# Comparison of Two Different Shape Descriptions in the Half-Space Inverse Problem

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*Abstract* — **The problem of determining the two different shape description of a perfectly conducting cylinder buried in a half-space by the genetic algorithm is investigated. Assume that a cylinder of unknown shape is buried in one half-space and scatters the field incident from another half-space where the scattered field is measured. Based on the boundary condition and the measured scattered field, a set of nonlinear integral equations is derived and the imaging problem is reformulated into an optimization problem. As a result, the shape of the scatterer which is described by using cubic-spline can be reconstructed. In such a case, fourier series expansion will fail.**

*Index Terms* — **Inverse Problem, Cubic-spline, Fourier series.**

## I. INTRODUCTION

Due to large domain of applications such as non-destructive problem, geophysical prospecting and determination of underground tunnels and pipelines, etc, the inverse scattering problems related to the buried bodies has a particular importance in the scattering theory. In the past 20 years, many rigorous methods have been developed to solve the exact equations. However, inverse problems of this type are difficult to solve because they are illposed and nonlinear. As a result, many inverse problems are reformulated into optimization ones and then numerically solved by iterative methods such as Newton-Kantorovitch method [1], [2]. Recently, researchers have applied GA together with electromagnetic solver to attack the inverse scattering problem mainly in two ways. One is surface reconstruction approach, Chiu [3], [4] first applied the GA for the inversion of a perfectly conducing cylinder with the geometry described by a Fourier series (surface reconstruction approach), the other is volume reconstruction approach [5]. The 2-D perfectly conducting cylinders are denoted by local shape functions $\rho = F(\theta)$ with respect to their local origins which can be continuous or discrete. However, to the best of our knowledge, there are still no numerical results which compared the cubic-spline and Fourier-series shape description with the genetic algorithm for the buried conducting scatterers. In this paper, we present a computational method based on the genetic algorithm to recover the shape of a buried cylinder. In Section II, a theoretical formulation for the inverse scattering and the general principles of genetic algorithms are described. Numerical results for reconstructing objects of different shapes are given in Section III. Finally, some conclusions are drawn in Section IV.

## II. THEORETICAL FORMULATION

Let us consider a perfectly conducting cylinder buried in a lossy homogeneous half-space, as shown in Fig 1. Media in regions 1 and 2 are characterized by permittivity and conductivity $(\varepsilon_1, \sigma_1)$ and $(\varepsilon_2, \sigma_2)$ respectively. The metallic cylinder with cross section described in polar coordinates in xy plane by the equation $\rho = F(\theta)$ is illuminated by transverse magnetic (TM) waves. We assume that time dependence of the field is harmonic with the factor exp( $j\omega t$ ). Let $E^{inc}$ denote the incident field from region 1 with incident angle $\phi_1$. A reflected wave (for $y \le -a$) and a transmitted wave (for $y > -a$).

$$E_i(x,y) = \begin{cases} E_1(x,y) = e^{-jk_1[x\sin\phi_1 + (y+a)\cos\phi_1]} + R_1 e^{-jk_1[x\sin\phi_1 - (y+a)\cos\phi_1]} & , y \le -a \\ E_2(x,y) = Te^{-jk_2[x\sin\phi_2 + (y+a)\cos\phi_2]} & , y > -a \end{cases}$$

$$R_1 = \frac{1-n}{1+n}, T = \frac{2}{1+n}, n = \frac{\cos\phi_2}{\cos\phi_1}\sqrt{\frac{\varepsilon_2 - j\sigma_2/\omega}{\varepsilon_1 - j\sigma_1/\omega}}$$

$$k_1 \sin\phi_1 = k_2 \sin\phi_2$$

$$k_i^2 = \omega^2 \varepsilon_i \mu_0 - j\omega\mu_0\sigma_i, i = 1,2 \quad \mathrm{Im}(k_i) \le 0$$

