2002 3rd International Conference on Microwave and Millimeter Wave Technology Proceedings

Imaging Reconstruction of a Partially Immersed Perfect Conductor by the Genetic Algorithm

Chih-Ming Yang and Chien-Ching Chiu
Electrical Engineering Department, Tamkang University
Tamsui, Taiwan, China
Email: Chiu@ee.tku.edu.tw

Abstract -The problem of determining the shape of a perfectly conducting cylinder which is partially immersed in the half-space by the genetic algorithm is investigated. Assume that a conducting cylinder of unknown shape partially immersed in the half-space and scatters the incident field. Based on the measured scattered field and boundary condition the, a set of nonlinear integral equations is derived and the imaging problem is reformulated into an optimization problem.

Introduction

The image problem of conducting objects has been a subject of considerable importance in noninvasive measurement, medical imaging, and biological application. In the past 20 years, many rigorous methods have been developed to solve the exact equation. However, inverse problem of this type are difficult to solve because they are ill-posed and nonlinear. As a result, many inverse problems are reformulated as optimization problems. General speaking, two main kinds of approaches have been developed. The first is based on gradient search approach such as the Newton-Kantorovitch method [1], the Levenberg-Marguart algorithm [2] and the successive-overrelaxation method [3]. This method is highly dependent on the initial guess and tends to get trapped in a local extreme. In contrast, the second approach is based on the genetic algorithm [4]. It usually converges to the global extreme of the problem, no matter what the initial estimate is.

Theoretical Formulation

Let us consider a perfectly conducting cylinder which is partially immersed in a lossy homogeneous half-space, as shown in Fig 1. Media in regions 1 and 2 are characterized by permittivity and conductivity (ε_1,σ_1) and (ε_2,σ_2) respectively. The metallic cylinder with cross section described in polar coordinates in x-y plane by the equation $\rho = F(\theta)$ is illuminated by transverse magnetic (TM) waves. We assume that time dependence of the field is harmonic with the factor $\exp(j\omega t)$. Let E_i denote the incident field from region 1 with incident angle ϕ_1. A reflected wave (for $y \le 0$) and a transmitted wave (for $y > 0$).

$$E_i(x,y)=\begin{cases} E_1(x,y)=e^{-jk_1[x\sin\phi_1+(y+a)\cos\phi_1]} & ,y\le 0 \\ \qquad +\mathrm{R}e^{-jk_1[x\sin\phi_1-(y+a)\cos\phi_1]} & \\ E_2(x,y)=Te^{-jk_2[x\sin\phi_2+(y+a)\cos\phi_2]} & ,y>0 \end{cases} \tag{1}$$

where

$$R=\frac{1-n}{1+n},T=\frac{2}{1+n},n=\frac{\cos\phi_2}{\cos\phi_1}\sqrt{\frac{\varepsilon_2-j\sigma_2/\omega}{\varepsilon_1-j\sigma_1/\omega}}$$

$$k_1\sin\phi_1=k_2\sin\phi_2 \qquad \text{(Snell's law)}$$

$$k_i^2=\omega^2\varepsilon_i\mu_0-j\omega\mu_0\sigma_i,\ \mathrm{Im}(k_i)\le 0$$

For a TM incident .wave, the scattered field can be expressed as

$$E_s(\bar r)=-\int_0^{\pi}G_1(\bar r,F(\theta'),\theta')J(\theta')d\theta'-\int_{\pi}^{2\pi}G_2(\bar r,F(\theta'),\theta')J(\theta')d\theta' \tag{2}$$

with

$$J(\theta)=-j\omega\mu_0\sqrt{F^2(\theta)+F'^2(\theta)}J_s(\theta)$$

$$G_1(x,y;x',y')=\begin{cases} G_{21}(x,y;x',y') & ,y>0 \\ G_{11}(x,y;x',y')=G_{f11}(x,y;x',y') & \\ \qquad +G_{s11}(x,y;x',y') & ,y\le 0 \end{cases} \tag{3}$$

where

$$G_{21}(x,y;x',y')=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{j}{\gamma_1+\gamma_2}e^{-j\gamma_2 y}e^{j\gamma_1 y'}e^{-j\alpha(x-x')}d\alpha \tag{3a}$$

$$G_{f11}(x,y;x',y') = \frac{j}{4} H_0^{(2)}[k_1\sqrt{(x-x')^2+(y-y')^2}] \quad (3b)$$

$$G_{s11}(x,y;x',y') = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{j}{2\gamma_1}\left(\frac{\gamma_1-\gamma_2}{\gamma_1+\gamma_2}\right) e^{j\gamma_1(y+y')} e^{-j\alpha(x-x')} d\alpha \quad (3c)$$

