Performance of the MIMO CS-PRP-OFDM Systems with Complementary Codes

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***Abstract*—This paper presents a new transceiver framework of the MIMO-OFDM systems, with the space-time block code (ST-BC). Unlike the conventional Pseudo-Random Postfix (PRP) - OFDM in our proposed framework the null samples of zero-padding (ZP)-OFDM is replaced by known cyclic postfix sequence (CPS), weighted by a pseudo-random (PR) scalar. Since the CPS is implemented by the cyclic-shift (CS) complementary code (CC) sequences, the proposed transceiver scheme is referred to as the MIMO CS-PRP-OFDM systems. By exploring the useful property of CC sequences, convolved with channel information, the receiver design associated with the semi-blind channel estimation of the proposed MIMO CS-PRP-OFDM systems is affected only by the background noise. It avoids the interference of the transmitted signals, and yields achieving better system performance, in terms of symbol error rate, compared with the conventional PRP-OFDM based systems, with less complexity. This is especially true when the signal-to-noise ratio is increased.**

Keywords- MIMO-OFDM; space-time encoder; redundant sequence; complementary codes; cyclic-postfix

I. INTRODUCTION

Multiple-input multiple-output (MIMO) systems provide a very promising means to increase the spectral efficiency. Equipped with space-time block encoding (ST-BC) [10] in the transmitter and applied the intelligent signal processing at the receiver [6], MIMO systems can provide diversity and coding gains over single antenna system. The orthogonal frequency division multiplexing (OFDM) technique has been widely adopted for high-speed wireless communications, due to its robustness of multipath propagation [1]-[5]. In OFDM system to maintain the orthogonality among subcarriers and to prevent the inter-block-interference (IBI), a redundant sequence named as the cyclic prefix (CP), zero-padding (ZP) and pseudo-random postfix (PRP), with length no less than the order of channel impulse response (CIR), is inserted between consecutive block symbols in the transmitter [1][4][8][9]. Eventually, the ZP-OFDM [4] system was proposed to circumvent the problem of channel null occurred when the CP-OFDM system was employed, with the price of increased receiver complexity. Although the appended zero-samples can be used to eliminate the channel induced IBI, it seems a little waste. Hence, in the PRP-OFDM system [8][9] the PRP is used to replace the guard interval contents of ZP-OFDM. It capitalizes on the advantages of ZP-OFDM, but unlike the ZP-OFDM it exploits additional information, i.e. pseudo-randomly weighted postfix sequences, for semi-blindly channel estimation, with order-one statistics of the received signal [7]. At the receiver end, by removing the added redundant sequence the effect of IBI could be alleviated, effectively. The PRP-OFDM system with single antenna was extended to the MIMO case, by encoding the transmitted signal and postfixes vectors with two different space-time encoder [13][14], and is referred to as the ST-BC MIMO PRP-OFDM systems. With specific design of the space-time (ST) encoder for the PRP sequences, the MIMO PRP-OFDM system is capable to perform a semi-blind estimation for the entire MIMO-channel, by exploiting the order-one statistics of the received signals.

In the aforementioned PRP-OFDM systems [8][9][13][14], to extract the channel information it assumed that the transmitted signals and background noise are zero-mean, and is performed by averaging a number of collected signal-blocks in the receiver to avoid the effect due to the interfered transmitted signal. The zero-mean assumptions do not hold, due to the fact that the collection of sufficient signal-blocks may not be possible, in practical situations. Especially, when the signal-to-noise ratio (SNR) is increased, the effect of the interfered transmitted signal becomes significant. To circumvent the above mentioned problem, in this paper, we propose a new transceiver framework of the ST-BC MIMO-OFDM systems to further enhance the system reliability and with less complexity compared to the PRP-OFDM based systems. In our proposed scheme, the null samples of ZP-OFDM is replaced by the known cyclic-postfix sequences (CPS), weighted by a pseudo-random scalar. Since the CPS is implemented by cyclic-shift (CS) complementary codes (CC) [15][16], it is referred to as the MIMO CS-PRP-OFDM system. By exploring the useful property of CC sequences convolved with channel information, at the receiver we can enhance the semi-blind channel estimation, effectively. The resulting channel estimation is affected simply by the background noise; it avoids the interference of the transmitted signals, yields better system performance achievement. This is especially true when the signal-to-noise ratio (SNR) is increased.

