thus insensitive to the variation of shape and conductivity resulting in larger errors. In other words, the reconstruction is good in the resonant frequency. It is worth noting that in these cases the present work provided not only comparative information but also quantitative information.

### V. REFERENCES

- [1] A. Roger, "Newton-Kantorovitch algorithm applied to an electromagnetic inverse problem," IEEE Trans. Antennas Propagat., vol. 29, pp. 232-238, 1981.
- [2] W. Tobocman, "Inverse acoustic wave scattering in two dimensions from impenetrable targets," Inverse Problems, vol. 5, pp. 1131-1144, Dec. 1989.
- [3] C. C. Chiu and Y. W. Kiang, "Image reconstruction for an imperfectly conducting cylinder buried in half-space," Journal of Electromagnetic Waves and Application, vol. 6, pp.

- 1413-1431, Oct. 1992.
- [4] A. Kirsch, R. Kress, P. Monk, and A. Zinn, "Two methods for solving the inverse acoustic scattering problem," *Inverse Problems*, vol. 4, pp. 749-770, Aug. 1988.
- [5] F. Hettlich, "Two methods for solving an inverse conductive scattering problem," *Inverse Problems*, vol. 10, pp. 375-385, 1994.
- [6] C. C. Chiu and P. T. Liu, "Image reconstruction of a perfectly conducting cylinder by the genetic algorithm," *IEE Proc.-Micro. Antennas Propagat.*, vol. 143, pp.249-253, June 1996.
- [7] Z. Q. Meng, T. Takenaka and T. Tanaka, "Image reconstruction of two-dimensional impenetrable objects using genetic algorithm," *Journal of Electromagnetic Waves and Application*, vol. 13, pp. 95- 118, 1999.
- [8] F. Xiao and H. Yabe, "Microwave imaging of perfectly conducting cylinders from real data by micro genetic algorithm coupled with deterministic method." *IEICE Trans. Electron.* , vol. E81-C, pp. 784- 1792, Dec. 1998.
- [9] D. E. Goldberg, *Genetic Algorithm in Search, Optimization and Machine Learning*. Addison-Wesley, 1989.
- [10] F. M. Tesche, "On the inclusion of loss in time domain solutions of electromagnetic interaction problems," *IEEE Trans. Electromagn. Compat.*, vol. 32, pp. 1-4, 1990.