For a TM incident wave, the scattered field can be expressed as

$$E_s(x,y) = -\int_0^{2\pi} G(x,y;F(\theta'),\theta')J(\theta')d\theta' \qquad (1)$$

where

$$J(\theta) = -j\omega\mu_0\sqrt{F^2(\theta) + F'^2(\theta)}J_s(\theta)$$

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$$G(x,y,x',y')=\begin{cases} G_1(x,y,x',y') & ,y\le -a \\ G_2(x,y,x',y')=G_f(x,y,x',y')+G_s(x,y,x',y'), & y>-a \end{cases} \quad (2)$$

where

$$G_1(x,y,x',y')=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{j}{\gamma_1+\gamma_2}e^{j\gamma_1(y+a)}e^{-j\gamma_2(y'+a)}e^{-j\alpha(x-x')}d\alpha \quad (2a)$$

$$G_f(x,y,x',y')=\frac{j}{4}H_0^{(2)}[k_2\sqrt{(x-x')^2+(y-y')^2}] \quad (2b)$$

$$G_s(x,y,x',y')=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{j}{2\gamma_2}\left(\frac{\gamma_2-\gamma_1}{\gamma_2+\gamma_1}\right)e^{-j\gamma_2(y+2a+y')}e^{-j\alpha(x-x')}d\alpha \quad (2c)$$

$$\gamma_i^2=k_i^2-\alpha^2, i=1,2, \mathrm{Im}(\gamma_i)\le 0, y'>a$$

Here $J_s(\theta)$ is the induced surface current density which is proportional to the normal derivative of electric field on the conductor surface. $H_0^{(2)}$ is the Hankel function of the second kind of order zero. The boundary condition at the surface of the scatterer given by [5] then yield an integral equation for $J(\theta)$:

$$E_2(F(\theta),\theta)=\int_0^{2\pi}G_2(F(\theta),\theta;F(\theta'),\theta')J(\theta')d\theta' \quad (3)$$

For the direct scattering problem, the scattered field, $E_s$, is calculated by assuming that the shape and the conductivity of the object are known. This can be achieved by first solving $J$ in (3) and calculating $E_s$ in (1).

Let us consider the following inverse problem, given the scattered electric field $E_s$ measured outside the scatterer, and determine the shape $F(\theta)$ of the object.

(A) Using Fourier-series to describe the shape:

Assume the approximate center of the scatterer, which in fact can be any point inside the scatterer, is known. Then the shape function $F(\theta)$ can be expanded as:

$$F(\theta)\cong\sum_{n=0}^{\frac{N}{2}}B_n\cos(n\theta)+\sum_{n=1}^{\frac{N}{2}}C_n\sin(n\theta) \quad (4)$$

where $B_n$, and $C_n$,are real coefficients to be determined, and $N+1$ is the number of expanded terms.

(B) Using Cubic-spline to describe the shape:

The geometry of the cubic-spline is shown in Fig. 2. First, we separate the boundary of the shape with $N$ pieces and we have $N+1$ separated points. We denote the separated points by polarized-coordinate expression $(\rho_0,\theta_0)$, $(\rho_1,\theta_1)$ ,…, $(\rho_N,\theta_N)$, where $(0°\le\theta_i\le 360°)$,i=0…N, $\theta_0=0°$, $\theta_N=360°$ and. $\theta_0<\theta_1<\ldots<\theta_N$ $\rho_i$ is the distance from point $(\rho_i,\theta_i)$ to the center point $(x_0,y_0)$

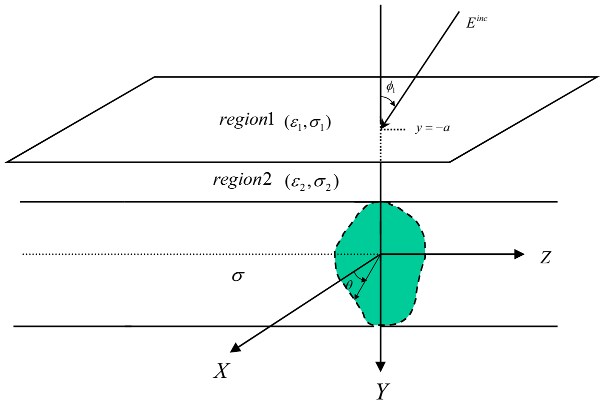


Fig. 1 Geometry of the problem in (x,y) plane

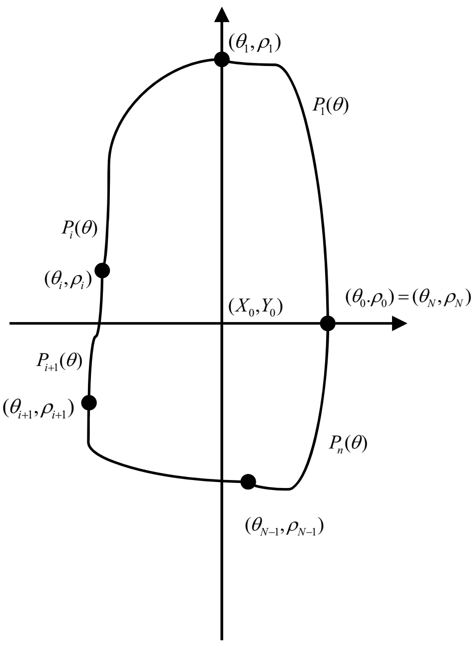


Fig. 2 Geometry of the cubic-spline

The genetic algorithm is used to minimize the following cost function:

