Modeling Reconstruction of Coupled Transmission Lines Using Time-Domain Characterization in High-Speed Digital System

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Abstract —In this paper, a novel method is developed to reconstruct the physically structures of a nonuniform couple transmission lines from layer peeling algorithm and genetic algorithm. Base on the time domain reflection (TDR) measurement, the impedance profile of the device under test (D.U.T) is first derived by layer peeling transmission line synthesis. Then, the genetic algorithm (G.A.) is employed to extract the parameter of the lumped/distributed circuits in high-speed digital circuit. As a result, the system characteristic can be easily obtained by the extracted model and SPICE circuit simulation software.

I. INTRODUCTION

Recently, interconnection lines on integrated circuit (IC), multichip module (MCM), and printed circuit board (PCB) have become critical elements for determining the performance of current high-speed integrated circuits and systems. As low-swing components, clock rates and bus speeds have increased dramatically, packaging and interconnections have greater importance, and in some cases they actually limit the system performance. Due to high-speed signal propagating on these interconnections, electrical design issues such as signal integrity, delay, via-holes and cross-talk become critical. In order to accurately model the effect of interconnection in high-speed system, EDA simulation software requires more accurate equivalent circuit model of component to eliminate the debugging procedure and reduce the EMI/SI problem, as well as to shorten the circuit design cycle.

Two methods which are in frequency domain and time domain measurement are currently available for extracting the equivalent electrical parameters of interconnection based on measurement data. Base on vector network analyzer (V.N.A), the problem deals with in frequency domain is to develop an equivalent circuit of interconnection, which was approximated the correct frequency domain measurement data and optimized to fit the corresponding coefficients. Particularly, an overly simplified model gives rise to nonphysical phenomenon.

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In this paper, we use excitation of odd-mode and even-mode to reconstruct the equivalent circuit model of nonuniform couple transmission lines, the interconnecting discontinuity in high-speed digital circuit is investigated; the parameters of equivalent circuit model will be generated by layer peeling transmission line synthesis [1]-[2] and genetic algorithms [3]-[4], based on the measured data from TDR [5].The extraction model parameters combined into SPICE circuit simulation, one can easily obtain system characteristic. The results of this study will help in understanding the effects of the characteristic of the interconnection discontinuity on system performance, and reduce the undesirable ring and/or delay in time.

II. THEORY

Before describing the method for extracting a model for an actual IC package, it is useful to first develop the method for the simpler cases of an D.U.T to ground. TDR measurement system is used to get the transient response (V_{tdr}) of the unknown circuits. Once V_{tdr} is obtained, the characteristic impedance profile of the D.U.T is firstly derived by layer peeling transmission line synthesis, then the genetic algorithm is used to find the parameters of the lumped circuit in the D.U.T.

Let the nonuniform line be represented by a series of N+1 uniform impedance section of equal electrical length, as shown in Fig.(1). Each section is characterized by its impedance Z(x), current I(x,t), and voltage V(x,t), then the incident waves and reflected waves can be expressed as follows[6]:

$$a(x,t)=\frac{1}{2}\left[\frac{V(x,t)}{\sqrt{Z(x)}}+I(x,t)\sqrt{Z(x)}\right] \quad (1)$$

$$b(x,t)=\frac{1}{2}\left[\frac{V(x,t)}{\sqrt{Z(x)}}-I(x,t)\sqrt{Z(x)}\right] \quad (2)$$

