Computational Approach based on Some Population-Based Optimization Algorithms for Inverse Scattering of a Metallic Cylinder

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Abstract—**The paper presents a study of time domain inverse scattering for a metallic cylinder based on the finite difference time domain (FDTD) method and the evolutionary algorithms (EAs). The FDTD method is employed to calculate the scattered *E* fields for the forward scattering, while the inverse scattering problem is transformed into an optimization one. The idea is to perform the image reconstruction by utilization of some optimization schemes to minimize the discrepancy between the measured and calculated scattered field data. The schemes tested include differential evolution (DE), dynamic differential evolution (DDE), self-adaptive differential evolution (SADE) and self-adaptive dynamic differential evolution (SADDE). The suitability and efficiency of the above methods applied to microwave imaging of a 2-D metallic cylinder are examined. Numerical results show that good reconstruction can be obtained by all optimization methods tested. However, SADDE outperforms DE, DDE and SADE regarding the reconstruction accuracy and the convergent speed in terms of the number of the function calls.**

***Keywords*- Inverse Scattering, Time Domain, Finite Difference Time Domain (FDTD), Differential Evolution (DE) Algorithm, Dynamic Differential Evolution (DDE), Self-Adaptive Differential Evolution (SADE) and Self-Adaptive Dynamic Differential Evolution (SADDE), Cubic-Spline.**

I. INTRODUCTION

In recent decades, the scientific community has witnessed addressed a growing interest inthe detection and imaging of unknown objects located in inaccessible domains by means of electromagnetic fields in microwave range. As a matter of fact, the propagation of microwave is significantly affected by the characteristics of the medium. Therefore, it is profitable to exploit such a phenomenon in order to sense an unknown scenario in a non-invasive fashion. Toward this end, several researches have been pursuing in the framework of non-destructive evaluation and testing and biomedical diagnostics [1]-[4].

However, it is well known that ill-posedness and non-uniqueness are the major difficulties of inverse scattering problems [5]. The inverse scattering problems are usually treated by traditional deterministic methods which are founded on a functional minimization via some gradient-type scheme. The major drawback of the traditional deterministic methods is that the final reconstructed image is highly dependent on the initial trial guess [6]. In general, they tend to get trapped in local minima when the initial trial solution is far from the exact one. Thus, some population-based stochastic methods, such as GA [7], PSO [8], and DE [9], are proposed to search the global extreme of the inverse problems to overcome the major drawback of the deterministic methods. The advantages of applying the algorithms based on stochastic strategies include strong search ability, simplicity, robustness, and insensitivity to ill-posedness. As compared with GA, the algorithm of DE is much easier to implement and converge faster. Moreover, it has been shown that DE outperforms real-coded GA and PSO in terms of convergence speed [10], [11]. In recent years, some papers have compared different algorithm in inverse scattering [10]-[13]. However, these methods exhibit certain drawbacks usually related to the intensive computational effort they demand in order to achieve the global optimum and still the possibility of premature convergence to a local optimum. Hence, it is seemingly natural to use evolutionary algorithms, not only to find the solutions of a problem but also to tune these algorithms for the particular problem. Technically speaking, this requires extra efforts to modify the values of certain control parameters during the course of searching process. The proof of convergence of EAs with self-adaptation is difficult because the control parameters are changed randomly and the selection does not affect their evolution directly [14]. Usually, the best settings of the control parameter for DE and DDE are problem dependent. However, it is interesting to investigate how self-adaptivity can be applied to DE and DDE. It should be noted that a good test of parameter tuning usually requires multiple runs of the algorithm, which could be time consuming and not feasible for certain problems. The proposed self-adaptive concept is able to overcome the disadvantage such that there is no need for multiple runs in order to adjust and/or find good control parameters.

Application of self-adaptive differential evolution (SADE) to real-valued antenna and microwave design problems was investigated [15]. To the best of our knowledge, there are still no research work on self-adaptivity in different differential evolutionary algorithms (DEAs) applied to inverse scattering problems.

In this paper, the computational method combining the FDTD method and DE algorithm is studied first. The forward problem is solved by the FDTD method, for which the subgridding technique [16] is implemented to closely describe the fine structure of the cylinder. The cubic spline [17] is more efficient in terms of the unknown number required to describe a cylinder of arbitrary cross section. In addition, by using the cubic spline, the coordinates of local origin inside the cylinder can serve as the searching parameters and move around the searching space, which is very hard to achieve, if not impossible when the trigonometric series expansion is used in the inversion procedure. In sections II and III, the forward scattering and inverse problems are presented, respectively. In sections IV and V, evolutionary algorithms and the numerical results of the proposed inverse schemes are given, respectively. Finally, in VI section some conclusions are drawn.

II. Theoretical Formulation

Let us consider a two-dimensional metallic cylinder in a free space as shown in Figure 1. The cylinder is parallel to z axis, while the cross-section of the cylinder is arbitrary. The object is illuminated by a Gaussian pulse line source located at the points denoted by Tx, sequentially, while reflected waves are recorded at those points denoted by Rx. The computational domain is discretized by Yee cells. It should be mentioned that the computational domain is surrounded by the optimized perfect matching layers (PML) absorber to reduce the reflection from the environment-PML interface.