$$SF=\left\{\sum_{x=1}^{X_t}\left|E_s^{\exp}(\vec{r}_x)-E_s^{cal}(\vec{r}_x)\right|^2\Big/\sum_{x=1}^{X_t}\left|E_s^{\exp}(\vec{r}_x)\right|^2+\beta\left|F'(\theta)\right|^2\right\}^{-1/2} \quad (5)$$

where $X_T$ is the total number of measured points. $E_s^{\exp}(\bar{r})$ and $E_s^{cal}(\bar{r})$ are the measured scattered field and the calculated scattered field respectively. The minimization of $\beta|F'(\theta)|^2$ can, to a certain extent, be interpreted as the smoothness requirement for the boundary of $F(\theta)$. The basic GA for which a flowchart is shown in Fig. 3 starts with a large population containing a total of X candidates. Each candidate is described by a chromosome. Then the initial population can simply be created by taking X random chromosomes. Finally, the GA iteratively generates a new population which is derived from the previous population through the application of the reproduction, crossover, and mutation operators.

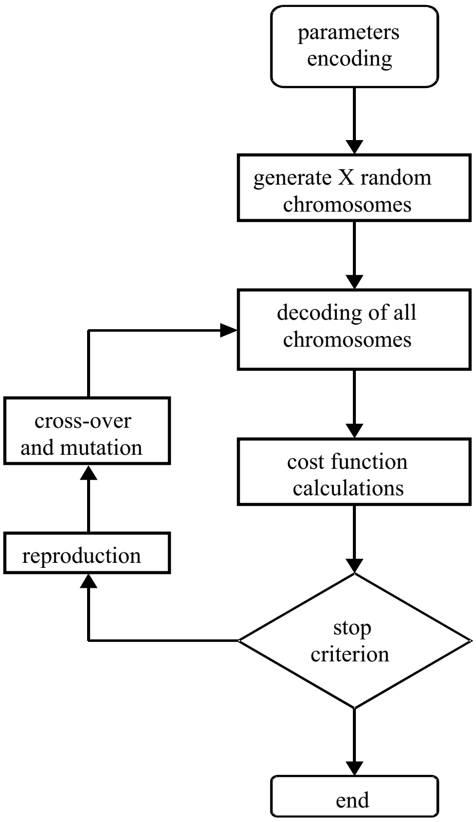


Fig. 3 The flowchart of GA

## III. NUMERICAL RESULT

Let us consider a perfectly conducting cylinder buried in a lossless half-space ( $\sigma_1 = \sigma_2 = 0$ ). The permittivities in region 1 and region 2 are characterized by $\varepsilon_1 = \varepsilon_0$ and $\varepsilon_2 = 2.56\varepsilon_0$ respectively. A TM polarization plane wave of unit amplitude is incident from region 1 upon the object in region 2 as shown in Fig. 1. The frequency of the incident wave is chosen to be 3GHz, of which the wavelength $\lambda_0$ in free space is 0.1m. The object is buried at a depth $a \cong \lambda_0$ and the scattered fields are measured on a probing line along the interface between region 1 and region 2. Our purpose is using the Fourier-series and cubic-spline shape expressions to reconstruct the shape and comparing which is better in the inverse problem. The object is illuminated by three incident waves from different directions, while 20 measurement points at equal spacing are used along the interface $y = -a$ for each incident angle. There are 60 measurement points in each simulation. The measurement is taken from $x$ =0 to 0.2m for incident angle $\phi_1 = -60°$ , from $x$ =-0.1 to 0.1m for incident angle $\phi_1 = 0°$ , and from $x$ =-0.2 to 0m for incident angle $\phi_1 = 60°$ . To save computing time, the number of unknowns is set to be 7, and the population size is chosen as 300. The binary string length of the unknown coefficient, $B_n$ ($C_n$, and $\rho_i$), is set to 20 bit (i.e., L=20). The search range for the unknown coefficient of the shape function is chosen to be from 0 to 0.1. The extreme values of the coefficients of the shape function and can be determined by some priori knowledge of the objects. Here, the prior knowledge means that we can get the approximate position and the size of the buried cylinder by first using tomography technique, and then get the exact solution by the genetic algorithm. The crossover probability $p_c$ and mutation probability $p_m$ are set to be 0.8 and 0.1, respectively.