II. SYSTEM MODEL DESCRIPTION

In Fig.1, we consider the discrete-time transceiver model of the MIMO CS-PRP-OFDM system, with two transmit-antennas

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and M_r receive-antennas. As an introduction, we assume that the transmitting symbol block is with length M, the $M \times 1$ signal block $\mathbf{s}(n)$ is first modulated by the $M \times M$ inverse fast Fourier transform (IFFT) matrix F^{-1} (or F^H), to obtain the resulting time-domain signal block vector $\mathbf{u}(n)=\mathbf{F}^H\mathbf{s}(n)$, denoting as $\mathbf{u}(n) = [u_0(n)\ u_1(n)\ \ldots u_{M-1}(n)]^H$, and H denotes the Hermitian transpose. As in [13][14] the signal-blocks and postfix vectors are, independently, encoded by two STC-encoders, where the new proposed cyclic-postfix sequences (or cyclic complementary-code sequences) will be introduced later. First, at transmitter, the corresponding $M \times 1$ successive precoded blocks $\mathbf{u}(2i)$ and $\mathbf{u}(2i+1)$ (or $\mathbf{u}(n)$, $n = 2i + k$, $k = 0$ and 1) are sent to the ST encoder $M(.)$, to output the following $2M \times 2$ ST-coded matrix: (Lee [12]).

$$\begin{bmatrix} \bar{\mathbf{u}}^{(1)}(2i) & \bar{\mathbf{u}}^{(1)}(2i+1) \\ \bar{\mathbf{u}}^{(2)}(2i) & \bar{\mathbf{u}}^{(2)}(2i+1) \end{bmatrix} = \begin{bmatrix} \mathbf{u}(2i) & -\mathbf{P}_M^{(0)}\mathbf{u}^*(2i+1) \\ \mathbf{u}(2i+1) & \mathbf{P}_M^{(0)}\mathbf{u}^*(2i) \end{bmatrix} \quad (1)$$

Where the superscript * denotes the complex conjugate. At ith time block interval, the symbol-blocks $\bar{\mathbf{u}}^{(1)}(i)$ and $\bar{\mathbf{u}}^{(2)}(i)$ are transmitted through the first and second antenna, respectively. In (1) the function of $M \times M$ permutation matrix (or reverse matrix) $\mathbf{P}_M^{(n)}$ is such that for vector $\mathbf{a}=[a(0)\ a(1)\ldots a(M-1)]^T$ with this operation, the pth element becomes $[\mathbf{P}_M^{(n)}\ \mathbf{a}]_p = a((M-p+n) \bmod M)$. Thus in (1) $\mathbf{P}_M^{(0)}\mathbf{u}^*(n) = [u_{M-1}(n)\ u_{M-2}(n)\ \ldots\ u_0(n)]^H$ is simply a reversed version of $\mathbf{u}^*(n)$.

As described earlier, the CC sequence is adopted to build up the modulator of the MIMO CS-PRP-OFDM systems. Basically, the CC sequences or series have the important property that their periodic autocorrelation series sum is zero everywhere, except at the zero shifts [15][16]. For exposition, let us consider a pair of CC sequence $\mathbf{a} = [a_0\ a_1 \ldots a_n]$ and $\mathbf{b} = [b_0\ b_1\ \ldots\ b_n]$ with equal length n+1. By definition, the respective autocorrelation series are defined as

$$p_j = \sum_{i=1}^{n} a_{i-1}a_{(i+j-1) \bmod n} \text{ and } q_j = \sum_{i=1}^{n} b_{i-1}b_{(i+j-1) \bmod n}.$$

As described in [15][16], the pair of complementary codes **a** and **b** satisfies the following property; $p_j + q_j=0$ for $j\neq 0$ and $p_0 + q_0=2n$. For example, if $\mathbf{a}$ =[-1 -1 -1 1 1 1 -1 1] and $\mathbf{b}$ =[-1 -1 -1 1 -1 1 -1], with equal length (D = 8), are selected, we have $p_j + q_j=0$ for $j\neq 0$ and $p_0 + q_0=16$. With these properties, during the semi-blind channel estimation, we could sum the autocorrelation series of two complementary codes to create a diagonal matrix. It has desired noise suppression capability, and hence improving the system performance. Based on the above discussion, we can introduce the cyclic-shift sequences used as the postfix sequence in the proposed CS-PRP-OFDM system.

Let us consider the $D\times 1$ cyclic postfix sequence $\mathbf{c} = [c_0(i)\ c_1(i)\ldots c_{D-1}(i)]^T$, before ST encoder $W(.)$; it consists of two equal length CC sequences. At the beginning, we insert the encoded postfix sequence, which uses one of the first complementary sequences **a**, with zero-shift, behind the transmitting signal block encoded by the ST encoder, $M(.)$. For the next transmitting signal block, do the same processing except that the postfix sequence is obtained by cyclic-shifting **a** once, before it is encoded by the ST encoder $W(.)$. Again, repeating the same processes until the first D transmitting signal blocks, encoded by the ST encoder, are appended with the ST-coded cyclic-shifting postfix sequences. Following the same manner as the first complementary code sequence **a**, and do the same thing with the second complementary code sequence **b,** with zero-shift, for the next D transmitting signal blocks to complete the whole 2D transmitted signal blocks for transmission. Due to this fact, we refer it as the CS-PRP-OFDM system, to distinct it from the conventional PRP-OFDM system. To be more specific, we define the cyclic postfix sequences as