By the continuity of voltage and current at $x=x_i$ we have

$$\sqrt{Z_{i-1}}(a_i^- + b_i^-)=\sqrt{Z_i}(a_i^+ + b_i^+) \quad (3)$$

$$\sqrt{Z_i}(a_i^- - b_i^-)=\sqrt{Z_{i-1}}(a_i^+ - b_i^+) \quad (4)$$

where a_i^- and a_i^+ are the incident waves at the interface of the ith section and $(i+1)$th section respectively, and the reflected waves b_i^- and b_i^+ are similarly represented. We solve for the wave variables a_i^+ and b_i^+ as:

$$\begin{bmatrix} a_i^+ \\ b_i^+ \end{bmatrix} = \frac{1}{\sqrt{1-S_i^2}} \begin{bmatrix} 1 & -S_i \\ -S_i & 1 \end{bmatrix} \begin{bmatrix} a_i^- \\ b_i^- \end{bmatrix} \quad (5)$$

where S_i is the reflection coefficient. Let the incident waves and reflected waves at x_i be defined as the piecewise constant functions; then we can represent (1) and (2) in the first section at x_1 as follows:

$$a_{1,j}^- = \frac{1}{2}\left[\frac{V(2(j-1)\Delta t)}{\sqrt{Z_0}} + I(2(j-1)\Delta t)\sqrt{Z_0}\right] \quad (6)$$

$$b_{1,j}^- = \frac{1}{2}\left[\frac{V(2(j-1)\Delta t)}{\sqrt{Z_0}} - I(2(j-1)\Delta t)\sqrt{Z_0}\right] \quad (7)$$

here j=1,2,3,...N, z_0 is the source impedance and $\Delta t = \Delta x / c$, c being the wave propagation velocity; The time interval (2Δt) is because a change in the reflected wave caused by the junction $i+1$ occurs no sooner than 2Δt after that due to junction i. We can represent (5) as following:

$$\begin{bmatrix} a_{i,j}^+ \\ b_{i,j}^+ \end{bmatrix} = \frac{1}{\sqrt{1-S_i^2}} \begin{bmatrix} 1 & -S_i \\ -S_i & 1 \end{bmatrix} \begin{bmatrix} a_{i,j}^- \\ b_{i,j}^- \end{bmatrix} \quad (8)$$

the relation of incident and reflected at the interface of x_i and x_{i+1} section

$$a_{i+1,j}^- = a_{i,j}^+ \qquad \text{for j=1,2,3}\ldots N-i \quad (9)$$
$$b_{i+1,j}^- = a_{i,j+1}^+$$

By using the prior equation and lattice diagram analysis Fig.(1), we can extract the nonuniform transmission line.

Genetic algorithms are the global numerical optimization methods based on genetic recombination and evolution in nature. They use the iterative optimization procedures that start with a randomly selected population of potential solutions, and then gradually evolve toward a better solution through the application of the genetic operators: reproduction, crossover and mutation operators. In this paper, genetic algorithm is used to find the parameters of the lumped circuit in the D.U.T by minimizing the following cost function:

$$CS = \left\{\frac{1}{K}\sum_{i=1}^{K}\left|V_{tdr}^{\exp}(t) - V_{tdr}^{cal}(t)\right|^2\right\}^{1/2} \quad (10)$$

where K is the total number of time steps of V_{tdr} measured by TDR. $V_{tdr}^{\exp}(t)$ and $V_{tdr}^{cal}(t)$ are the measured voltage and calculated voltage, respectively. To calibrate the multi-reflection effect in the TDR measurement data, the $V_{tdr}^{cal}(t)$ is recombined by $a_{i,j}$ and $b_{i,j}$ at discontinuity section x_i:

$$V_{tdr}^{cal}(j) = \left(\frac{a_{1,j}^-}{a_{i,j}^-}\right)\left(a_{i,j}^- + b_{i,j}^-\right)\sqrt{z_0} \quad (11)$$

In our problem, parameters L, Lm, C, Cm and R are coded by the following equations:

$$x = p_{\min} + \frac{(p_{\max} - p_{\min})}{2^l - 1} \times \sum_{i=0}^{l-1} b_i 2^i \quad (12)$$

Where x represents the value of our parameters L, Lm, C, Cm and R; b_i is the l-bit string of the binary representation of x; $p_{\min}$ and $p_{\max}$ are the minimum and maximum value admissible for x, respectively. Here, $p_{\min}$ and $p_{\max}$ can be

