

Differential BPSK Modulation for Cooperative Communication Systems in Time-Selective Rayleigh Fading Channels

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Abstract—In this paper, we consider the amplify-and-forward (AF) relaying cooperative communication system employing differential binary phase-shift-keying (DBPSK) modulation in time-selective (fast) Rayleigh fading channels. When channel gains are fast-varying, it is well known that differential modulation can ease the channel estimation process and reduce the power and bandwidth overhead occurred in coherent modulation schemes. Unlike the previous work on this topic which always assumed the channel gains are the same over two adjacent symbol periods, we model the channel time-selectivity exactly in our formulation and derive the optimal diversity combining weights for the AF relaying system based on the maximum likelihood criterion. Since the optimum combining rule depends on the channel gains of the relay-to-destination links which are usually unavailable in the context of differential modulation, we propose a suboptimal diversity combining rule which replaces the instantaneous channel gains by their second order statistics. Compared with the performance of the diversity combining rule without taking the time-selectivity into account, computer simulation results show that the proposed diversity combining rule has superior performance for the AF relaying system in time-selective Rayleigh fading channels, especially when the relay and destination nodes have different Doppler spreads.

I. INTRODUCTION

Cooperative communications can achieve spatial diversity gains without the physical size limitation existing in conventional multiple-antenna communications systems [1][2]. The key idea of cooperative communications is to create independent diversity receptions at the destination node via spatially separated relays. Many different protocols have been proposed to perform signal processing at the relay node [3]. In this paper, we focus our study on amplify-and-forward (AF) relaying communication systems due to its lower complexity and superior performance.

When coherent modulation is employed at the transmitter, the receiver of a diversity communication system suffering channel fading and additive white Gaussian noise (AWGN) must acquire channel state information (CSI) to perform maximal ratio combining (MRC) which is optimal in the sense of minimum error probability. However, accurate CSI estimates via pilots or training sequences are difficult and costly to obtain in a rapidly changing mobile environment [4]. This difficulty becomes more significant in cooperative communication systems because the MRC receiver for the AF relaying system also requires the knowledge of the individual

link equivalent noise variance to perform diversity combining [5]-[8].

To ease the CSI estimation process and reduce the power and bandwidth overhead occurred in coherent modulation schemes over time-selective fading channels, one alternative option is to employ differential phase-shift-keying (DPSK) modulation in cooperative communication systems [9][10]. At the destination node, the diversity combining differential receiver takes the previous received signals as the references and computes the weighted sum of the phase differences between two adjacent received samples over different propagation paths to detect the modulated symbol.

For mobile fading channels, if the normalized Doppler spread of the channel is small enough, the channel is called slow fading in which the channel gains over two successive symbol periods are almost constant. On the other hand, if the normalized Doppler spread of the channel is large due to high mobility and/or low data rate, the channel is called fast or time-selective fading [11]. The effect of time-selectivity on the performance of conventional DPSK modulation in generalized diversity Rayleigh was analyzed in [12] and [13].

The previous work on the DPSK modulation for cooperative communication systems assumed the channel gains are the same over two adjacent symbol periods [9][10]. Although this assumption can simplify the derivation of diversity combining rule and make the analysis tractable, it ignores the channel mismatch effect in the differential demodulation due to channel time-selectivity. Since differential modulation is often adopted in a fast-varying fading environment, it is important to assess the effect of fast channel gain variation on the performance of the differential receiver. In this paper, we model the channel time-selectivity exactly in our formulation and derive the optimum diversity combining rule for cooperation communication systems with differential BPSK (DBPSK) modulation in time-selective Rayleigh fading channels. Since the optimum diversity combining rule involves the instantaneous relay-to-destination (RD) channel gains which are usually hard to acquire in the context of DBPSK modulation, we devise a suboptimum detection rule which only requires the second-order statistics of the RD link CSI. The performance of the proposed detection rules is compared and characterized by computer simulation.