$$\mathbf{c}(i) = [a_{(D-i) \bmod D}\ a_{(D-i+1) \bmod D}\ \cdots\ a_{(D-i-1) \bmod D}]^T \quad (2a)$$

for i mod 2D to be 0, …,D-1, and

$$\mathbf{c}(i) = [b_{(D-i) \bmod D}\ b_{(D-i+1) \bmod D}\ \cdots\ b_{(D-i-1) \bmod D}]^T \quad (2b)$$

for i mod 2D to be D, …,2D-1, where D is no less than L+1 (L is the channel order), and for simplicity, we let $D = L$+1. The purpose of the cyclic-shifting the CPS is to form a full rank matrix for semi-blind channel estimation, at the receiver. First, we would like to introduce the ST-BC encoder for the CS-PRP-OFDM system. As depicted in Fig. 1, the corresponding cyclic-postfix sequences are applied to the ST-BC encoder $W(.)$ to construct the $2D\times 2$ cyclic-postfix matrix, i.e.,

$$\begin{bmatrix} \bar{\mathbf{c}}^{(1)}(2i) & \bar{\mathbf{c}}^{(1)}(2i+1) \\ \bar{\mathbf{c}}^{(2)}(2i) & \bar{\mathbf{c}}^{(2)}(2i+1) \end{bmatrix} = \begin{bmatrix} w^{(1)}\alpha(2i) & w^{(1)}\alpha(2i+1) \\ w^{(2)}\alpha(2i) & w^{(2)}\alpha(2i+1) \end{bmatrix} \otimes \mathbf{c}(i) \quad (3)$$

where $\bar{\mathbf{c}}^{(l)}(n)$ represents the cyclic-postfix vector of nth block for the lth transmit-antenna (l =1, and 2), with index $n = 2i + k$, k = 0 and 1), and ⊗ denotes the Kronecker product. The function of $w^{(l)}$ in (3) is used to identify the all MIMO channels for two transmit-antennas case, at receiver. In consequence, the 2 ×2 matrix **W,** which gathers the corresponding deterministic weighting factor $w^{(l)}(.)$ is denoted as

$$\mathbf{W} = \begin{bmatrix} w^{(1)}(0) & w^{(2)}(0) \\ w^{(1)}(1) & w^{(2)}(1) \end{bmatrix} \quad (4)$$

By choosing matrix **W** to be a full rank orthogonal matrix ($\mathbf{W}^H\mathbf{W}=2\mathbf{I}_2$), we are able to separate all transmitted-postfixes, completely [13][14]. It is notice that the use of the pseudo random- weight $\alpha(n)$ can avoid the cyclo-stationary in the postfix sequences [8][9]. With $P=M+D$, the $P\times 1$ signal vector $\mathbf{x}^{(l)}(n)$, transmitted from the lth antenna is represented as

$$\mathbf{x}^{(l)}(n) = \mathbf{T}_{ZP}\bar{\mathbf{u}}^{(l)}(n) + \mathbf{T}_P\bar{\mathbf{c}}^{(l)}(n) \quad (5)$$

where the $P\times D$ and $P\times M$ matrices, $\mathbf{T}_P$ and $\mathbf{T}_{ZP}$, are defined, respectively, as

$$\mathbf{T}_P = \begin{bmatrix} \mathbf{0}_{M\times D} \\ \mathbf{I}_{D\times D} \end{bmatrix} \text{ and } \mathbf{T}_{ZP} = \begin{bmatrix} \mathbf{I}_{M\times M} \\ \mathbf{0}_{D\times M} \end{bmatrix}.$$

Also, the channel response between the lth transmit-antenna and mth receive-antenna is modeled as a FIR filter denoted as $\mathbf{h}^{(lm)} = [h_0^{(lm)}\ h_1^{(lm)} \ldots h_L^{(lm)}]^T$, where L is the channel order. The $P\times 1$ signal vector received at the mth receive-antenna is defined as

$$\mathbf{r}^{(m)}(n) = \sum_{l=1}^{2}[\mathbf{H}_0^{(lm)}\mathbf{x}^{(l)}(n) + \mathbf{H}_1^{(lm)}\mathbf{x}^{(l)}(n-1)] + \mathbf{v}^{(m)}(n) \quad (6)$$

In (6), channel matrices $\mathbf{H}_0^{(lm)}$ and $\mathbf{H}_1^{(lm)}$ are denoted as the upper triangular Toeplitz and lower triangular matrices, respectively. The noise vector $\mathbf{v}^{(m)}=[v_0^{(m)}\ v_1^{(m)} \ldots v_{P-1}^{(m)}]^T$ has the same dimension with $\mathbf{r}^{(m)}(n)$, and the elements are assumed to be with variance σ_v^2.

III. SEMI-BLIND CHANNEL ESTIMATION AND EQUALIZATION OF ST-BC MIMO-OFDM SYSTEM

We first develop the semi-blind channel estimation for the MIMO CS-PRP-OFDM system and after the equalization will be performed with the estimated channel information.