$$\gamma_i^2 = k_i^2 - \alpha^2, i=1,2, \mathrm{Im}(\gamma_i) \le 0, y' < 0$$

$$G_2(x,y;x',y') = \begin{cases} G_{12}(x,y;x',y') & ,y \le 0 \\ G_{22}(x,y;x',y') = G_{f22}(x,y;x',y') + G_{s22}(x,y;x',y') & ,y > 0 \end{cases} \quad (4)$$

where

$$G_{12}(x,y;x',y') = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{j}{\gamma_1+\gamma_2} e^{j\gamma_1 y} e^{-j\gamma_2 y'} e^{-j\alpha(x-x')} d\alpha \quad (4a)$$

$$G_{f22}(x,y;x',y') = \frac{j}{4} H_0^{(2)}[k_2\sqrt{(x-x')^2+(y-y')^2}] \quad (4b)$$

$$G_{s22}(x,y;x',y') = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{j}{2\gamma_2}\left(\frac{\gamma_2-\gamma_1}{\gamma_2+\gamma_1}\right) e^{-j\gamma_2(y+y')} e^{-j\alpha(x-x')} d\alpha \quad (4c)$$

$$\gamma_i^2 = k_i^2 - \alpha^2, i=1,2, \mathrm{Im}(\gamma_i) \le 0, y' > 0$$

Here $J_s(\theta)$ is the induced current density which is proportional to the normal derivative of electric field on the conductor surface. Note that G_1 and G_2 denote the Green's function for the line source in the region 1 and region 2 respectively. $H_0^{(2)}$ is the Hankel function of the second kind of order zero. The boundary condition on the surface of the scatter states that the total tangential electric field must be zero and this yield an integral equation for $J(\theta)$:

$$\begin{aligned} E_1(\bar{r}) &= \int_0^{\pi} G_{12}(\bar{r}, F(\theta'), \theta') J(\theta') d\theta \\ &\quad - \int_{\pi}^{2\pi} G_{11}(\bar{r}, F(\theta'), \theta') J(\theta') d\theta' \qquad , y < 0 \\ E_2(\bar{r}) &= \int_0^{\pi} G_{22}(\bar{r}, F(\theta'), \theta') J(\theta') d\theta' \\ &\quad - \int_{\pi}^{2\pi} G_{21}(\bar{r}, F(\theta'), \theta') J(\theta') d\theta' \qquad , y > 0 \end{aligned} \quad (5)$$

For the direct scattering problem, the scattered field E_S, is calculated by assuming that the shape are known. This can be achieved by first solving J in (4) and calculating E_S in (2). The shape $F(\theta)$ function can be expanded as:

$$F(\theta) = \sum_0^{N/2} B_n \cos(n\theta) + \sum_{n=1}^{N/2} C_n \sin(n\theta) \quad (6)$$

where B_n and C_n are real coefficient to be determined, and N+1 is the number of unknowns for shape function. The genetic algorithm is used to minimize the following cost function:

$$CF = \left\{ \frac{1}{M_t} \sum_{m=1}^{M_t} \left| E_s^{\exp}(\bar{r}_m) - E_s^{cal}(\bar{r}_m) \right|^2 / \left| E_s^{\exp}(\bar{r}_m) \right|^2 + \alpha \left| F'(\theta) \right|^2 \right\}^{1/2} \quad (7)$$

where M_t is the total number of measured points. $E_s^{\exp}(\bar{r})$ and $E_s^{cal}(\bar{r})$ are the measured scattered field and the calculated scattered field respectively. The minimization of $\alpha|F'(\theta)|^2$ can be interpreted as the smoothness requirement for the boundary of $F(\theta)$. The basic GA for which a flowchart is shown in Fig. 2.

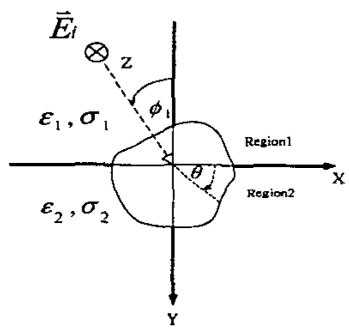

Fig. 1 Geometry of the problem in (x,y) plane

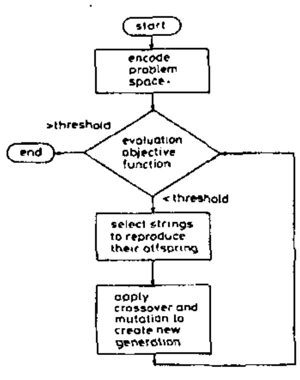

Fig. 2 Flow chart for the genetic algorithm

Numerical Results

Let us consider an perfectly conducting cylinder which is partially immersed in a half-space ($\sigma_1 = \sigma_2 = 0$). The permittivity in region 1 and region 2 is characterized by $\varepsilon_1 = \varepsilon_0$ and $\varepsilon_2 = 2.56\varepsilon_0$ respectively. The frequency of the incident wave are chosen to be 1 GHz and their incident angles (ϕ_1) are 45° and 315° . Eight points with radius of 3m in region 1 are measured for each incident wave. As a result, there are totally 16 measurement points in each simulation. The object is between the region 1 and region 2.