The remainder of this paper is organized as follows. In Section II, we describe the system model. The optimum and sub-optimum detection rules for the cooperative communications system with DBPSK modulation in time-selective Rayleigh fading channels are presented in Section III. Numerical results are given in Section IV and conclusions are drawn in Section V.

Notation: Symbols for matrices and vectors are in boldface. The symbol $(\cdot)^H$ stands for Hermitian transpose. \mathbb{Z} is the set of integers. $X \sim \mathcal{CN}(\mu, \sigma^2)$ means the random variable X is a circularly symmetric complex Gaussian random variable with mean μ and variance σ^2 . We use the symbols $|\cdot|$, $*$, and $\mathbb{E}[\cdot]$ to denote the Euclidean norm, the complex conjugation, and the statistical expectation, respectively.

II. SYSTEM MODEL

The two-hop AF relay system with one relay node is shown in Figure 1. During the phase I period, the source node transmits signals to the relay and destination nodes. In phase II period, the relay node amplifies the received signal and forwards it to the destination node. Transmissions in different phases are taken in orthogonal channels to avoid interfering with each other at the destination node. Furthermore, we assume the source node is a fixed base station and the relay and destination nodes are mobile devices. This scenario can be used to model the downlink transmission in a cellular communication system.

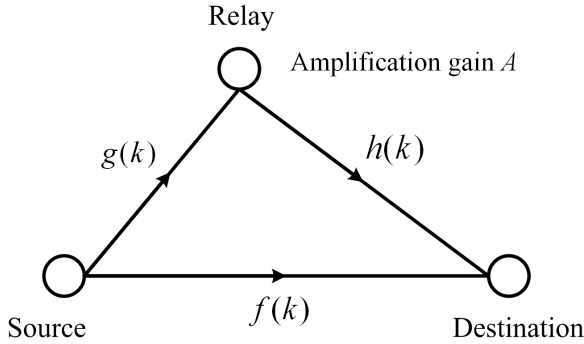


Fig. 1. Diagram of a dual-hop relay system.

The received discrete-time baseband signals at the destination and relay nodes during phase I are given by

$$y_1(k) = \sqrt{P_1}f(k)x(k) + n_1(k), \quad k = 0, 1, \dots \quad (1)$$

$$r(k) = \sqrt{P_1}g(k)x(k) + n_r(k), \quad k = 0, 1, \dots \quad (2)$$

where k is the symbol index, $y_1(k)$ and $r(k)$ represent the received signal for the source-to-destination (SD) link and the source-to-relay (SR) link, respectively. P_1 is the transmitted power at the source node. The source node employs DBPSK modulation for signal transmission, that is

$$x(k) = x(k-1)d(k), \quad k = 1, 2, \dots \quad (3)$$

where $d(k)$ is chosen from the signal constellation $\mathbb{S} = \{-1, +1\}$ with equal probability and $x(0) = 1$ is the initial reference symbol. The additive white Gaussian noises (AWGNs) $n_1(k)$ and $n_r(k)$ are independent, identically distributed (i.i.d.) from $\mathcal{CN}(0, N_0)$. Following the Clarke's two-dimensional isotropic scattering model [11], the channel gains $f(k)$ and $g(k)$ are modeled by independent zero-mean complex Gaussian random processes with discrete autocorrelation functions

$$R_f(l) = \sigma_f^2 J_0(2\pi l f_{d_1} T_s) \quad (4)$$

$$R_g(l) = \sigma_g^2 J_0(2\pi l f_{d_2} T_s) \quad (5)$$

where $J_0(\cdot)$ is the zeroth order Bessel function of the first kind, f_{d_1} and f_{d_2} are the Doppler spreads associated with the corresponding channels, T_s is the symbol period, σ_f^2 and σ_g^2 are the variances of $f(k)$ and $g(k)$, respectively.

