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Image Reconstruction of Buried Uniaxial Dielectric Cylinders

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Abstract - The inverse scattering of buried inhomogeneous uniaxial dielectric cylinders is investigated. Dielectric cylinders of unknown permittivities are buried in one half space and scatter a group of unrelated waves incident from another half space where the scattered field is recorded. By proper arrangement of the various unrelated incident fields, the difficulties of ill-posedness and nonlinearity are circumvented, and the permittivity distribution can be reconstructed through simple matrix operations. The algorithm is based on the moment method and the unrelated illumination method. Numerical results are given to demonstrate the capability of the inverse algorithm. Good reconstruction is obtained even in the presence of additive random noise in measured data. In addition, the effect of noise on the reconstruction result is also investigated.

I .INTRODUCTION

In the last few years, electromagnetic inverse scattering problems of underground composite objects have been a growing importance in many different fields of applied science. However, the composite materials are electrically anisotropic. The permittivity of these materials depends on the chosen coordinates. This problem is more difficult and complex than that of isotropic materials.

In this paper, the inverse scattering of buried inhomogeneous uniaxial dielectric cylinders is investigated. An efficient algorithm is proposed to reconstruct the permittivity distribution of the objects by using only the scattered field measured outside. The algorithm is based on the unrelated illumination method [1] - [3].

II .THEORETICAL FORMULATION

Let us consider uniaxial dielectric cylinders buried in a lossless homogeneous half-space as shown in Fig. 1. Media in regions 1 and 2 are characterized by permittivities å$_1$ and å$_2$, respectively. The permeability is ì$_0$ for all material including the scatterers. The axis of the buried cylinder is the z-axis; that is, the properties of the scatterer may vary with the transverse coordinates only. The dielectric permittivities of the object are ε_a and ε_b in the x (or y) and z direction. The object is illuminated by the following two different polarized incident waves:

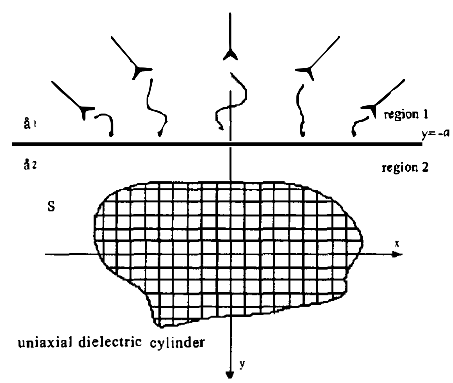


Fig. 1 Geometry of problem in the (x,y) plane

(1) TM Waves: A group of unrelated incident wave with electric field parallel to the z-axis (i.e., transverse magnetic) is illuminated upon the scatterers. Then the internal total electric field inside the uniaxial object, $\overline{E}(x,y) = E(x,y)\hat{z}$, can be expressed by the following integral equation:

$$E_z^i(\bar{r}) = \int_s G(r,r')k_2^2[\varepsilon_b(r') - 1]E(r')ds', \qquad y > -a \tag{1}$$

The scattered field, $\overline{E}_z(x,y) = E_z(x,y)\hat{z}$, can be expressed as

$$E_z^s(\bar{r}) = -\int_s G(r,r')k_2^2[\varepsilon_b(r') - 1]E_z(r')ds' \tag{2}$$

Here $G(r,r')$ is the half-space Green's function, which can be obtained by the Fourier transform [4].

(2) TE Waves: A group of unrelated incident wave with magnetic field parallel to the z-axis (i.e., transverse electric) is illuminated upon the object. By using Hertz vectorial potential techniques [3], the internal total electric field $\overline{E}(x,y) = E_x(x,y)\hat{x} + E_y(x,y)\hat{y}$ and the external scattered field, $\overline{E}^s(x,y) = E_x^s(x,y)\hat{x} + E_y^s(x,y)\hat{y}$ can be expressed by the following equations:

$$E_x(\bar{r}) = E_x^s(\bar{r}) + E_x^i(\bar{r}) \tag{3}$$

$$E_y(\bar{r}) = E_y^s(\bar{r}) + E_y^i(\bar{r}) \tag{4}$$

$$\begin{aligned} E_x^s(\bar{r}) = &-\left(\frac{\partial^2}{\partial x^2} + k_2^2\right)\left[\int_s G(\bar{r},\bar{r}')(\varepsilon_a(\bar{r}') - 1)E_x(\bar{r}')ds'\right] \\ &-\frac{\partial^2}{\partial x \partial y}\left[\int_s G(\bar{r},\bar{r}')(\varepsilon_a(\bar{r}') - 1)E_y(\bar{r}')ds'\right] \end{aligned} \tag{5}$$

$$\begin{aligned} E_y^s(\bar{r}) = &-\frac{\partial^2}{\partial x \partial y}\left[\int_s G(\bar{r},\bar{r}')(\varepsilon_a(\bar{r}') - 1)E_x(\bar{r}')ds'\right] \\ &-\left(\frac{\partial^2}{\partial y^2} + k_2^2\right)\left[\int_s G(\bar{r},\bar{r}')(\varepsilon_a(\bar{r}') - 1)E_y(\bar{r}')ds'\right] \end{aligned} \tag{6}$$

For the direct problem, the scattered field is computed by giving the permittivity distribution of the buried uniaxial objects. This can be achieved by using (1), (3) and (4) to solve the total field inside the materials $\overline{E}$ and calculating $\overline{E}^s$ by (2), (5) and (6).