In the first example, the shape function is chosen to be $F(\theta) = 0.1 + 0.02\cos 2\theta + 0.02\sin\theta - 0.025\sin 2\theta$ m. The reconstructed shape function for the best population member is plotted in Fig. 3(a) with the error shown in Fig. 3(b).In the second example, the shape function is chosen to be $F(\theta) = 0.05 - 0.01\cos 3\theta + 0.01\sin 3\theta$ m. Good results are obtained in Fig. 4(a) and Fig. 4(b).

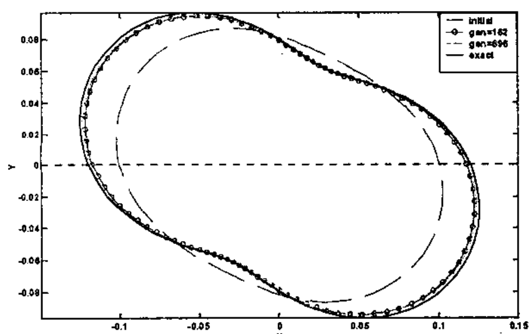

Fig. 3(a) Shape function. The solid curve represents the exact shape, and the others represent the best shape function of each generation.

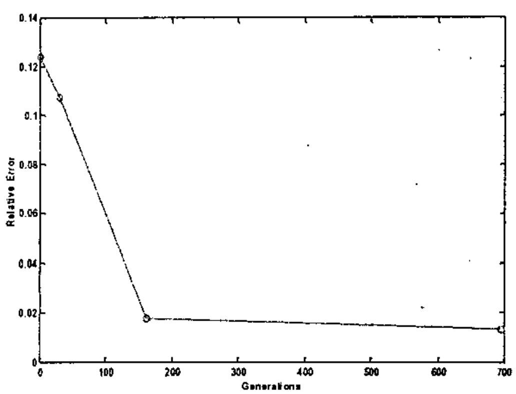

Fig. 3 (b) Shape function error

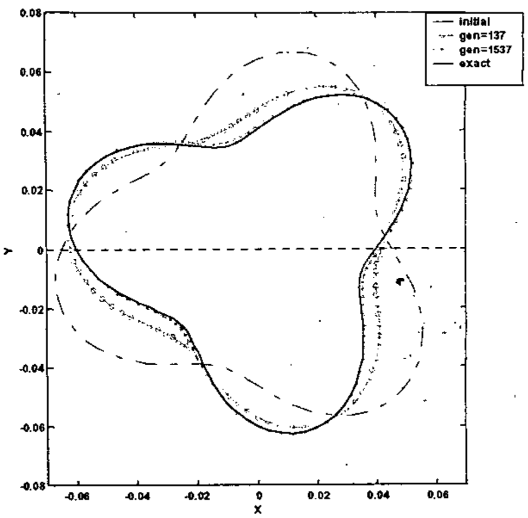

Fig. 4(a) Shape function. The solid curve represents the exact shape, the solid curve represents the exact shape, and the others represent the best shape function of each generation.

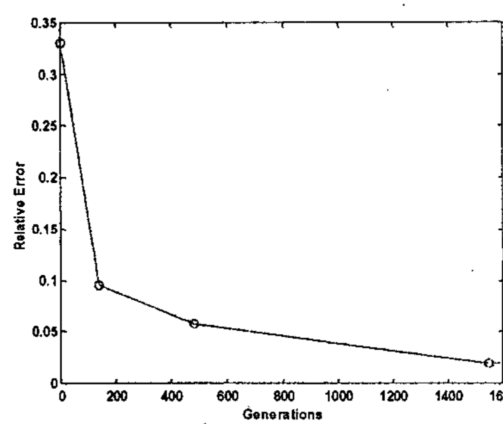

Fig. 4(b) Shape function error

Conclusions

We have presented a study of applying the genetic algorithm to reconstruct the shape of partially immersed conductor through knowledge of scattered field. Based on the boundary condition and measured scattered field, we have derived a set of nonlinear integral equations and reformulated the imaging problem into an optimization problem. By using the genetic algorithm, the shape of the object can be reconstructed from the scattered fields. Numerical results also illustrate that the shape function error decreases with the generation increases.

References

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