Image Reconstruction for Inhomogeneous Biaxial Dielectric Cylinders

Chien-Ching Chiu and Rong-Hwa Yang

Department of Electrical Engineering,Tamkang University, Tamsui,Taiwan, R.O.C.

Abstract The inverse scattering of inhomogeneous biaxial dielectric cylinders is investigated. Based on the properly arrangement of the direction and polarization of the incident field,a set of integral equations which can be solved by moment method and unrelated illumination method is derived. Numerical results are given to demonstrate the capability of the inverse algorithm.

I. Introduction

The electromagnetic inverse scattering of anisotropic objects have attracted increasing attention due to the development of compiste material. However, inverse scattering of this type is considerably more difficult than those of isotropic objects. This is due to the fact that the dielectric constant of anisotropic materials vary when the direction of the applied field changes. Hence, there is still no rigorous algorithm to handle this type of problems up to now. Most papers concerning the inverse scattering calculation dealt with the case of isotropic objects only. Generally speeking,two kinds of approaches have been developed. The first is an approximate approach [1]-[2]. It makes use of Fourier diffraction tomography to reconstruct the permittivities of dielectric objects. Since the Fourier diffraction tomography is generally based on the Born or the Rytov approximation, it usually fails when applied to the case of strong scattering. In contrast, the second approach is a rigorous one [3]-[6]. These techniques need no approximation in formula, but the calculation is more complex than the approximate approach stated above. In this paper, unrelated illumination method [6] is employed to reconstruct the permittivities of biaxial dielectric cylinders. In section II, the theoretical formulation for microwave imaging is briefly described. Numerical results are given in section III. Finally some conclusions are drawn in section IV.

II. Theoretical Formulation

Let us consider cylinderical biaxial dielectric objects in the free space,as shown in *Fig. 1*. The relative permittivity tensors $\overline{\overline{\varepsilon_r}}$ of the biaxial objects are characterized by a diagonal matrix in the Cartesian Coordinate system (x,y,z)

$$\overline{\overline{\varepsilon_r}}(x,y)=\begin{pmatrix} \varepsilon_1(x,y) & 0 & 0 \\ 0 & \varepsilon_2(x,y) & 0 \\ 0 & 0 & \varepsilon_3(x,y) \end{pmatrix}_{xyz}$$

The permeabilities of the objects are μ_0, i.e., non-magnetic objects are concerned here. The properties of the scatterers may vary with the transverse coordinates only. The scatterers are then illuminated by the following incident waves:

(i) TM (Transverse Magnetic) waves:

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Let $\overline{E}^i$ denote the incident field whose field vector is parallel to the z axis, i.e., $\overline{E}^i = E_z^i(\bar{r})\bar{z}$, where $\bar{r} = (x, y)$. Then the internal total field $\overline{E} = E_z\bar{z}$ and the external scattered field $\overline{E}^s = E_z^s\bar{z}$ can be written as follows:

$$E_z(\bar{r}) = \int_s G(\bar{r},\bar{r}')k_0^2(\varepsilon_3(\bar{r}') - 1)E_z(\bar{r}')ds' + E_z^i(\bar{r}) \tag{1}$$

$$E_z^s(\bar{r}) = \int_s G(\bar{r},\bar{r}')k_0^2(\varepsilon_3(\bar{r}') - 1)E_z(\bar{r}')ds' \tag{2}$$

where $G(\bar{r},\bar{r}') = -\frac{j}{4}H_0^{(2)}(k_0|\bar{r}-\bar{r}'|)$ and k_0 denotes the free-space wavenumber.

(ii) TE (Transverse Electric) waves:

Let $\overline{E}^i$ denote the incident field, $\overline{E}^i(\bar{r}) = E_x^i(\bar{r})\bar{x} + E_y^i(\bar{r})\bar{y}$, then the internal total field, $\overline{E}(\bar{r}) = E_x(\bar{r})\bar{x} + E_y(\bar{r})\bar{y}$, and the external scattered field, $\overline{E}^s(\bar{r}) = E_x^s(\bar{r})\bar{x} + E_y^s(\bar{r})\bar{y}$, can be written as follows:

$$E_x(\bar{r}) = \left(\frac{\partial^2}{\partial x^2} + k_0^2\right)\left[\int_s G(\bar{r},\bar{r}')(\varepsilon_1(\bar{r}') - 1)E_x(\bar{r}')ds'\right] + \frac{\partial^2}{\partial x \partial y}\left[\int_s G(\bar{r},\bar{r}')(\varepsilon_2(\bar{r}') - 1)E_y(\bar{r}')ds'\right] + E_x^i(\bar{r}) \tag{3}$$