A. Order-one MIMO CS-PRP-OFDM Semi-blind Channel Estimation

First, by observing the signal vector received of the mth receive-antenna in (6), we found that the last element of received signal vector consists of the postfix-sequence convolved with channel information, and interfered with background noise. Thus, we may collect the last element of two consecutive blocks, $\mathbf{r}^{(m)}(2i)$ and $\mathbf{r}^{(m)}(2i+1)$, and after some mathematical manipulation, the pth entry of both vectors are given by

$$[\mathbf{r}^{(m)}(2i)]_P = \sum_{l=1}^{2}\alpha(2i)w^{(l)}(0)[\bar{\mathbf{h}}^{(lm)}]^T\mathbf{c}(i) + [\mathbf{v}^{(m)}(2i)]_P \quad (7)$$

and

$$[\mathbf{r}^{(m)}(2i+1)]_P = \sum_{l=1}^{2}\alpha(2i+1)w^{(l)}(1)[\bar{\mathbf{h}}^{(lm)}]^T\mathbf{c}(i) + [\mathbf{v}^{(m)}(2i+1)]_P \quad (8)$$

Where in (7) and (8) $\bar{\mathbf{h}}^{(lm)} = [h_0^{(lm)}\ h_1^{(lm)} \ldots h_L^{(lm)}]^T$ is the reverse version of $\mathbf{h}^{(lm)}$. Next, dividing (7) and (8) by $\alpha(2i)$ and $\alpha(2i+1)$, respectively, we can remove the factor of pseudo random scalar, and the related signal vectors can be written as

$$d^{(m)}(2i) = \frac{[\mathbf{r}^{(m)}(2i)]_P}{\alpha(2i)} = \sum_{l=1}^{2}w^{(l)}(0)(\bar{\mathbf{h}}^{(lm)})^T\mathbf{c}(i) + \frac{[\mathbf{v}^{(m)}(2i)]_P}{\alpha(2i)} \quad (9)$$

and

$$d^{(m)}(2i+1) = \frac{[\mathbf{r}^{(m)}(2i+1)]_P}{\alpha(2i+1)} = \sum_{l=1}^{2}w^{(l)}(1)(\bar{\mathbf{h}}^{(lm)})^T\mathbf{c}(i) + \frac{[\mathbf{v}^{(m)}(2i+1)]_P}{\alpha(2i+1)} \quad (10)$$

Now, by stacking both $d^{(m)}(2i)$ and $d^{(m)}(2i+1)$ into a 2×1 vector $\mathbf{d}^{(m)}(2i)=[d^{(m)}(2i)\ d^{(m)}(2i+1)]^T$, and after some manipulation, we get

$$\mathbf{d}^{(m)}(i) = \sum_{l=1}^{2}\begin{bmatrix} w^{(l)}(0)\left(\bar{\mathbf{h}}^{(lm)}\right)^T\mathbf{c}(i) \\ w^{(l)}(1)\left(\bar{\mathbf{h}}^{(lm)}\right)^T\mathbf{c}(i)\end{bmatrix} + \begin{bmatrix}[\mathbf{v}^{(m)}(2i)]_P/\alpha(2i) \\ [\mathbf{v}^{(m)}(2i+1)]_P/\alpha(2i+1)\end{bmatrix}$$

$$= \mathbf{W}\begin{bmatrix}(\bar{\mathbf{h}}^{(1m)})^T\mathbf{c}(i) \\ (\bar{\mathbf{h}}^{(2m)})^T\mathbf{c}(i)\end{bmatrix} + \begin{bmatrix}[\mathbf{v}^{(m)}(2i)]_P/\alpha(2i) \\ [\mathbf{v}^{(m)}(2i+1)]_P/\alpha(2i+1)\end{bmatrix} \quad (11)$$

We note that $\mathbf{W}$ was defined in (4). By multiplying $\mathbf{W}^H$ in both sides of (11), we can isolate each individual channel contribution to $\bar{\mathbf{h}}^{(lm)}$, and divide it by two to get

$$\mathbf{z}^{(m)}(i) = \mathbf{W}^H\mathbf{d}^{(m)}(i)/2$$

$$= \begin{bmatrix}(\bar{\mathbf{h}}^{(1m)})^T\mathbf{c}(i) \\ (\bar{\mathbf{h}}^{(2m)})^T\mathbf{c}(i)\end{bmatrix} + \frac{1}{2}\mathbf{W}^H\begin{bmatrix}\frac{1}{\alpha(2i)}[\mathbf{v}^{(m)}(2i)]_P \\ \frac{1}{\alpha(2i+1)}[\mathbf{v}^{(m)}(2i+1)]_P\end{bmatrix}$$

$$= \begin{bmatrix}(\mathbf{c}(i))^T\bar{\mathbf{h}}^{(1m)} \\ (\mathbf{c}(i))^T\bar{\mathbf{h}}^{(2m)}\end{bmatrix} + \tilde{\mathbf{v}}^{(m)}(i) \quad (12)$$