We consider the following inverse problem: the permittivity distribution of the uniaxial dielectric objects is to be computed by the knowledge of the scattered field measured in region 1. Note that the only unknown permittivity in the TM case is $\varepsilon_b(r)$, and $\varepsilon_a(r)$ in the TE case. The moment method is used to solve Eqs. (1), (2), and (3) - (6) with a pulse basis function for expansion and point matching for testing [2]. In the inversion procedure, we choose N different incident column vectors for the TM case and $2N$ different incident column vectors for the TE case. Then [ô$_b$] and [ô$_t$] can be found by solving the following equations:

$$[\Psi_b][\tau_b] = [\Phi_b] \tag{7}$$

$$[\Psi_t][\tau_t] = [\Phi_t] \tag{8}$$

where

$$[\tau_t] = \begin{bmatrix} [\tau_a] & 0 \\ 0 & [\tau_a] \end{bmatrix}$$

$$[\Phi_b] = -[E_z^s][E_z^i]^{-1} \qquad [\Psi_b] = [E_z^s][E_z^i]^{-1}[G_2] + [G_1]$$

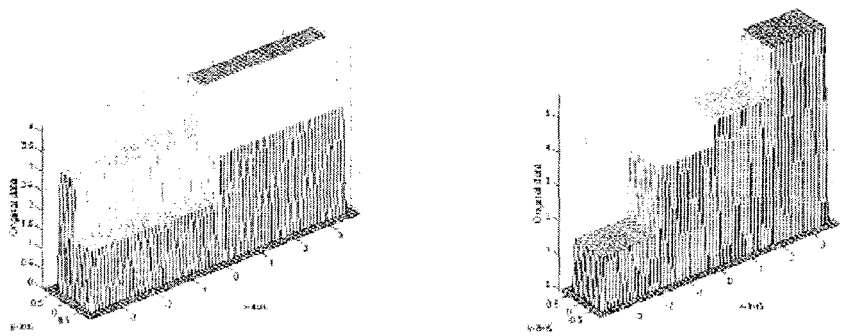
$$[\Phi_t] = -[E_t^s][E_t^i]^{-1} \qquad [\Psi_t] = [E_t^s][E_t^i]^{-1}[G_{t1}] + [G_{t2}]$$

Here $[E_z^i]$ and $[E_z]$ are both $N \times N$ matrices. $[E_z^s]$ is a $M \times N$ matrix. $[E_t^i]$ and $[E_t]$ are both $2N \times 2N$ matrices. $[E_t^s]$ is a $M \times 2N$ matrix. $[G_1]$ is a $N \times N$ matrix. $[G_2]$ is a $M \times N$ matrix. $[G_{t1}]$ is a $2N \times 2N$ matrix. $[G_{t2}]$ is a $M \times 2N$ matrix. $[\tau_a]$ and $[\tau_b]$ are $N \times N$ diagonal matrixes whose diagonal element are formed from the permittivities of each cell minus one. From (7) (or (8)), all the diagonal elements in the matrix $[\hat{o}_b]$ (or $[\hat{o}_a]$) can be determined by comparing the element with the same subscripts, which may be any row of both $[\Psi_b]$ (or $[\Psi_a]$) and $[\Phi_b]$ (or $[\Phi_a]$).Note that there are a total of M possible values for each element of $\hat{o}_b$. Therefore, the average value of these M data is computed and chosen as final reconstruction result in the simulation. Similarly, there are a total of 2M possible values for each element of $\hat{o}_a$. The average value of these 2M data is computed and chosen as final reconstruction result in the simulation.

