Image Reconstruction for a Partially Immersed Conducting Cylinder by Transverse Electric Wave Illumination

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***Abstract* — In this paper, we present a computational approach to the imaging of a partially immersed perfectly conducting cylinder by the steady-state genetic algorithm. A conducting cylinder of unknown shape scatters the incident transverse electric (TE) wave in free space while the scattered field is recorded outside. Based on the boundary condition and the measured scattered field, a set of nonlinear integral equations is derived and the imaging problem is reformulated into an optimization problem. An improved steady-state genetic algorithm is employed to search for the global extreme solution. Numerical results demonstrate that, even when the initial guess is far away from the exact one, good reconstruction can be obtained.**

***Index Terms* — inverse problem, partially immersed, conductor, steady-state genetic algorithm, transverse electric wave.**

I. INTRODUCTION

The inverse scattering techniques have been widely investigated in recent years. It has been one of the most challenging problems with considerable practical applications in many areas of technology such as medical imaging, biological application and etc. It is well known that the main difficulties of inverse problems are the nonlinearity and ill-posedness. Generally speaking, two kinds of approaches have been developed to solve the problem. The first is based on gradient searching schemes such as the Newton-Kantrovitch method [1]-[2], the Levenberg-Marguart algorithm [3]-[4] and successive-overelaxation method [5]. In these methods, the initial guess is very important and they tend to get trapped in a local extreme. The second method is contrast to last one and is based on genetic algorithm. It tends to converge to the global extreme of the problem, no matter what the initial guess is [6]-[8].

Most researches are developed for transverse magnetic illuminations in which the vectorial problem can be simplified to a scalar one. Only a fewer papers have been reported on the more complicated TE case [9]-[11]. In the TE wave excitation-case, the presence of polarization charges makes the inverse problem more nonlinear. As a result, the reconstruction of inverse scattering becomes more difficult. However, the TE polarization case is useful because it provides additional information about the object. To the best of our knowledge, there is no paper dealing with the inverse scattering problem of the partially immersed perfectly conducting cylinder by TE wave illumination.

In this paper, the inverse scattering problem of the partially immersed perfectly conducting cylinder by TE wave illumination is investigated. We use the steady-state genetic algorithm to recover the shape of a partially immersed perfectly conducting cylinder.. In section II, the theoretical formulation for the inverse scattering is derived. The numerical results for various objects of different shapes are presented in section III. Section IV gives the conclusions.

II. THEORETICAL FORMULATION

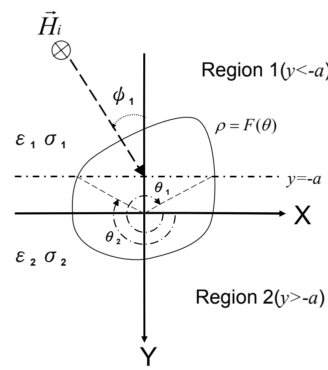

Fig. 1 Geometry of the problem in (x,y) plane

Let us consider a perfectly conducting cylinder which is partially immersed in a lossy homogeneous half-space, as shown in Fig. 1. Media in regions 1 and 2 are characterized by permittivities and conductivities (ε_1, σ_1) and (ε_2, σ_2) respectively. A perfectly conducting cylinder is illuminated by a transverse electric (TE) plane wave. The cylinder is of an infinite extent in the z direction, and its cross-section is described in polar coordinates in the x, y plane by the equation $\rho = F(\theta)$. We assume that the time dependence of the field is harmonic with the factor $e^{j\omega t}$. Let $\vec{H}^{in}$ denote the incident field form region 1 with incident angle ϕ_1 as follow:

$$\vec{H}^{in} = e^{-jk_1(y\cos\phi_1 + x\sin\phi_1)}\hat{z} \tag{1}$$

Owing to the interface between regions 1 and 2, the incident plane wave generates two waves that would exist in the absence of the conducting object. Thus, the unperturbed field is given by