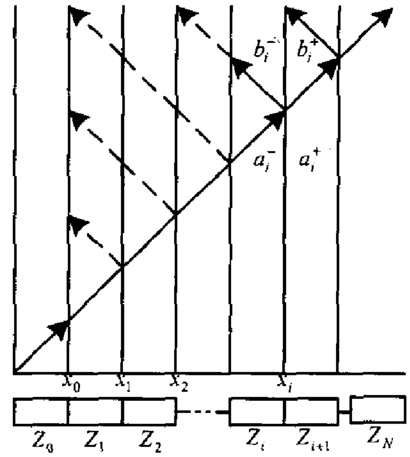


Fig.1 Lattice diagram analysis for cascaded transmission line

determined by experience and actual physics quantity in the high-speed digital circuit. Also, the finite resolution with which L, Lm, C, Cm, and R can be tuned in practice is reflected in the number of bits assigned to it. The total unknown coefficients in (11) would then be described by a $(n \times l)$ bit string (chromosome), where n is total number of unknown parameters in the equivalent circuit of lumped circuit in the D.U.T.. The G.A. starts with a large population containing a total of M candidates. Each candidate is described by a chromosome. Then the initial population can simply be created by taking M random chromosomes. Finally, the GA iteratively generates a new population, which is derived from the previous population through the application of the reproduction, crossover, and mutation operators[2]. The new populations will contain increasingly better chromosomes and will eventually converge to an optimal population that consists of the optimal chromosomes. As soon as the cost function satisfies our condition, the algorithm will be terminated and a solution is then obtained.

A lossless couple transmission lines circuit, as show in Fig.(2), the characteristic impedance of a transmission line can be described using its inductance and capacitance and conductance per unit length[6]:

$$Z = \sqrt{\frac{L}{C}} \quad (13)$$

The electrical length of such a line can be determined by using:

$$t = l\sqrt{LC} \quad (14)$$

Here l is the physical length of the line

we can use the even mode and odd mode excitation, as shown in Fig.(3). The LC matrices can be easily extracted from the even and odd TDR impedance profiles using:

$$L_s = \frac{1}{2\Delta l}(Z_{even}T_{even} + Z_{odd}T_{odd}) \quad (15)$$

$$L_m = \frac{1}{2\Delta l}(Z_{even}T_{even} - Z_{odd}T_{odd}) \quad (16)$$

$$C_s = \frac{1}{2\Delta l}\left(\frac{T_{odd}}{Z_{odd}} + \frac{T_{even}}{Z_{even}}\right) \quad (17)$$

$$C_m = \frac{1}{2\Delta l}\left(\frac{T_{odd}}{Z_{odd}} - \frac{T_{even}}{Z_{even}}\right) \quad (18)$$

The L_s, L_m, C_s, and C_m are self-inductance, mutual-inductance, self-capacitance and mutual- capacitance, respectively; the Z_{even}, Z_{odd}, T_{even}, and T_{odd} are even mode impedance, odd-mode impedance and even-mode time delay, respectively. Even-mode and odd-mode impedances for a differential pair can be computed using layer-peeling algorithm, Even-mode and odd-mode time delay for a differential pair can be obtained using SIPCE simulation output waveform. Finally, SPICE simulation is used to reconstruct time domain waveform to verify the accuracy of theorem.

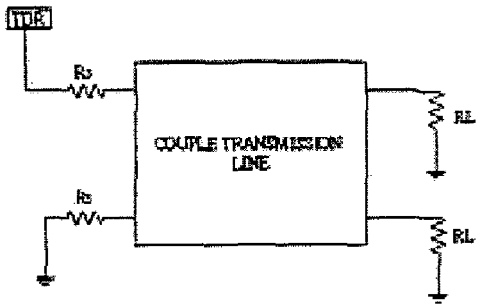
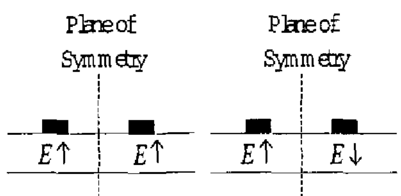


