

## Image Reconstruction for a perfectly Conducting Cylinder Buried in a Three-layer Structure by TE Wave Illumination

Yi-shian Lin, Chien-Ching Chiu and Yueh-Cheng Chen

Electrical Engineering Department, Tamkang University

Tamsui, Taiwan, R.O.C.

e-mail : [691350150@s91.tku.edu.tw](mailto:691350150@s91.tku.edu.tw); [chiu@ee.tku.edu.tw](mailto:chiu@ee.tku.edu.tw); [mdl-862@yahoo.com.tw](mailto:mdl-862@yahoo.com.tw)

### Abstract

The imaging of a perfectly conducting cylinder illuminated by transverse electric (TE) wave illumination is investigated. A perfectly conducting cylinder of unknown shape buried in the second layer scatters the incident wave from the first layer or the third layer. The metallic cylinder with cross section described by the equation is illuminated by an incident plane wave whose magnetic field vector is perpendicular to the  $z$  axis (i.e. TE polarization). Based on the boundary condition and the recorded scattered field, a set of nonlinear integral equations is derived and the imaging problem is reformulated into an optimization problem.

### I. Introduction

The inverse scattering techniques for imaging the shape of perfectly conducting objects have attracted considerable attention in recent years. They can apply in noninvasive measurement, medical imaging, and biological application. However, inverse problems of this type are difficult to solve because they are ill-posed and nonlinear. As a result, many inverse problems are reformulated as

optimization problems. Generally speaking, two main kinds of approaches have been developed. The first is based on gradient searching schemes such as the Newton-Kantorovitch method [1], the Levenberg-Marguart algorithm and the successive-overrelaxation method. These methods are highly dependent on the initial guess and tend to get trapped in a local extreme. In contrast, the second approach is based on the evolutionary searching schemes [2]. They tend to converge to the global extreme of the problem, no matter what the initial estimate is [3]. Owing to the difficulties in computing the Green's function by numerical method, the problem of inverse scattering in a three-layer structure has seldom been attacked.

Most microwave inverse scattering algorithms developed are for TM wave illumination in which vector problem can be simplified to a scalar one, which only a few papers works have been reported on the more complicated TE case. In the TE case, the presence of polarization charges makes the inverse problem more nonlinear. As a result, the reconstruction becomes more difficult. In this paper, image reconstruction of a perfectly conducting cylinder buried in a

three-layer structure by TE case is investigated. The steady state genetic algorithm is used to recover the shape of the scatterer. It is found the steady-state genetic algorithm [5] can reduce the calculation time of the image problem compared with the generational genetic algorithm. In Section II, the theoretical formulation for the electromagnetic imaging is presented. The general principle of the genetic algorithm and the way we applied them to the imaging problem are described. Numerical results are given in Section III. Section IV is the conclusion.

## II. Theoretical Formulation

Let us consider a two-dimensional three-layer structure as shown in Fig.1, where  $(\epsilon_i, \sigma_i)$   $i = 1, 2, 3$ , denote the permittivities and conductivities in each layer and an perfectly conducting cylinder is buried in second layer. The metallic cylinder with cross section described by the equation  $\rho = F(\theta)$  is illuminated by an incident plane wave whose magnetic field vector is parallel to the z axis (i.e., TE polarization).

At an arbitrary point  $(X, Y)$  (or  $(r, \theta)$

in polar coordinates) in regions 1 and 3 the scattered field as

$$H_s(\vec{r}) = - \int_0^{2\pi} G(\vec{r}, F(\theta'), \theta') J_m(\theta') d\theta' \quad (1)$$

where  $G$  that denotes Green's function which can be obtained by tedious mathematic manipulation for the line

source in region 2.  $J_m(\theta)$  is the induced surface magnetic current density, which is proportional to the normal derivative of the magnetic field on the conductor surface. For a perfectly conducting scatterer, the total tangential electric field at the surface of the scatterer is equal to zero.

$$\hat{n} \times \left( \frac{1}{j\omega\epsilon} \nabla \times \vec{H} \right) = 0 \quad (2)$$

where  $\hat{n}$  is the outward unit vector normal to the surface of the scatterer. For the direct scattering problem, the scattered field  $H_s$  is calculated by assuming that the shape are known. This can be achieved by first solving  $J_m$  in (2) and then calculating  $H_s$  using (1). For the inverse problem, assume the approximate center of scatterer, which in fact can be any point inside the scatterer, is known. Then the shape function  $F(\theta)$  can be expanded as:

$$F(\theta) = \sum_{n=0}^{N/2} B_n \cos(n\theta) + \sum_{n=1}^{N/2} C_n \sin(n\theta) \quad (3)$$

where  $B_n$  and  $C_n$  are real coefficients to be determined, and  $N+1$  is the number of unknowns for the shape function. In the inversion procedure, the steady state genetic algorithm is used to minimize the following cost function:

$$CF = \left\{ \frac{1}{M_t} \sum_{m=1}^{M_t} \left| H_s^{exp}(\vec{r}_m) - H_s^{cd}(\vec{r}_m) \right|^2 \left| H_s^{exp}(\vec{r}_m) \right|^2 + d \left| F(\theta) \right|^2 \right\}^{1/2} \quad (4)$$

where  $M_t$  is the total number of measurement points.  $H_s^{exp}(\vec{r})$  and  $H_s^{cd}(\vec{r})$  are the measured and calculated scattered

fields, respectively. The factor  $\alpha|F'(\theta)|^2$  can be interpreted as the smoothness requirement for the boundary  $F(\theta)$ .