In phase II period, the relay node amplifies and forwards the received signal $r(k)$ to the destination node. With some abuse of the symbol index notation k , the received signal at the destination node is given by

$$\begin{aligned} y_2(k) &= Ah(k)r(k) + n_2(k), \quad k = 0, 1, \dots \\ &= A\sqrt{P_1}h(k)g(k)x(k) + Ah(k)n_1(k) + n_2(k) \end{aligned} \quad (6)$$

where the constant amplification gain A is determined by the average transmitted power constraint P_2 at the relay node as [2]

$$A = \sqrt{\frac{P_2}{P_1\sigma_g^2 + N_0}}. \quad (7)$$

The mobile-to-mobile channel gains of the RD link $h(k)$ are sampled from a zero-mean Gaussian random process with autocorrelation function [14]

$$R_h(l) = \sigma_h^2 J_0(2\pi l f_{d_1} T_s) J_0(2\pi l f_{d_2} T_s) \quad (8)$$

where σ_h^2 is the variance of the channel gain $h(k)$. The AWGNs $n_2(k)$ are i.i.d. from $\mathcal{CN}(0, N_0)$. Finally, all channel gains and AWGNs in different channels are assumed to be independent to each other.

III. OPTIMUM AND SUBOPTIMUM DETECTION RULES

A. Optimal Detection Rule

At time index k , the receiver at the destination node detects the transmitted data symbol $d(k)$ based on the received signals $y_1(k-1), y_1(k), y_2(k-1)$ and $y_2(k)$ in two consecutive symbol periods. The maximum a posteriori (MAP) detection rule minimizing the detection error probability is given by

$$\begin{aligned} \hat{d}(k) &= \arg \max_{d(k) \in \mathbb{S}} p[d(k) | y_1(k-1), y_1(k), y_2(k-1), y_2(k)] \\ &= \arg \max_{d(k) \in \mathbb{S}} p[y_1(k-1), y_1(k), y_2(k-1), y_2(k) | d(k)] \\ &= \arg \max_{d(k) \in \mathbb{S}} p[y_1(k), y_2(k) | y_1(k-1), y_2(k-1), d(k)] \\ &= \arg \max_{d(k) \in \mathbb{S}} p[y_1(k) | y_1(k-1), d(k)] p[y_2(k) | y_2(k-1), d(k)] \end{aligned} \quad (9)$$

where we use the facts that the *a priori* probabilities of $d(k)$ are equal and the statistics of different channels are independent. Besides, the expressions $p[y_1(k)|y_1(k-1), d(k)]$ and $p[y_2(k)|y_2(k-1), d(k)]$ are the conditional probability density functions (PDFs) of $y_1(k)$ and $y_2(k)$ given $y_1(k-1), d(k)$ and $y_2(k-1), d(k)$, respectively.

First, we derive the explicit form of the conditional PDF $p[y_1(k)|y_1(k-1), d(k)]$. Given $y_1(k-1)$ and $d(k)$, $y_1(k)$ is a complex Gaussian random variable with the mean $\mu_1(k)$ and variance η_1^2 . By using the results given in [15], the mean $\mu_1(k)$ is given by

$$\begin{aligned}\mu_1(k) &= \frac{\mathbb{E}[y_1(k)y_1^*(k-1)|d(k)]}{\mathbb{E}[y_1(k-1)y_1^*(k-1)]} y_1(k-1) \\ &= \frac{\rho_f P_1 \sigma_f^2}{P_1 \sigma_f^2 + N_0} y_1(k-1) d(k) \\ &= \frac{\rho_f \gamma_f}{\gamma_f + 1} y_1(k-1) d(k)\end{aligned}\quad (10)$$

where $\rho_f = J_0(2\pi f_{d_1} T_s)$ is the correlation coefficient of the channel gains $f(k-1)$ and $f(k)$ and $\gamma_f = P_1 \sigma_f^2 / N_0$ is the average received SNR of the SD link. In addition, the variance η_1^2 is given by

$$\begin{aligned}\eta_1^2 &= \mathbb{E}[y_1(k)y_1^*(k)] - \frac{|\mathbb{E}[y_1(k)y_1^*(k-1)|d(k)]|^2}{\mathbb{E}[y_1(k-1)y_1^*(k-1)]} \\ &= P_1 \sigma_f^2 + N_0 - \frac{\rho_f^2 P_1^2 \sigma_f^4}{P_1 \sigma_f^2 + N_0} \\ &= N_0 \left[\frac{(1 - \rho_f^2) \gamma_f^2 + 2\gamma_f + 1}{\gamma_f + 1} \right].\end{aligned}\quad (11)$$