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$$\vec{H}^i = \begin{cases} [e^{-jk_1(y\cos\phi_1 + x\sin\phi_1)} + H_1 e^{jk_1(y\cos\phi_1 - x\sin\phi_1)}]\hat{Z} & , y \le -a \\ H_2 e^{-jk_2(y\cos\phi_2 + x\sin\phi_2)}\hat{Z} & , y > -a \end{cases} \tag{2}$$

where

$$H_1 = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)e^{2jk_1 a\cos\phi_1} \qquad \phi_2 = \sin^{-1}\frac{k_1}{k_2}\sin\phi_1$$

$$H_2 = \frac{2Z_1 e^{jk_1 a\cos\phi_1} e^{-jk_2 a\cos\phi_2}}{Z_1 + Z_2}, \quad Z_1 = \eta_1\cos\theta_1, \; Z_2 = \eta_2\cos\theta_2, \; \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}, \; \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$$

Since the cylinder is partially immersed, the equivalent current exists both in the upper half space and the lower half space. As a result, the details of Green's function are given first as follows:

(a) When the equivalent current exists in the upper half space, the Green's function for the line source in the region 1, can be expressed as

$$G_1(x,y;x',y') = \begin{cases} G_{21}(x,y;x',y') & , y > -a \\ G_{11}(x,y;x',y') = G_{f11}(x,y;x',y') + G_{s11}(x,y;x',y') & , y \le -a \end{cases} \tag{3}$$

where

$$G_{21}(x,y;x',y') = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{j}{\gamma_1+\gamma_2}e^{-j\gamma_2(y+a)}e^{j\gamma_1(y'+a)}e^{-j\alpha(x-x')}d\alpha$$

$$G_{f11}(x,y;x',y') = \frac{j}{4}H_0^{(2)}[k_1\sqrt{(x-x')^2+(y-y')^2}]$$

$$G_{s11}(x,y;x',y') = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{j}{2\gamma_1}\left(\frac{\gamma_1-\gamma_2}{\gamma_1+\gamma_2}\right)e^{j\gamma_1(y+2a+y')}e^{-j\alpha(x-x')}d\alpha$$

$\gamma_i^2 = k_i^2 - \alpha^2$, $i = 1,2$, $\mathrm{Im}(\gamma_i) \le 0$, $y' < -a$

(b) When the equivalent current exists in the lower half space, the Green's function for the line source in the region 2, can be expressed as

$$G_2(x,y;x',y') = \begin{cases} G_{12}(x,y;x',y') & , y \le -a \\ G_{22}(x,y;x',y') = G_{f22}(x,y;x',y') + G_{s22}(x,y;x',y'), & y > -a \end{cases} \tag{4}$$

where

$$G_{12}(x,y;x',y') = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{j}{\gamma_1+\gamma_2}e^{j\gamma_1(y+a)}e^{-j\gamma_2(y'+a)}e^{-j\alpha(x-x')}d\alpha$$

$$G_{f22}(x,y;x',y') = \frac{j}{4}H_0^{(2)}[k_2\sqrt{(x-x')^2+(y-y')^2}]$$

$$G_{s22}(x,y;x',y') = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{j}{2\gamma_2}\left(\frac{\gamma_2-\gamma_1}{\gamma_2+\gamma_1}\right)e^{-j\gamma_2(y+y'+2a)}e^{-j\alpha(x-x')}d\alpha$$

$\gamma_i^2 = k_i^2 - \alpha^2$, $i = 1,2$, $\mathrm{Im}(\gamma_i) \le 0$, $y' > -a$

The scattered field in each region can be expressed by

$$H^S(\vec{r}) = \begin{cases} H_1^S(\vec{r}) = \int_{\theta_1-2\pi}^{\theta_2} G_{12}(x,y;x',y')J_m(\theta')d\theta' + \int_{\theta_2}^{\theta_1} G_{11}(x,y;x',y')J_m(\theta')d\theta', & y \le -a \\ H_2^S(\vec{r}) = \int_{\theta_1-2\pi}^{\theta_2} G_{22}(x,y;x',y')J_m(\theta')d\theta' + \int_{\theta_2}^{\theta_1} G_{21}(x,y;x',y')J_m(\theta')d\theta', & y > -a \end{cases} \tag{5}$$

with

$$J_m(\theta) = -j\omega\sqrt{F^2(\theta)+F'^2(\theta)}\,J_{sm}(\theta)$$

Here, $J_{sm}(\theta)$ is the induced surface magnetic current density, which is proportional to the normal derivative of the magnetic field on the conductor surface. $G_1(x,y;x',y')$ and $G_2(x,y;x',y')$ denote the Green's function for the line source in the region 1 and region 2 , respectively. $H_0^{(2)}$ is the Hankel function of the second kind of order zero.