where $\mathbf{z}^{(m)} = [z_1^{(m)}(i)\ z_2^{(m)}(i)]^T$ is a 2×1 vector, and

$$\tilde{\mathbf{v}}^{(m)}(i) = [\tilde{v}_1^{(m)}(i)\ \ \tilde{v}_2^{(m)}(i)]^T = \frac{1}{2}\mathbf{W}^H\begin{bmatrix}[\mathbf{v}^{(m)}(2i)]_P/\alpha(2i) \\ [\mathbf{v}^{(m)}(2i+1)]_P/\alpha(2i+1)\end{bmatrix} \quad (13)$$

Next, we stack $z_1^{(m)}(i)$ and $z_2^{(m)}(i)$ for $i= 0 \sim 2L+1$, to get

$$\mathbf{z}_l^{(m)} = \mathbf{C}\mathbf{h}^{(lm)} + \tilde{\mathbf{v}}_l^{(m)} \quad (14)$$

In (14) $\mathbf{z}_l^{(m)} = [z_l^{(m)}(0)\ z_l^{(m)}(1) \ldots z_l^{(m)}(2L+1)]^T$, $\mathbf{h}^{(lm)} = [h_L^{(lm)}\ h_{L-1}^{(lm)} \ldots h_0^{(lm)}]^T$, $\tilde{\mathbf{v}}_l^{(m)} = \left[\tilde{v}_l^{(m)}(0)\ \tilde{v}_l^{(m)}(1) \ldots \tilde{v}_l^{(m)}(2L+1)\right]^T$, and $\mathbf{C} = \begin{bmatrix}\mathbf{C}_{1,CIRC} \\ ---- \\ \mathbf{C}_{2,CIRC}\end{bmatrix}$, where matrices $\mathbf{C}_{1,CIRC}$ and $\mathbf{C}_{2,CIRC}$ are circulant matrices with their first rows to be denoted, respectively, as $[a_0\ a_1 \ldots a_{D-1}]$ and $[b_0\ b_1 \ldots b_{D-1}]$. Due to the inherent property of CC sequences, we have $(\mathbf{C}_{1,CIRC})^H \mathbf{C}_{1,CIRC} + (\mathbf{C}_{2,CIRC})^H \mathbf{C}_{2,CIRC} = 2D\times\mathbf{I}_D$ [15][16]. With this fact, we obtain an estimated channel vector of $\bar{\mathbf{h}}^{(lm)}$, that is

$$\hat{\bar{\mathbf{h}}}^{(lm)} = \frac{1}{2D}\left(\mathbf{C}_{1,CIRC}^H \times \begin{bmatrix} z_l^{(m)}(0) \\ z_l^{(m)}(1) \\ \vdots \\ z_l^{(m)}(L)\end{bmatrix} + \mathbf{C}_{2,CIRC}^H \times \begin{bmatrix} z_l^{(m)}(L+1) \\ z_l^{(m)}(L+2) \\ \vdots \\ z_l^{(m)}(2L+1)\end{bmatrix}\right) \quad (15)$$

If the noise signal is assumed to be zero-mean and the cyclic-postfix sequence is transmitted periodically. We may collect sufficient symbol blocks (e.g., N_b) for averaging to reduce the effect due to background noise. Thus, (14) can be rewritten as

$$\mathbf{z}_l^{(m)} = \mathbf{C}\begin{bmatrix} h_L^{(lm)} \\ h_{L-1}^{(lm)} \\ \vdots \\ h_0^{(lm)} \end{bmatrix} + \frac{1}{\beta}\sum_{j=0}^{\beta-1}\begin{bmatrix} \tilde{v}_l^{(m)}(j(L+1)) \\ \tilde{v}_l^{(m)}(j(L+1)+1) \\ \vdots \\ \tilde{v}_l^{(m)}(j(L+1)+2L+1) \end{bmatrix} \quad (16)$$

where $D = L+1$ and $\beta = N_b / 4(L+1)$.

B. Equalization of the ST-BC MIMO CS-PRP-OFDM Receiver

After obtaining the estimated channel response, we may remove the postfix sequences, and convert the signals received of (6) into the form of ST-BC MIMO ZP-OFDM, i.e.

$$\mathbf{r}_{ZP}^{(m)}(n) = \mathbf{r}^{(m)}(n) - \sum_{l=1}^{2}[\mathbf{H}_0^{(lm)}\mathbf{T}_P\bar{\mathbf{c}}^{(l)}(n) + \mathbf{H}_1^{(lm)}\mathbf{T}_P\bar{\mathbf{c}}^{(l)}(n-1)] \quad (17a)$$

or

$$\mathbf{r}_{ZP}^{(m)}(n) = \sum_{l=1}^{2}\mathbf{H}_0^{(lm)}\mathbf{T}_{ZP}\bar{\mathbf{u}}^{(l)}(n) + \mathbf{v}^{(m)}(n) \quad (17b)$$