### III. Numerical Results

In the example, the shape function is chosen to be  $F(\theta) = (0.03 + 0.01\cos 2\theta + 0.01\sin 2\theta)$  m. The reconstructed shape function for the best population member is plotted in Fig. 2 (a) with the shape error shown in Fig. 2 (b). The reconstructed shape error is about 2%. For investigating the sensitivity of the imaging algorithm against random noise, we added the uniform noise to the real and imaginary parts of the simulated scattered fields. Normalized standard deviations of  $10^{-5}$ ,  $10^{-4}$ ,  $10^{-3}$ ,  $10^{-2}$  and  $10^{-1}$  are used in the simulations. The shape error versus normalized noise level is plotted in Fig. 2 (c). It is found that the effect of noise to the shape reconstruction is negligible for normalized standard deviations below  $10^{-3}$ . The reconstructed result is quite good. Here, the shape function discrepancy is defined as

$$DFR = \left\{ \frac{1}{N'} \sum_{i=1}^{N'} [F^{cal}(\theta_i) - F(\theta_i)]^2 / F^2(\theta_i) \right\}^{1/2} \quad (5)$$

where  $N'$  is set to 2000.

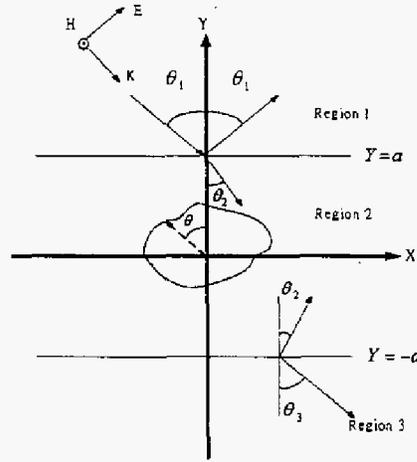


Fig. 1

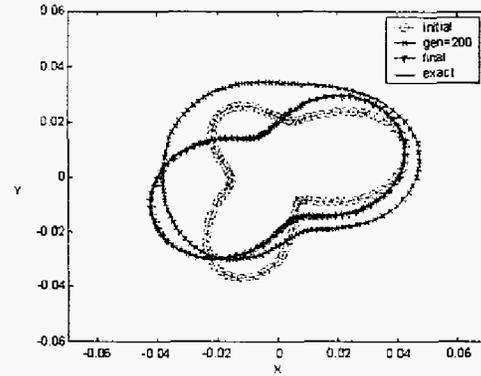


Fig. 2(a)

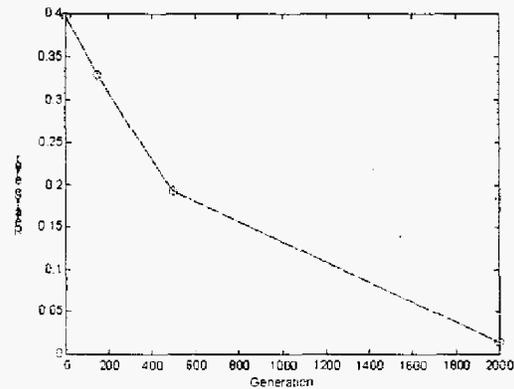


Fig. 2(b)

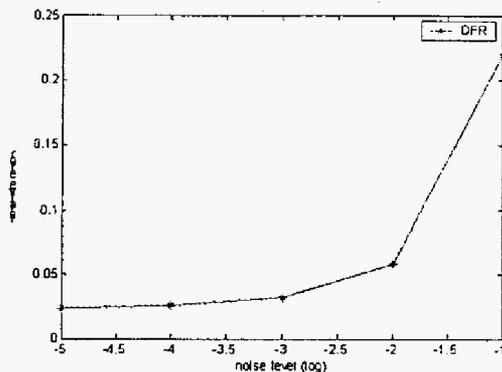


Fig. 2(c)

#### IV. Conclusions

We have reported a study of applying the genetic algorithm to reconstruct the shapes of an embedded conducting cylinder. Based on the boundary condition and measured scattered field, we have derived a set of nonlinear integral equations and reformulated the imaging problem into an optimization one. In the TE case, the presence of polarization charges makes the inverse problem more nonlinear. As a result, the reconstruction becomes more difficult. The genetic algorithm is then employed to de-embed the microwave image of metallic cylinder. In our experience, the main difficulties in applying the genetic algorithm to the problem are to choose the suitable

parameters, such as the population size, coding length of the string ( $L$ ), crossover probability ( $p_c$ ), and mutation probability ( $p_m$ ). Different parameter sets will affect the speed of convergence as well as the computation time. Numerical results show that good shape reconstruction can be achieved as long as the normalized noise level is  $< 10^{-3}$ .

#### References

- [1] C. C. Chiu and Y. W. Kiang, "Electromagnetic imaging for an imperfectly conducting cylinders," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 1632-1639, Sep. 1991.
- [2] C. C. Chiu and W. T. Chen, "Electromagnetic imaging for an imperfectly conducting cylinder by the genetic algorithm," *IEEE Trans. Microwave Theory and Tec.*, vol. 48, pp. 1901-1905, Nov. 2000.
- [3] Y. Rahmat-Samii and E. Michielelsen, "Electromagnetic Optimization by Genetic Algorithms," *Wiley Interscience*, 1999.
- [4] J. Michael Johnson and Yahya Rahmat-Samii, "Genetic algorithms in engineering electromagnetics," *IEEE Trans. Antennas Propagat.*, vol. 39, pp. 7-21, Aug. 1997.