Therefore, the conditional PDF $p[y_1(k)|y_1(k-1), d(k)]$ is given by

$$\begin{aligned}p[y_1(k)|y_1(k-1), d(k)] \\ = \frac{1}{\pi \eta_1^2} \exp \left\{ -\frac{1}{\eta_1^2} |y_1(k) - m_1 y_1(k-1) d(k)|^2 \right\}\end{aligned}\quad (12)$$

where

$$m_1 = \frac{\rho_f \gamma_f}{\gamma_f + 1}.\quad (13)$$

Next, we derive the explicit form of the conditional PDF $p[y_2(k)|y_2(k-1), d(k)]$ for the RD link. Conditioned on $y_2(k-1), d(k), h(k-1)$, and $h(k)$, $y_2(k)$ is a complex Gaussian random variable with mean $\mu_2(k)$ and variance $\eta_2^2(k)$. By using the formulas presented in [15], we have

$$\begin{aligned}\mu_2(k) &= \frac{\mathbb{E}[y_2(k)y_2^*(k-1)|d(k), h(k-1), h(k)]}{\mathbb{E}[y_2(k-1)y_2^*(k-1)|h(k-1)]} y_2(k-1) \\ &= \frac{A^2 P_1 \rho_g \sigma_g^2 h^*(k-1) h(k)}{A^2 |h(k-1)|^2 (P_1 \sigma_g^2 + N_0) + N_0} y_2(k-1) d(k) \\ &= \frac{A^2 \rho_g \gamma_g h^*(k-1) h(k)}{A^2 |h(k-1)|^2 (\gamma_g + 1) + 1} y_2(k-1) d(k)\end{aligned}\quad (14)$$

where $\gamma_g = P_1 \sigma_g^2 / N_0$ is the average received SNR of the SR link. Substituting the amplification gain formula (7) into (14), we have

$$\mu_2(k) = \frac{\rho_g \gamma_g P_2 h^*(k-1) h(k) / N_0}{(\gamma_g + 1) \left(\frac{P_2 |h(k-1)|^2}{N_0} + 1 \right)} y_2(k-1) d(k).\quad (15)$$

Moreover, the variance $\eta_2^2(k)$ is given by

$$\begin{aligned}\eta_2^2(k) &= \mathbb{E}[y_2(k)y_2^*(k)|h(k)] \\ &\quad - \frac{|\mathbb{E}[y_2(k)y_2^*(k-1)|d(k), h(k-1), h(k)]|^2}{\mathbb{E}[y_2(k-1)y_2^*(k-1)|h(k-1)]} \\ &= A^2 |h(k)|^2 (P_1 \sigma_g^2 + N_0) + N_0 \\ &\quad - \frac{A^4 P_1^2 \rho_g^2 \sigma_g^4 |h(k-1)|^2 |h(k)|^2}{A^2 |h(k-1)|^2 (P_1 \sigma_g^2 + N_0) + N_0}\end{aligned}\quad (16)$$

where $\rho_g = J_0(2\pi f_{d_2} T_s)$ is the correlation coefficient of the channel gains $g(k-1)$ and $g(k)$. Finally, the conditional PDF $p[y_2(k)|y_2(k-1), d(k), h(k-1), h(k)]$ is

$$\begin{aligned}p[y_2(k)|y_2(k-1), d(k), h(k-1), h(k)] \\ = \frac{1}{\pi \eta_2^2(k)} \exp \left\{ -\frac{1}{\eta_2^2(k)} |y_2(k) - m_2(k) y_2(k-1) d(k)|^2 \right\}\end{aligned}\quad (17)$$

where

$$m_2(k) = \frac{\rho_g \gamma_g P_2 h^*(k-1) h(k) / N_0}{(\gamma_g + 1) \left(\frac{P_2 |h(k-1)|^2}{N_0} + 1 \right)}.\quad (18)$$