Fig.2 Couple lossless transmission line circuit



(a) even mode (b) odd mode

Fig.3 Even mode and odd mode excitation

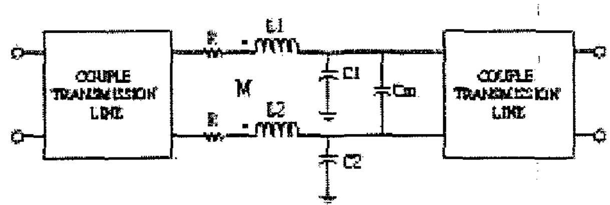
Ⅲ. NUMERICAL RESULTS

By numerical simulating, we illustrate the performance of the proposed inversion algorithm and its sensitivity to random error. Let us consider the models of interconnection discontinuity, as shown in Fig.(4). The impedance profile of distributed circuit in equivalent circuit model will be determined by layer peeling transmission line synthesis and the parameters (self inductance, mutual inductance, self capacitance, mutual capacitance and resistance) of the lumped circuit in equivalent circuit model will be determined by the genetic algorithm. We use TDR to measure those circuits .The input of TDR is a step-like signal with rising time 100ps, and its voltage at high state is 0.5V.

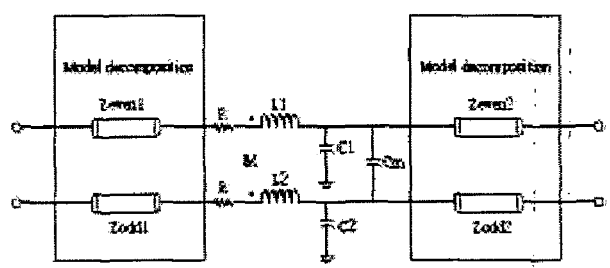
In the examples, the couple transmission lines can be easily analysis by the equation (15)-(18)[2][6][7]; the self inductance L_1=L_2=6nH, self capacitive C_1=C_2=3pF, mutual inductance L_m=3nH, mutual capacitive C_m=1pF and resistance R=0.05Ω are specified in Fig.(4). Fig.(5) shows the resultant impedance profile of equivalent circuit model ; It is seen that there is a discontinuous at 0.77ns by even-mode excitation and one at 0.69ns by odd-mode excitation. Note that, the time delay is twice of the specified value because of the round trip time. The characteristic impedance of Z_{even1} and Z_{even2} are all 56Ω shown in Fig. 5.(a), the characteristic impedance of Z_{odd1} and Z_{odd2} are all 46.5Ω shown in Fig. 5.(b).

To get the parameter of the lumped circuit of this interconnection discontinuity by genetic algorithm, we select the population size as 100 (i.e., M =100); the binary string length of those parameter are set to be 16 bit (i.e., l=16). Note that there are five unknown parameters (n=5) in Fig.(4) ; As a result, the bit number of a chromosome is 32 bits in Fig.(4). The search range for the unknown resistance R is chosen to be from 0 to 0.1Ω, the search range for the unknown self and mutual inductance is chosen to be from 0 to 10 nH, the search range for the unknown self and mutual capacitor is chosen to be from 0 to 10 pF. The extreme value of the coefficient of those parameters can be determined by the prior knowledge. The crossover probability and mutation probability are set to be 0.7 and 0.03, respectively. For the equivalent model in Fig. 4.(b), The measurement data of TDR and the reconstructed result are plotted in Fig. (6). The final solution of parameters are: L1=L2=5.899nH, C1=C2=3.089pF, L_m=2.89nH, C_m=0.98pF, R= 4.65×10^{-2} Ω, the R.M.S. error of the voltage is 3.856×10^{-2}. Here, the R.M.S. error for voltage of TDR is defined as:

$$RMSE=\left\{\frac{1}{K}\sum_{i=1}^{K}\frac{\left|V_{tdr}^{\exp}(t)-V_{tdr}^{cal}(t)\right|^2}{\left|V_{tdr}^{\exp}(t)\right|^2}\right\}^{1/2} \tag{19}$$