Using the fact that $\mathbf{H}_0\mathbf{T}_{zp} = \mathbf{H}_{CIRC}\,\mathbf{T}_{zp}$, the above equation can be rewritten as

$$\mathbf{r}_{ZP}^{(m)}(n) = \sum_{l=1}^{2}\mathbf{H}_{CIRC}^{(lm)}\mathbf{T}_{ZP}\bar{\mathbf{u}}^{(l)}(n) + \mathbf{v}^{(m)}(n) \quad (18)$$

where $\mathbf{H}_{CIRC}^{(lm)}$ is a $P \times P$ circulant matrix whose first row is given as $\begin{bmatrix} h_0^{(lm)} & 0 & \cdots & 0 & h_L^{(lm)} & \cdots & h_1^{(lm)} \end{bmatrix}$. Next, using the fact that $\mathbf{P}_M^{(n)}\ \mathbf{T}_{zp} = \mathbf{T}_{zp}\,\mathbf{P}_M^{(0)}$, the two consecutive blocks of (18) are written as

$$\mathbf{r}_{ZP}^{(m)}(2i) = \mathbf{H}_{CIRC}^{(1m)}\mathbf{T}_{ZP}\mathbf{u}(2i) + \mathbf{H}_{CIRC}^{(2m)}\mathbf{T}_{ZP}\mathbf{u}(2i+1) + \mathbf{v}^{(m)}(2i) \quad (19)$$

and

$$\mathbf{r}_{ZP}^{(m)}(2i+1) = -\mathbf{H}_{CIRC}^{(1m)}\mathbf{P}_P^{(M)}\mathbf{T}_{ZP}\mathbf{u}^*(2i+1) + \mathbf{H}_{CIRC}^{(2m)}\mathbf{P}_P^{(M)}\mathbf{T}_{ZP}\mathbf{u}^*(2i) + \mathbf{v}^{(m)}(2i+1) \quad (20)$$

Similar to [12], the corresponding frequency domain signals of (19) and (20) can be obtained, that is, $\mathbf{y}^{(m)}(2i) = \mathbf{F}_P\mathbf{r}_{ZP}^{(m)}(2i)$ and $\mathbf{y}^{(m)}(2i+1) = \mathbf{F}_P\mathbf{P}_P^{(M)}(\mathbf{r}_{ZP}^{(m)}(2i+1))^*$. Stacking both terms, we get

$$\bar{\mathbf{y}}^{(m)}(i) = \begin{bmatrix} \mathbf{y}^{(m)}(2i) \\ \mathbf{y}^{(m)}(2i+1) \end{bmatrix} = \mathbf{D}\begin{bmatrix} \mathbf{\Theta}\mathbf{s}(2i) \\ \mathbf{\Theta}\mathbf{s}(2i+1) \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{v}}^{(m)}(2i) \\ \tilde{\mathbf{v}}^{(m)}(2i+1) \end{bmatrix} \quad (21)$$

where

$$\mathbf{D} = \begin{bmatrix} \mathbf{H}_{DIAG}^{(1m)} & \mathbf{H}_{DIAG}^{(2m)} \\ (\mathbf{H}_{DIAG}^{(2m)})^* & -(\mathbf{H}_{DIAG}^{(1m)})^* \end{bmatrix}, \quad \mathbf{\Theta} = \mathbf{F}_P\mathbf{T}_{ZP}\mathbf{F}^H,$$

$\tilde{\mathbf{v}}^{(m)}(2i) = \mathbf{F}_P\mathbf{v}^{(m)}(2i)$ and $\tilde{\mathbf{v}}^{(m)}(2i+1) = \mathbf{F}_P\mathbf{P}_P^{(M)}(\mathbf{v}^{(m)}(2i+1))^*$. We then demodulate $\bar{\mathbf{y}}^{(m)}(i)$ with diversity gains by a simple matrix multiplication

$$\bar{\mathbf{e}}^{(m)}(i) = \mathbf{D}^H\bar{\mathbf{y}}^{(m)}(i) = \begin{bmatrix} \bar{\mathbf{D}}^{(m)}\mathbf{\Theta}\mathbf{s}(2i) \\ \bar{\mathbf{D}}^{(m)}\mathbf{\Theta}\mathbf{s}(2i+1) \end{bmatrix} + \mathbf{D}^H\begin{bmatrix} \tilde{\mathbf{v}}^{(m)}(2i) \\ \tilde{\mathbf{v}}^{(m)}(2i+1) \end{bmatrix} \quad (22)$$

where $\bar{\mathbf{D}}^{(m)} = \mathbf{H}_{DIAG}^{(1m)}\left(\mathbf{H}_{DIAG}^{(1m)}\right)^* + \mathbf{H}_{DIAG}^{(2m)}\left(\mathbf{H}_{DIAG}^{(2m)}\right)^*$. We note that multi-antenna diversity of order two can be achieved.