Substituting (12) and (17) into (9) and ignoring those terms independent of $d(k)$, we have

$$\begin{aligned}\hat{d}(k) &= \arg \max_{d(k) \in \mathbb{S}} \Re \left\{ \left[\frac{2m_1}{\eta_1^2} y_1(k-1) y_1^*(k) + \frac{2m_2(k)}{\eta_2^2(k)} y_2(k-1) y_2^*(k) \right] d(k) \right\} \\ &= \text{sgn} \{ \Re [w_1 y_1(k-1) y_1^*(k) + w_2(k) y_2(k-1) y_2^*(k)] \}\end{aligned}\quad (19)$$

where $\Re(z)$ denotes the real part of z , $\text{sgn}(\cdot)$ is the sign function, and the combination weights $w_1 = 2N_0 m_1 / \eta_1^2$ and $w_2(k) = 2N_0 m_2(k) / \eta_2^2(k)$ can be explicitly expressed as

$$w_1 = \frac{2\rho_f \gamma_f}{(1 - \rho_f^2) \gamma_f^2 + 2\gamma_f + 1}\quad (20)$$

$$w_2(k) = \frac{2\rho_g \gamma_g (\gamma_g + 1)}{\psi_1(k) + \psi_2(k)}\quad (21)$$

where

$$\psi_1(k) = [\gamma_g^2 (1 - \rho_g^2) + 2\gamma_g + 1] \frac{P_2 h(k-1) h^*(k)}{N_0}\quad (22)$$

$$\psi_2(k) = (\gamma_g + 1)^2 \frac{|h(k-1)|^2 + |h(k)|^2 + N_0 / P_2}{h^*(k-1) h(k)}.\quad (23)$$

To implement the optimum detection rule (19), the receiver at the destination node requires the knowledge of the instantaneous channel gains $h(k-1)$ and $h(k)$, which are usually unavailable in the context of differential modulation. However, the performance of the optimum detection rule can still be used

as the performance benchmark, which can be compared with some more practical, but suboptimal detection rules.

B. Suboptimal Detection Rule I

Since the instantaneous channel gains $h(k-1)$ and $h(k)$ are not available at the receiver, one can obtain the conditional PDF $p[y_2(k)|y_2(k-1), d(k)]$ from the conditional PDF $p[y_2(k)|y_2(k-1), d(k), h(k-1), h(k)]$ by taking expectation with respect to the joint PDF of $h(k-1)$ and $h(k)$. Due to the complicated expression of $p[y_2(k)|y_2(k-1), d(k), h(k-1), h(k)]$ in terms of $h(k-1)$ and $h(k)$, it is difficult to derive the decision metric in closed form.

Alternatively, we consider to use the second order statistic $\rho_h \sigma_h^2$ to replace the terms $h(k-1)h^*(k)$ and $h^*(k-1)h(k)$ in (21) where $\rho_h = J_0(2\pi f_{d1} T_s) J_0(2\pi f_{d2} T_s)$ is the correlation coefficient of $h(k-1)$ and $h(k)$. Moreover, the squares of the instantaneous channel gains $|h(k-1)|^2$ and $|h(k)|^2$ are replaced by their average value σ_h^2 . The resulting combination weights are given by

$$\hat{w}_1 = \frac{2\rho_f \gamma_f}{(1 - \rho_f^2) \gamma_f^2 + 2\gamma_f + 1} \quad (24)$$

$$\hat{w}_2 = \frac{2\rho_g \rho_h \gamma_h \gamma_g (\gamma_g + 1)}{[\gamma_g^2 (1 - \rho_g^2) + 2\gamma_g + 1] \rho_h^2 \gamma_h^2 + (\gamma_g + 1)^2 (2\gamma_h + 1)} \quad (25)$$

where $\gamma_h = P_2 \sigma_h^2 / N_0$ is the average received SNR of the RD link.