In the first example, the shape function is given by $F(\theta) = (0.03 + 0.015\cos 3\theta)$ m and we use Fourier-series and cubic-spline expressions to recover it. The reconstructed shape function for the best population member (chromosome) is plotted in Fig. 4. From Fig. 4, it is clear that reconstruction of the shape function is quite good for both Fourier-series and cubic-spline expressions

In the second example, we selected cubic-spline to describe the shape $\rho_1 = 0.02$ m, $\rho_2 = 0.02\sqrt{3}$ m, $\rho_3 = 0.02\sqrt{3}$ m, $\rho_4 = 0.02$ m, $\rho_5 = 0.02\sqrt{3}$ m, $\rho_6 = 0.02\sqrt{3}$ m. We can see that the 7-terms Fourier-series expression can not recover the shape. The purpose of this example is to show that cubic-spline method is able to reconstruct a scatter while the Fourier-series fails. The shape results are shown in Fig. 5.

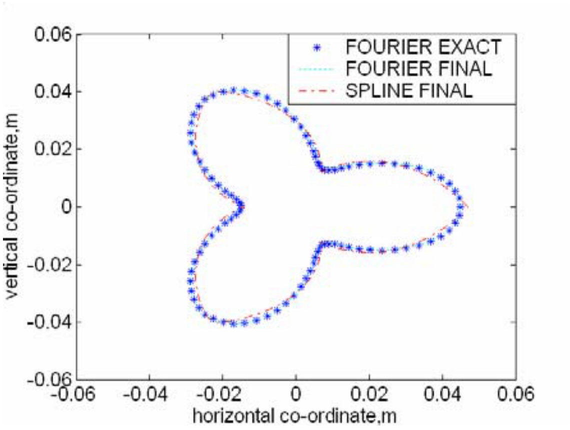


Fig. 4 Shape function for example 1. The star curve represents the exact shape by the Fourier-series, while the curve of short imaginary line is calculated shape by the Fourier-series and the curve of long imaginary line represents calculated shape by the cubic-spline in final result

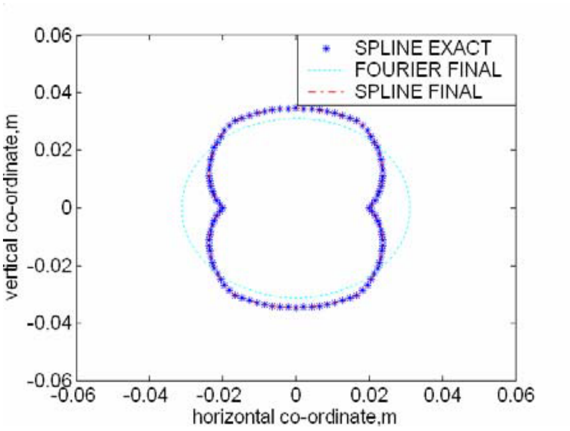


Fig. 5 Shape function for example 2. The star curve represents the exact shape by the Fourier-series, while the curve of short imaginary line is calculated shape by the Fourier-series and the curve of long imaginary line represents calculated shape by the cubic-spline in final result.

## III. CONCLUSION

We have presented a study of applying the genetic algorithm to reconstruct the shape of a buried metallic object through the measured of scattered $E$ fields. Based on the boundary condition and the measured scattered fields, we have derived a set of nonlinear integral equations and reformulated the imaging problem into an optimization one. The contours of the cylinders are denoted by cubic-spline local shape functions in local polar coordinate instead of trigonometric series local functions to guarantee the nonnegative definiteness. Experiment results show that the variable searching ability of GA has it's limitation, and Fourier-series expression can not recover the arbitrary shape in finite terms. In our numerical results, it is shown that using cubic-spline expand to describe the shape in the half-space inverse problem is more suitable than Fourier-series expression.

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