$$E_y(\bar{r}) = \frac{\partial^2}{\partial x \partial y}\left[\int_s G(\bar{r},\bar{r}')(\varepsilon_1(\bar{r}') - 1)E_x(\bar{r}')ds'\right] + \left(\frac{\partial^2}{\partial y^2} + k_0^2\right)\left[\int_s G(\bar{r},\bar{r}')(\varepsilon_2(\bar{r}') - 1)E_y(\bar{r}')ds'\right] + E_y^i(\bar{r}) \tag{4}$$

$$E_x^s(\bar{r}) = \left(\frac{\partial^2}{\partial x^2} + k_0^2\right)\left[\int_s G(\bar{r},\bar{r}')(\varepsilon_1(\bar{r}') - 1)E_x(\bar{r}')ds'\right] + \frac{\partial^2}{\partial x \partial y}\left[\int_s G(\bar{r},\bar{r}')(\varepsilon_2(\bar{r}') - 1)E_y(\bar{r}')ds'\right] \tag{5}$$

$$E_y^s(\bar{r}) = \frac{\partial^2}{\partial x \partial y}\left[\int_s G(\bar{r},\bar{r}')(\varepsilon_1(\bar{r}') - 1)E_x(\bar{r}')ds'\right] + \left(\frac{\partial 2}{\partial y^2} + k_0^2\right)\left[\int_s G(\bar{r},\bar{r}')(\varepsilon_2(\bar{r}') - 1)E_y(\bar{r}')ds'\right] \tag{6}$$

For the direct problem, the scattered field is calculated by assuming that the permittivity tensor distribution of the scatterers are known. This can be achieved by first solving $\overline{E}$ in Eq. (1) or Eqs. (3) and (4), and calculating $\overline{E}^s$ in Eq. (2) or Eqs. (5) and (6).

Next, we consider the following inverse problem, given the scattered field measured outside the scatterers, determine the permittivity tensor distribution of the scatterers. Note that the only unknown permittivities are $\varepsilon_3(\bar{r})$ for TM case and similarly the unknown permittivities are $\varepsilon_1(\bar{r})$ $\varepsilon_2(\bar{r})$ for TE case. Thus, we can solve the permittivity distribution $\varepsilon_1(\bar{r})$, $\varepsilon_2(\bar{r})$ and $\varepsilon_3(\bar{r})$ for the TE and TM cases respectively. Now we divide the objects into sufficiently small cells so that the electric field and the dielectric constant can be assumed to be constant in each cell. Then the moment method with pulse basis function and point matching techniques are used to transform integral equations into matrix forms. Finally, the unrelated illuminated method is used to reconstruct the permittivity tensors of the scatterers. We refer the reader to [6] for details.

III. Numerical Results

The reconstruction of a biaxial object illuminated by the beam focusing irradiation scheme is presented in the simulation. Many radiators outside the scatterers are used in the same time. By changing the beam directions and tuning the phase of radiators, one can focus all the incident beams in turn at each cell of the body. This procedure is called beam focusing scheme. The frequency of the incident wave is chosen to be 3GHz i.e. the wavelength is 0.1m and the measurement is taken on a circle of radius 0.3m at equal spacing. The number of measurement points is set to be 9 for each incident wave. The square cross-section of the object which is discretized into 4x4 cells is shown in Fig.2 and the corresponding dielectric permittivities are plotted in *Fig.3.* Each cell has 0.15cmx0.15cm cross section. For investigating the effect of noise, we add to each complex scattered field a quantity b+cj, where b and c are independent random numbers have a uniform distribution over 0 to the noise level times the R.M.S. value of the scattered field. The noise levels applied include 10^{-4}, 10^{-3} and 10^{-2} in the simulation. The reconstruction errors are shown in *Fig.4.* It shows the effect of noise is tolerable for noise levels below 10^{-3}. The reconstructed permittivity tensor distribution of the object corresponding to the noise level 10^{-3} is plotted in *Fig.5.* The R.M.S. error is about 0.173%, 0.089% and 1.143% for the dielectric permittivities ε_1, ε_2 and ε_3

respectively. It is clear that the reconstruction is good even in the presence of noise in measured data.