IV. SIMULATION RESULTS

To verify the merits of the new proposed ST-BC MIMO CS-PRP-OFDM system, computer simulation is carried out. As described earlier, by exploiting the cyclic postfix sequences, using the complementary code sequences, the semi-blind channel estimation could be performed, with low complexity. After obtaining the estimated CIR, we can remove the effect of IBI by removing the postfix sequences from the received signal blocks. At the same time, the received signal blocks are turn into the form of ZP-OFDM system. As compared with the conventional PRP-OFDM system, the proposed CS-PRP-OFDM system has the advantage that it avoids the influence of transmitted signals, during the channel estimation processes.

We first examine the accuracy of channel estimation using the proposed CS-PRP-OFDM approach, in terms of normalized mean square error (NMSE) and the symbol error rate (SER), for sufficient redundancy (P-$M \geqq L$+1) case. The channel coefficients are assumed to be time-invariant and randomly generated. Specifically, the parameter set (P, M, L) is chosen to be (40, 32,7). The channel coefficients are assumed to be time invariant and randomly generated. The complementary code sequences are chosen as **a**= [-1 -1 -1 1 1 1 -1 1] and **b** = [-1 -1 -1 1 -1 -1 1 -1], with equal length (D=8) to form the circulant matrices, $\mathbf{C}_{1,CIRC}$ and $\mathbf{C}_{2,CIRC}$.with the first row to be [-1 -1 -1 1 1 1 -1 1] and [-1 -1 -1 1 -1 -1 1 -1], respectively. In consequence, we have $\mathbf{C}_{1,CIRC}^H\mathbf{C}_{1,CIRC}$ + $\mathbf{C}_{2,CIRC}^H\mathbf{C}_{2,CIRC} = 16\mathbf{I}_8$.

To perform the semi-channel estimation, for fair comparison, we collect 96 blocks for averaging with the ST-BC PRP-MIMO-OFDM and ST-BC CS-PPR- MIMO-OFDM, to reduce the effect of interference. The NMSE of estimated channel coefficients is represented as

$$NMSE = \frac{1}{N_r N_t}\left(\sum_{m=1}^{N_r}\sum_{l=1}^{N_t}\frac{\left\|\mathbf{h}^{(lm)} - \hat{\mathbf{h}}^{(lm)}\right\|^2}{\left\|\mathbf{h}^{(lm)}\right\|^2}\right),$$

The simulation results are obtained by averaging 500 random channels. From Figure 2, we learn that the NMSE performance with the ST-BC CS-PRP MIMO-OFDM outperforms the conventional ST-BC PRP-MIMO-OFDM scheme, for SNR greater than 3 dB, due to less influence by the transmitted signal, and is performed similarly when SNR is less than 3 dB. Besides, by increasing the number of receive-antenna does not improve NMSE performance because the channel estimation does not apply multiple antenna diversity. That is to say, different receive antenna has different channel coefficients and we have to estimate these channel coefficients, individually. Therefore, the NMSE performance of two receive antenna is the same as one receive antenna. From Fig. 3, we learn that the SER performance with the ST-BC CS-PRP MIMO-OFDM outperforms the conventional ST-BC PRP-MIMO-OFDM scheme, especially for high SNR. Besides, by increasing the number of receive-antenna the better SER performance can be obtained.

Next, we consider the case with orthogonal cyclic-postfix sequences. If the channel order is not larger than 3, such as (P, M, L) is changed to (36, 32, 3). In this situation, we find that we can use an orthogonal cyclic sequence to simplify the process with CC cyclic sequences. Here, if we choose a deterministic sequence $\mathbf{c} = [c_0 \ \ c_1 \ c_3 \ \ c_4]^T = [1 \ 1 \ 1 \ \text{-}1 \ ]^T$ to be the postfix sequence, the cyclic-postfix sequence in (2-a) and (2-b) becomes

$$\mathbf{c}(i) = \left[c_{(D-i)\bmod D}, c_{(D-i+1)\bmod D}, \cdots, c_{(D-i-1)\bmod D} \right]^T$$

where $D = L+1$. With other parameters to be the same in previous case, the cyclic-postfix sequence is orthogonal to each other; the circulant matrix $\mathbf{C}_{CIRC}$ formed by the cyclic-postfix sequences is given by $\mathbf{C}_{CIRC}^H \mathbf{C}_{CIRC} = D \times \mathbf{I}_D = 4 \times \mathbf{I}_4$. The results are given in Fig. 4. From Fig. 4, we found that similar results as in Fig. 3 are observed.