When the channel gains are slowly fading (i.e., the normalized Doppler spreads $f_{d1} T_s$ and $f_{d2} T_s$ are small), the correlation coefficients ρ_f , ρ_g , and ρ_h are close to 1. In this case, the weights \hat{w}_1 and \hat{w}_2 can be well approximated as

$$\hat{w}_1 \approx \frac{2\gamma_f}{2\gamma_f + 1} \quad (26)$$

$$\hat{w}_2 \approx \frac{2\gamma_h \gamma_g (\gamma_g + 1)}{(2\gamma_g + 1) \gamma_h^2 + (\gamma_g + 1)^2 (2\gamma_h + 1)}. \quad (27)$$

If the SNRs are high enough (i.e., $\gamma_f \gg 1$, $\gamma_g \gg 1$, and $\gamma_h \gg 1$), then the weights (26) and (27) can be further approximated as

$$\hat{w}_1 \approx 1 \quad (28)$$

$$\hat{w}_2 \approx \frac{\gamma_g}{\gamma_g + \gamma_h}. \quad (29)$$

C. Suboptimal Detection Rule II

In previous works [9][10], the combination weights for AF cooperative communication systems with differential modulation were derived under the assumption that the channel gains are constant over two symbol periods. The performance of those combination weights in time-selective Rayleigh fading channels should be quantified and compared with the other suboptimal combination weights derived in this paper. For the purpose of comparison and completeness, we include those combination weights used in [9][10] as follows.

$$\tilde{w}_1 = 1 \quad (30)$$

$$\tilde{w}_2 = \frac{\gamma_g + 1}{\gamma_g + \gamma_h + 1}. \quad (31)$$

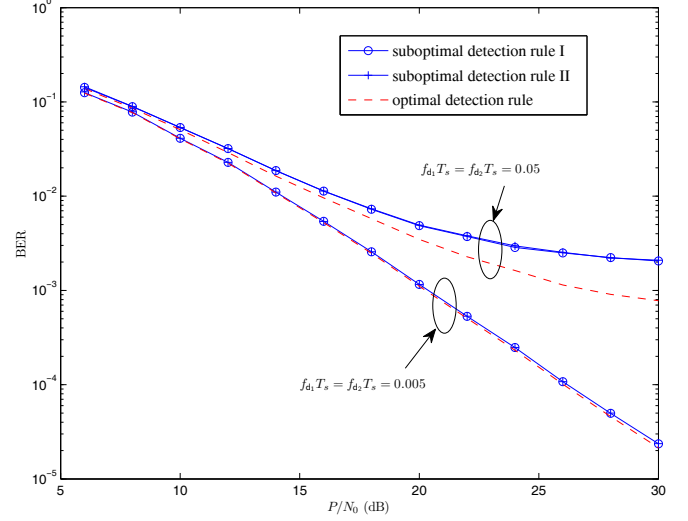


Fig. 2. BER performance of AF relaying systems with DBPSK modulation in time-selective Rayleigh fading channels. $P_1 = P_2 = P/2$ and $\sigma_f^2 = \sigma_g^2 = \sigma_h^2 = 1$.

IV. NUMERICAL RESULTS

In our numerical experiments, the Rayleigh fading channels of the SR and SD links are simulated based on the modified Jakes' model proposed in [16]. The mobile-to-mobile RD link is simulated based on the method presented in [17]. There are 1000 BPSK modulated symbols in one frame. The total transmitted power by the source and relay nodes is denoted by $P = P_1 + P_2$.

First, we consider the situation in which the channel variances σ_f^2 , σ_g^2 , and σ_h^2 are all normalized to 1. These parameters imply the source, relay, and destination nodes are located in an equilateral triangle. Besides, the total transmitted power P are equally distributed at the source and relay nodes (i.e., $P_1 = P_2 = 0.5P$). Moreover, two kinds of normalized Doppler spreads $f_{d1} T_s = f_{d2} T_s = 0.05$ and $f_{d1} T_s = f_{d2} T_s = 0.005$ are considered in our computer simulation. Fig. 2 shows the BER performance of the optimal and suboptimal detection rules for AF relaying systems with DBPSK modulation in time-selective Rayleigh fading channels. It can be clearly seen in the figure, the performance of the two suboptimal detection rules are almost the same under this parameter setting. When the normalized Doppler spreads are small (e.g., $f_{d1} T_s = f_{d2} T_s = 0.005$), the performance curves corresponding the suboptimal is very close to the optimal one. As the normalized Doppler spread becomes larger (e.g., $f_{d1} T_s = f_{d2} T_s = 0.05$), the difference between the performance curves corresponding to the optimal and suboptimal detection rules increases.