The direct scattering problem is to calculate the scattered electric fields while the shape and location of the scatterer is given. The shape function $F(\theta)$ of the scatterer is described by the trigonometric series in the direct scattering problem

$$F(\theta)=\sum_{n=0}^{N/2}B_n\cos(n\theta)+\sum_{n=1}^{N/2}C_n\sin(n\theta) \tag{1}$$

III. INVERSE PROBLEM

For the inverse scattering problem, the shape and location of the perfectly conducting cylinder are reconstructed by the given scattered electric field measured at the receivers. This problem is resolved by an optimization approach, for which the global searching evolutionary algorithms are employed to minimize the following objective function (OF):

$$OF=\frac{\sum_{n=1}^{N_i}\sum_{m=1}^{M}\sum_{q=0}^{Q}\left|E_z^{exp}(n,m,q\Delta t)-E_z^{cal}(n,m,q\Delta t)\right|}{\sum_{n=1}^{N_i}\sum_{m=1}^{M}\sum_{q=0}^{Q}\left|E_z^{exp}(n,m,q\Delta t)\right|} \tag{2}$$

where E_z^{exp} and E_z^{cal} are experimental electric fields and the calculated electric fields, respectively. N_i and M are the total number of the transmitters and receivers, respectively. Q is the total time step number of the recorded electric fields.

IV. EVOLUTIONARY ALGORITHMS

（A）*Differential Evolution (DE)*

DE algorithm starts with an initial population of potential solutions that is composed by a group of randomly generated individuals which represent the center position and the geometrical radiuses of the cylinder. Each individual in DE algorithm is a D-dimensional vector consisting of D optimization parameters. The initial population may be expressed by $\{x_i: \ i=1,\ 2,\cdots,Np\}$, where Np is the population size. After initialization, DE algorithm performs the genetic evolution until the termination criterion is met. DE algorithm, like other EAs, also relies on the genetic operations (mutation, crossover and selection) to evolve generation by generation. The mutant vector V_j^{k+1} is generated according to equation (3) for typical DE [18].

$$(V_j^{k+1})_i=(X_j^k)_i+\chi\cdot[(X_m^k)_i-(X_n^k)_i]$$
$$j,m,n\in[0,N_p-1]\text{，}m\neq n \tag{3}$$

where i=1~D and χ is the scaling factor associated with the vector difference $(X_m^k-X_n^k)$. A modified DE namely dynamic differential evolution, DDE, is proposed to speedup the convergence of the DE. The key distinction between a DDE and a typical DE is on the population updating mechanism. In a typical DE, all the update actions of the population are performed at the end of the generation of which the implementation is referred to as static updating mechanism. Alternatively, the updating mechanism of DDE is carried out in a dynamic way: each parent individual would be replaced by its offspring if the offspring has obtained a better objective function value than its parent. Thus, DDE can respond the progress of population status immediately and is expected to yield faster convergence speed than the typical DE.

SADE and SADDE are based on DE and DDE schemes, respectively. Each vector is extended to carry its own values of control parameters λ, F and CR. Moreover, the control parameters are self-adjusted in every generation for each individual according to the following scheme:

$$F_{i,G+1}=\begin{cases}F_l+rand_1*F_u, & if\quad rand_2<0.1\\ F_{i,G}, & otherwise\end{cases} \tag{4}$$

$$\lambda_{i,G+1} = \begin{cases} \lambda_l + rand_3 * \lambda_u, & if \quad rand_4 < 0.1 \\ \lambda_{i,G}, & otherwise \end{cases} \tag{5}$$

$$CR_{i,G+1} = \begin{cases} rand_5, & if \quad rand_6 < 0.1 \\ CR_{i,G}, & otherwise \end{cases} \tag{6}$$

where *rand1*, *rand2*, *rand3*, *rand4, rand5* and *rand6* are uniform random numbers with its value uniformly between 0 and 1. F_l, F_u, λ_l and λ_u are the lower and the upper limits of F and λ, respectively. Both F_l and λ_l are set to 0.1, while both F_u and λ_u are set to 0.9 [14]. Based on the self-adaptive concept, the F, λ and CR parameters adjust automatically while the time consumption does not increase. More details about the SADE and SADDE algorithm can be found in [3], [14].