(a) couple lump equivalent circuit

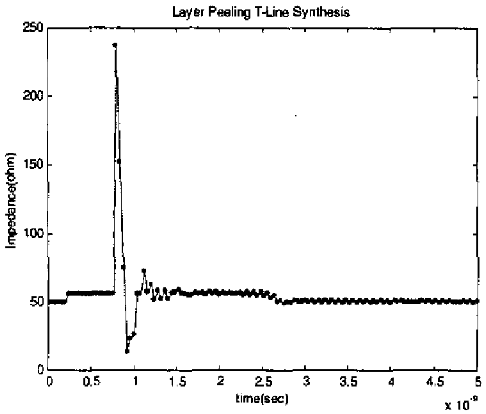


(b) model decomposition lump equivalent circuit

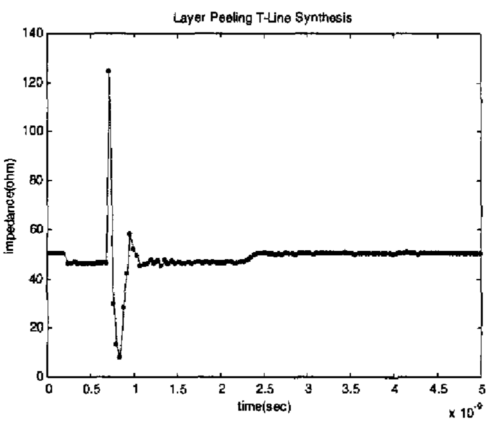
Fig.4 Lump equivalent circuit model

Ⅳ. CONCLUSION

Even-mode and odd-mode analysis can be utilized for characterization of lumped interconnect structures, even when single-ended signaling schemes are utilized. In this paper, we have presented a study that the novel algorithm and the even and odd mode analysis are applied to extract the parameters of equivalent circuit model of the D.U.T. by TDR measurement. Structures such as high speed connectors, ball-grid array packages and high performance automatic test equipment sockets can be easily modeled using even and odd TDR measurements. The resultant waveform in time domain got good agreement between specified circuit models and extracted one. Through the extraction modeling of interconnection, the design engineer will decide the proper stratagem to improve the signal quality and/or reduce the electromagnetic interference.



a) Impedance profile of the even mode excitation equivalent circuit model



b) Impedance profile of the odd mode excitation equivalent circuit model

Fig.5 Impedance profile of even and odd mode excitation equivalent circuit model

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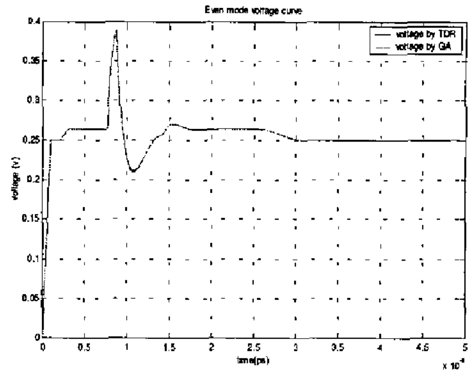
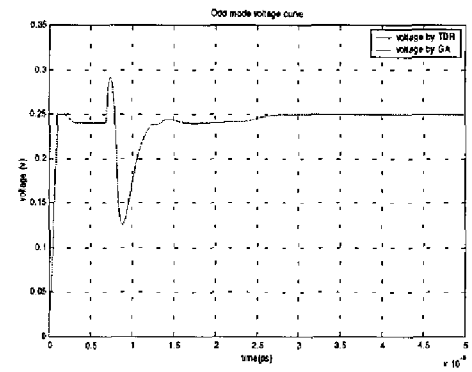


a) Even mode excitation



b) Odd mode excitation

Fig.6 Compare the measurement data of TDR with the reconstructed results by G.A.

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