IV. Conclusion

We have used the integral equation formulation to reconstruct the permittivities of biaxial dielectric objects by the knowledge of the scattered field measured outside. The unrelated illumination method and moment method are used to solve the nonlinear integral equations. Numerical simulation for imaging the permittivities of biaxial objects has been carried out and good reconstruction has been obtained.

Reference

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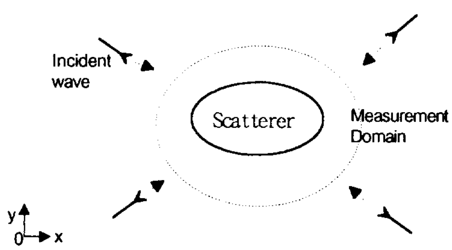
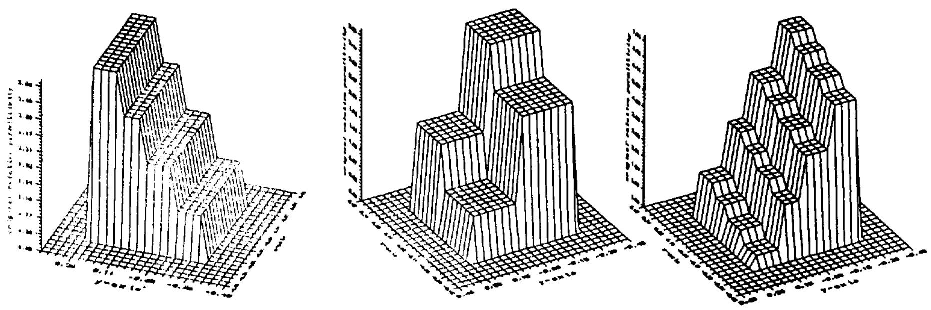


Fig.1 Geometry of the problem in the (x,y) plane

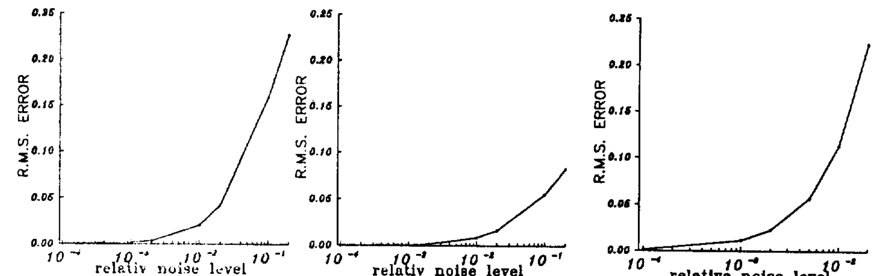
1	**2**	**3**	**4**
5	**6**	**7**	**8**
9	**10**	**11**	**12**
13	**14**	**15**	**16**

Fig.2 Geometry of the simulated experiment (the algebric number is cell number)



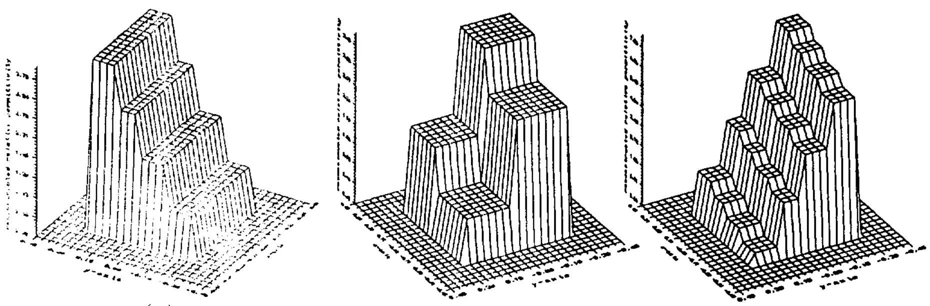
(a) (b) (c)

Fig.3 Original relative permittivity tensor distribution of the biaxial object
(a) $\varepsilon_1(x,y)$ (b) $\varepsilon_2(x,y)$ (c) $\varepsilon_3(x,y)$



(a) (b) (c)

Fig.4 Reconstruction error as a function of noise level (a) Reconstruction result for ε_1
(b) Reconstruction result for ε_2 (c) Reconstruction result for ε_3



(a) (b) (c)

Fig.5 Reconstructed relative permittivity tensor distribution of the biaxial object
(a) $\varepsilon_1(x,y)$ (b) $\varepsilon_2(x,y)$ (c) $\varepsilon_3(x,y)$