V. NUMERICAL RESULT

As shown in Figure 1, the problem space is divided in 68×68 grids with the grid size $\Delta x = \Delta y$ =5.95 mm. The metallic cylinder is located in free space. The cylindrical object is illuminated by a transmitter at four different positions, N_i=4. The scattered E fields for each illumination are collected at the eight receivers, M=8. Note that the simulated result using one incident wave is much worse than that by two incident waves. In order to get more accurate result, four transmitters are used here. The transmitters and receivers are collocated at a distance of 24 grids from the origin. The scatterer is illuminated by cylindrical waves with the electric field polarized along the axis, while the time dependence of the field is of a derivative Gaussian pulse. There are eleven unknown parameters to retrieve, which include the center position (X_O, Y_O), the radiuses ρ_i, $i = 1,2,\cdots,8$, of the shape function and the slope ρ'_N. Very wide searching ranges are used for the DDE to optimize the objective function given by (2). The parameters and the corresponding searching ranges are listed follows: $-47.6\text{mm} \le X_O \le 47.6\text{mm}$, $-47.6\text{mm} \le Y_O \le 47.6\text{mm}$, $5.95\text{mm} \le \rho_i \le 71.4\text{mm}$, $i = 1,2,\cdots,8$, and $-2 \le \rho'_N \le 2$. The crossover rate *CR* is set to be 0.8. Both parameters F and λ are set to be 0.8 in DE and DDE. The population size *Np* is set to be 110. In our simulation, DE, DDE, SADE and SADDE used the same population size and the same stopping criteria.

For the first example, the metallic cylinder with shape function $F(\theta) = 29.75 + 11.9\cos(2\theta)$ mm is considered. It is shown that the DE scheme is able to achieve good convergences. Here, the r.m.s. error is defined as

$$Error = \{\frac{1}{N'}\sum_{i=1}^{N'}[F^{cal}(\theta_i) - F(\theta_i)]^2 / F^2(\theta_i)\}^{1/2} \tag{7}$$

where the N' is set to 720.

The reconstructed images of the example are shown in Fig. 2 and Fig. 3, respectively. The DF value for DE, DDE, SADE and SADDE are about 10.8%, 9.6%, 2.1% and 1.3% in the final generation, respectively. Figure 3 shows that DDE and SADDE the relative errors of the shape decrease quickly and good convergences are achieved within 100 generation.

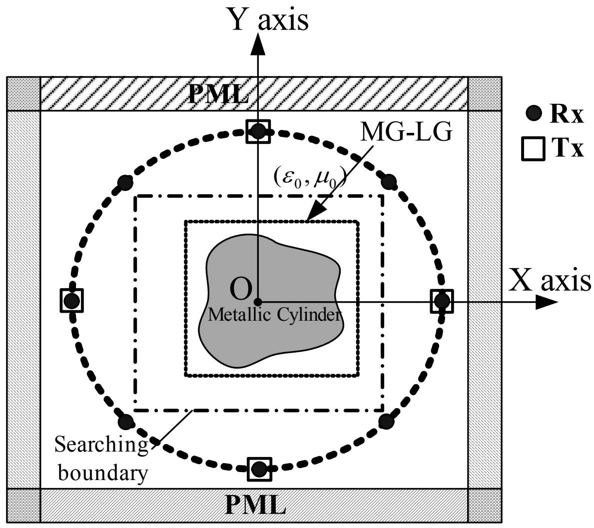

Fig. 1 Geometry for the inverse scattering of a metallic cylinder of arbitrary shape in free space.

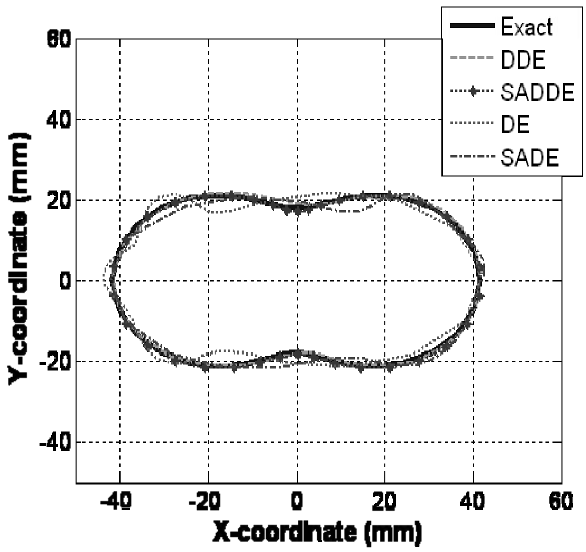

Fig. 2 The exact cross section of the cylinder and final reconstructed shape.

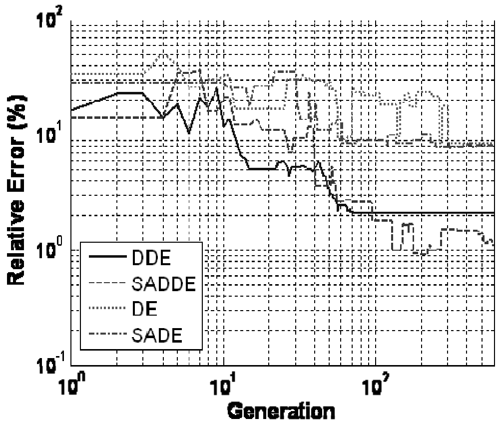

Fig. 3 The objective function (*OF*) versus numbers of function calls for the example.

VI. Conclusions

We present a study for the time domain inverse scattering of metallic cylinders. The inverse problem is reformulated into an optimization one. By combining the FDTD method and the evolutionary algorithms, good reconstructed results are obtained. Numerical results show that the SADDE has better reconstructed results as compared with the others when the same number of iterations is concerned. More test results about these evolutionary algorithms will be reported in the conference presentation.

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