To illustrate the advantage of the proposed suboptimal detection rule over its counterpart given in [9] and [10], we consider the case in which the relay and destination nodes have different Doppler spreads. Fig. 3 shows the BER performance curves for various detection rules given the normalized Doppler spreads $f_{d1} T_s = 0.001$ and $f_{d2} T_s = 0.05$. As the

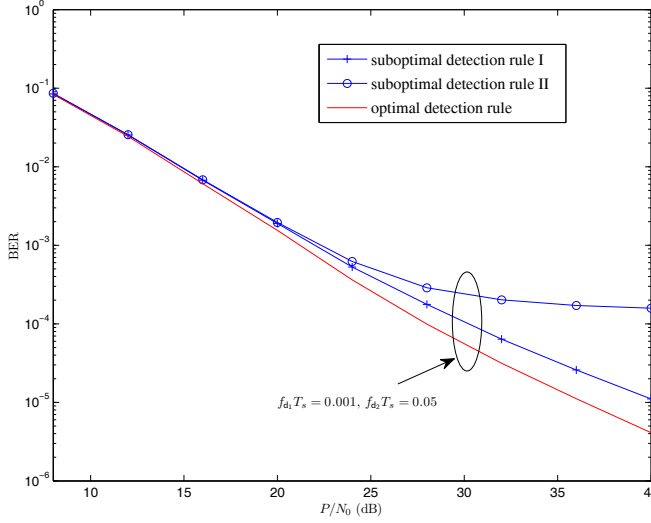


Fig. 3. BER performance of AF relaying systems with DBPSK modulation in time-selective Rayleigh fading channels. $P_1 = P_2 = P/2$, $\sigma_f^2 = \sigma_g^2 = \sigma_h^2 = 1$, $f_{d_1} T_s = 0.001$, and $f_{d_2} T_s = 0.05$.

SNR increases, the suboptimal detection rule II has an error floor around 1.5×10^{-4} . On the other hand, the BER of the proposed suboptimal detection rule I improves steadily as the SNR becomes larger. Since the derivation of the suboptimal detection rule II does not take the time-selectivity into account, it can not determine the proper weight of each individual link in the diversity combiner.

Finally, the BER performance of AF relay networks with unequal channel variance is shown in Fig. 4 where two channel variance settings $\sigma_f^2 = 0.1, \sigma_g^2 = 1, \sigma_h^2 = 1$ and $\sigma_f^2 = 1, \sigma_g^2 = 1, \sigma_h^2 = 0.1$ are used in our simulation. In both scenario, the suboptimal detection rule I outperforms the suboptimal detection rule II. When the variance of RD link σ_h^2 is equal to 0.1, the performance gap between the two suboptimal detection rules decreases because the smaller σ_h^2 mitigates the effect of time-selectivity in the mobile-to-mobile RD channel.

V. CONCLUSIONS

In this paper, we derived optimal and suboptimal detection rules for AF relaying cooperative communication systems employing DBPSK modulation in time-selective Rayleigh fading channels. Compared with the conventional detection rule in this context, the proposed suboptimal detection rule depending on the second-order statistics of each channels can yield good performance even when the relay and destination nodes have different Doppler spreads.

REFERENCES

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity - Part I: System description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927-1938, Nov. 2003.
 [2] M. Dohler and Y. Li, "Cooperative Communication - Hardware, Channel & PHY," England: John Wiley & Sons, 2010.