V. Conclusions

In this paper, we have proposed a novel ST-BC MIMO CS-PRP-OFDM scheme, where the pseudo random cyclic-postfix sequences, implementing with the complementary code (CC) sequences, was employed to replace the PRP in OFDM systems. The MIMO CS-PRP-OFDM systems with 2 transmit-antennas and 2 receive-antennas is considered to verify the merits of the proposed scheme. By exploring the property of CC sequence the semi-blind channel estimation, with first-order statistics of received signals, could be performed, effectively and with lower complexity, compared with the PRP-OFDM based system. However, with the proposed MIMO CS-PRP-OFDM systems, it affected only by the background noise, to avoid the interference of the transmitted signals. Thus, when the SNR is increased better performance achievement could be obtained compared with the conventional MIMO PRP-OFDM systems. Also, we found that the increasing of the numbers of the receive antenna, we could improve the system performance of MIMO CS-PRP-OFDM, accordingly, compared with that using the conventional MIMO-PRP-OFDM systems, associated with the space-time block coding. This proposed transceiver could be further extended for insufficient redundancy case, with the blind channel shortening algorithm.

Acknowledgment

The financial support of the National Science Council, Taiwan, R.O.C., under contract NSC 99-2221 E-032-073 is of great acknowledged.

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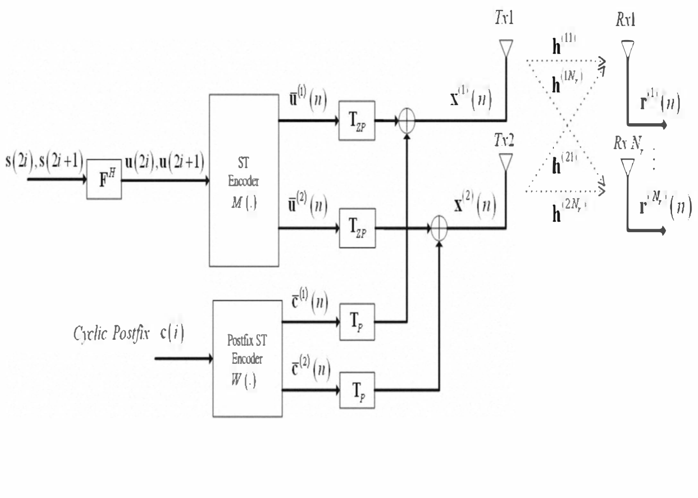

Figure 1 Block diagram of the ST-BC MIMO PRP-OFDM modulator

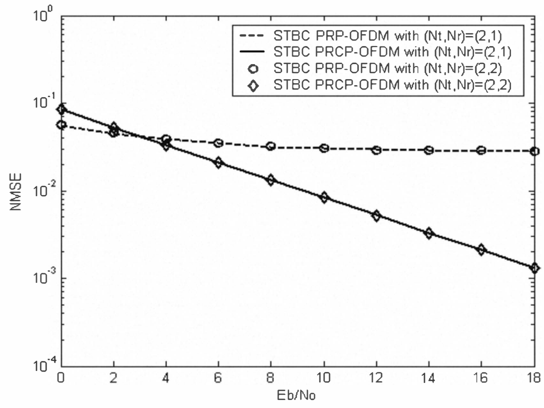

Figure 2 Comparison of NMSE for ST-BC MIMO PRP- OFDM and MIMO CS-PRP-OFDM system with complementary cyclic sequence.

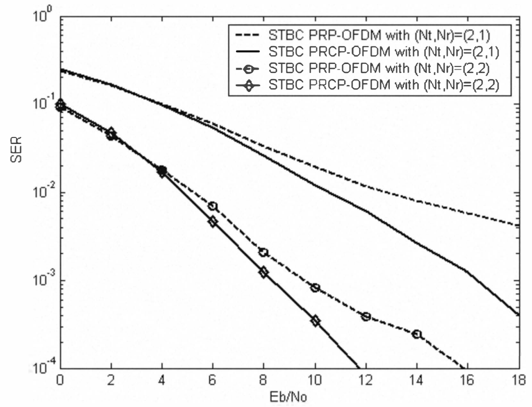

Figure 3 Comparison of SER for ST-BC PRP-OFDM and ST-BC CS-PRP–OFDM with orthogonal cyclic sequence.

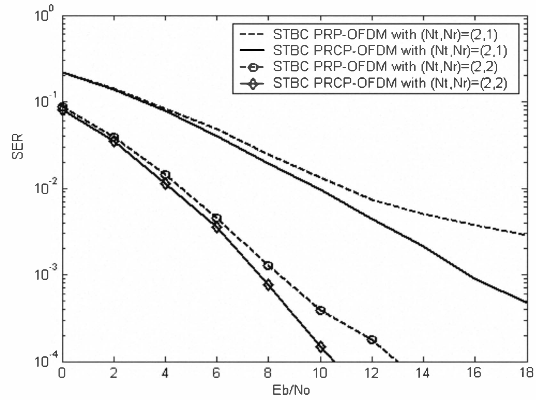

Figure 4 Comparison of SER for ST-BC PRP-OFDM and CS-PRP - OFDM with orthogonal cyclic sequence.