Ⅲ. NUMERICAL RESULTS

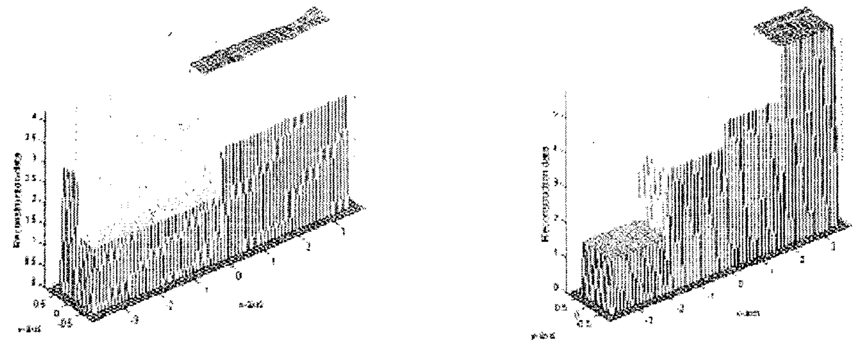
Let us consider a uniaxial dielectric cylinder buried at a depth of a = 4m in a lossless half space, as shown in Fig. 1. The permittivities in region 1 and 2 are characterized by $å_1 = å_0$ and $å_2 = 2.25å_0$. The frequency of the incident waves is chosen to be 30 MHz and the number of illuminations is the same as that of cells. The incident waves are generated by numerous groups of radiators operated simultaneously. Each group of radiators is restricted to transmit a narrow-bandwidth pattern that can be implemented by antenna array techniques. By changing the beam direction and tuning the phase of each group of radiators, one can focus all the incident beams in turn at each cell of the object. This procedure is named "beam focusing" [3]. Note that this focusing should be set when the scatterer is absent. Clearly, an incident matrix formed in this way is diagonally dominant and its inverse matrix exists. The measurement is taken on a half circle of radius 3m about (0, $-a$) at equal spacing. The number of measurement point is set to be 9 for each illumination. For avoiding trivial inversion of finite dimensional problems, the discretization number for the direct problem is four times that for the inverse problem in our numerical simulation.

In the example, the buried cylinder with a 7.2 × 1.2 m rectangular cross section is discretized into 24 × 4 cells, and the corresponding dielectric permittivities are plotted in Fig. 2. The model is characterized by simple step distribution of permittivity. Each cell has 0.3 × 0.3 m cross-sections. The reconstructed permittivity distributions of the object are plotted in Fig. 3. The root-mean-square (RMS) error is about 0.34 % and 0.86 % for the dielectric permittivity ε_a and ε_b receptively. It is clear that the reconstruction is good.



(a) (b)

Fig. 2 Original relative permittivity distribution. (a) $\varepsilon_a(x,y)$, (b) $\varepsilon_b(x,y)$.



(a) (b)

Fig. 3 Reconstructed relative permittivity distribution. (a) $\varepsilon_a(x,y)$, (b) $\varepsilon_b(x,y)$.

For investigating the effect of noise, we add to each complex scattered field a quantity b+cj, where b and c are independent random numbers having a uniform distribution over 0 to the noise level times the rms value of the scattered field. The noise levels applied include 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, and 10^{-1} in the simulations. The numerical results are plotted in Fig. 6. It shows the effect of noise is tolerable for noise levels below 1%.

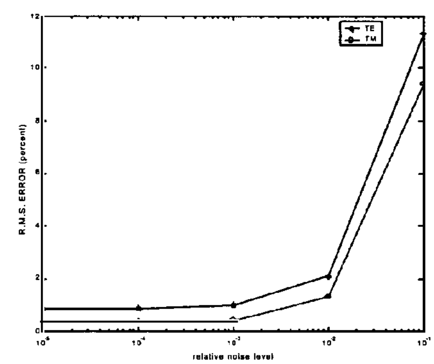


Fig. 6 Reconstructed error as a function of noise level

IV. CONCLUSIONS

An efficient algorithm for reconstructing the permittivity distribution of buried uniaxial dielectric cylinders has been proposed. By properly arranging the direction and the polarization of various unrelated waves, the difficulty of ill-posedness and nonlinearity is avoided. Thus, the permittivity distribution can be obtained by simple matrix operations. The moment method has been used to transform a set of integral equations into matrix form. Then these matrix equations are solved by the unrelated illumination method. Numerical simulation for imaging the permittivity distribution of a buried uniaxial dielectric object has been carried out and good reconstruction has been obtained even in the presence of random noise in measured data. This algorithm is very effective and efficient, since no iteration is required.

V. REFERENCES

[1] C. C. Chiu and P. T. Liu, "Image reconstruction of a complex cylinder illuminated by TE waves," *IEEE Trans. Microwave Theory Tech.*, Vol. 44, pp. 1921-1927, Oct. 1996

[2] C. C. Chiu and C. P. Huang, "Inverse scattering of dielectric cylinders buried in a half space," *Microwave and Optical Technology Letters*, Vol. 13, pp. 96-99, Oct. 1996.

[3] W. Wang and S. Zhang, "Unrelated illumination method for electromagnetic inverse scattering of inhomogeneous lossy dielectric bodies," *IEEE Trans. Antennas Propagat.*, Vol. AP-40, pp. 1292-1296, Nov. 1992.

[4] C. C. Chiu and Y. M. Kiang, "Inverse scattering of a buried conducting cylinder," *Inv. Prob.*, Vol.7, pp. 187-202, April 1991.