For a perfectly conducting scatterer, the total tangential electric field at the surface of the scatterer is equal to zero.

$$\hat{n} \times \left(\frac{1}{j\omega\varepsilon}\nabla \times \vec{H}^{tot}\right) = 0 \tag{6}$$

with $\vec{H}^{tot} = \vec{H}^i + H^s$

where $\hat{n}$ is the outward unit vector normal to the surface of the scatterer.

For the direct scattering problem, the scattered field H^s is calculated by assuming that the shape is known. This can be achieved by first solving $J_{sm}(\theta)$ in (6) and then calculating H^s using (5). Then the inverse problem is to determine the shapes and positions of the conducting cylinders when the scattered field H^s is measured outside the scatterer.

III. NUMERICAL RESULTS

Let us consider a perfectly conducting cylinder buried in a lossless half-space ($\sigma_1 = \sigma_2 = 0$). The permittivity in each region is characterized by $\varepsilon_1 = \varepsilon_0$ and $\varepsilon_2 = 2.56\varepsilon_0$ respectively. The frequency of the incident wave is chosen to be 1GHz with incident angles ϕ_1 equal to 45°, 315° and 360°, respectively. The wavelength λ_0 is 0.3m. For each incident wave, 8 measurements are made at the points equally separated on a semi-circle with the radius of 3m in region 1. There are 24 measurement points in each simulation. Our purpose is to reconstruct the shape of the object by using the scattered field at different incident angles. The number of unknowns is set to be 7 (i.e., N+1=7), and the coding length of each unknown coefficient is set to be 16 bits. The search range for the unknown coefficient of the shape function is chosen to be from 0 to 0.15. The population size is set 100 and the rank is set 70. The crossover probability is equal to 0.05 and mutation probability is set to be 0.025.

In the example, the shape function is chosen to be $F(\theta) = (0.1 + 0.025\cos 2\theta + 0.03\cos 3\theta)$ m. The reconstructed shape function for the best population member is plotted in Fig. 2 with the shape error shown in Fig. 3. The reconstructed shape error is 3%. It is clear that the reconstructed result is good.

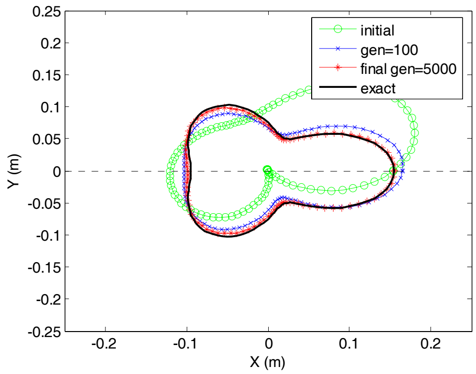

Fig. 2 The exact and reconstructed shape functions. The solid represents the exact shape, and the others are the calculated shape function in the iterative process

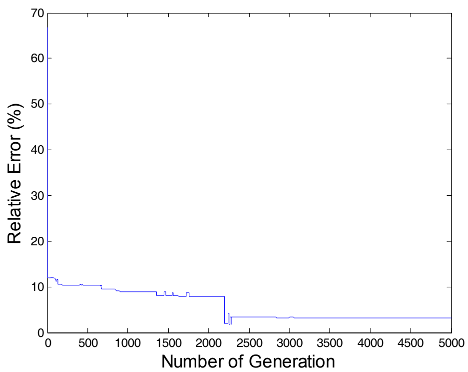

Fig. 3 Shape function error in each generation.

IV. CONCLUSION

We have presented a study of applying the steady state genetic algorithm to reconstruct the shapes of a partially immersed conducting cylinder illuminated by TE waves through the knowledge of scattered field. Based on the boundary condition and measured scattered field, we have derived a set of nonlinear integral equations and reformulated the imaging problem into an optimization one. The genetic algorithm is then employed to de-embed the microwave image of metallic cylinder in the TE case, where the presence of polarization charges makes the inverse problem more nonlinear and more difficult. In our experience, the main difficulties in applying the genetic algorithm to the problem are to choose the suitable parameters, such as the population size, coding length of the string (L), crossover probability (p_c), and mutation probability (p_m). Different parameter sets will affect the speed of convergence as well as the computation time.

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