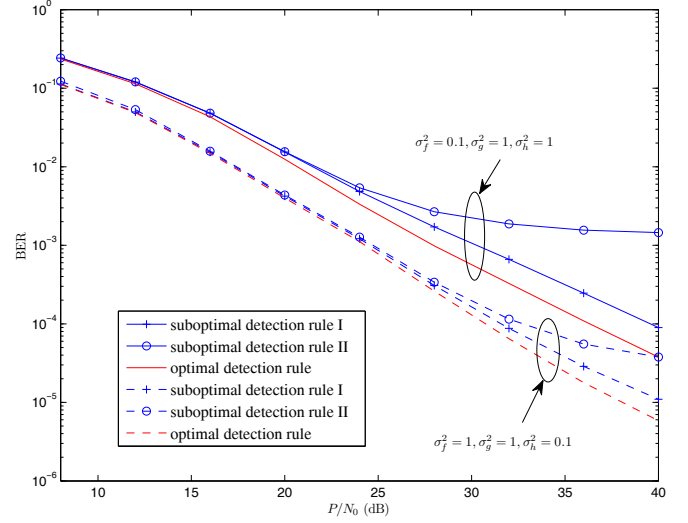


Fig. 4. BER performance of AF relaying systems with DBPSK modulation in time-selective Rayleigh fading channels. $P_1 = P_2 = P/2$, $f_{d_1} T_s = 0.001$, and $f_{d_2} T_s = 0.05$.

[3] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.
 [4] L. Tong, B. M. Sadler, and M. Dong, "Pilot-assisted wireless transmissions: general model, design criteria, and signal processing," *IEEE Signal Proc. Mag.*, vol. 21, no. 6, pp. 12-25, Nov. 2004.
 [5] C. S. Patel and G. L. Stuber, "Channel estimation for amplify and forward relay based cooperation diversity systems," *IEEE Trans. Wireless Commun.*, vol. 6, no. 6, pp. 2348-2356, June 2007.
 [6] F. Gao, T. Cui, and A. Nallanathan, "On channel estimation and optimal training design for amplify and forward relay networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 5, pp. 1907-1916, May 2008.
 [7] B. Gedik and M. Uysal, "Two channel estimation methods for amplify-and-forward relay networks," *Canadian Conference on Electrical and Computer Engineering*, pp. 615-618, May 2008.
 [8] C.-H. Yih, "LMMSE estimation of equivalent noise variance in amplify-and-forward relay communication systems," *International Journal of Electrical Engineering*, vol. 18, no. 5, pp. 235-243, Oct. 2011.
 [9] T. Himsoon, W. Su, and K. J. R. Liu, "Differential transmission for amplify-and-forward cooperative communications," *IEEE Signal Proc. Letters*, vol. 12, no. 9, pp. 597-560, Sep. 2005.
 [10] Q. Zhao and H. Li, "Differential modulation for cooperative wireless systems," *IEEE Trans. Signal Proc.*, vol. 55, no. 5, pp. 2273-2283, May 2007.
 [11] J. D. Parsons, *The Mobile Radio Propagation Channel*, England: John Wiley & Sons, 2000.
 [12] M. K. Varanasi, "A systematic approach to the design and analysis of optimum DPSK receivers for generalized diversity communications over Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 47, no. 9, pp. 1365-1375, Sep. 1999.
 [13] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, England: John Wiley & Sons, 2005.
 [14] A. S. Akki and F. Haber, "A statistical model for mobile-to-mobile land communication channel," *IEEE Trans. Veh. Technol.*, vol. 35, no. 1, pp. 2-7, Feb. 1986. 215-220.
 [15] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. New Jersey: Prentice-Hall, 1993.
 [16] Y. R. Zheng and C. Xiao, "Simulation models with correct statistical properties for Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 51, no. 6, pp. 920-928, June 2003.
 [17] C. S. Patel, G. L. Stüber, and T. G. Pratt, "Simulation of Rayleigh faded mobile-to-mobile communication channels," *IEEE Trans. Commun.*, vol. 53, no. 11, pp. 1876-1884, Nov. 2005.