Performance of OFDM QAM over Frequency-Selective Fading Channels

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Abstract—**We present exact symbol error rate (SER) performance analysis for *M*-QAM OFDM systems over Ricean and Rayleigh fading is analyzed. Both slow and fast quasi-static fading as well as frequency-selective and –nonselective channels are considered.**

Index Terms—**OFDM, QAM, symbol error rate, fading channels**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) employs parallel transmission of data carried by orthogonal subcarriers over overlapping subbands to avoid high speed equalization, combat impulse noise, mitigate multipath distortion, and fully use the available bandwidth [1]-[4]. For frequency-selective fading channels, the OFDM system can eliminated inter-symbol interference (ISI) caused by multipath delay spreads by use of guard intervals or cyclic prefix in data blocks [5].

However, the OFDM system is not without disadvantages. Multi-carrier systems are more sensitive to symbol timing errors and frequency offsets than single-carrier systems [5], [6]. Frequency offsets may arise from Doppler shift and mismatch of local oscillator (LO) carrier frequencies between the transmitter and the receiver. Both timing errors and frequency offsets will destruct the orthogonality between subcarriers resulting in inter-channel or inter-carrier interference (ICI) and hence degrading system performance [6]. Thus, timing and frequency estimation and synchronization are required in OFDM systems and numerous techniques have been proposed in the literature [6]-[11]. Frequency offsets and symbol timing errors are not the only causes for ICI. The ICI can also be caused by fading rate [2]. When fading is fast, ICI will exist [12]. But when fading is slow such that the fading is static over an OFDM symbol block, then no ICI will be present due to fading [12].

Analysis works on OFDM digital performance over slow or fast fading and frequency-selective or –non slective channels have appeared in the literature [13]-[17]. Most of these works treat uncoded OFDM using approximations. In this work, we shall analyze the exact SER performance for OFDM systems employing square *M*-QAM over Ricean and Rayleigh fading.

The paper is organized as follows. Section II gives the signal and channel model. Section III analyzes SER performance. Then, Section IV presents simulation results. Finally, Section VII draws conclusions.

II. Signal and Channel Model

For an OFDM system, N complex data symbols X_k over a time interval T constitute a data block, $k = 0,1,...,N-1$. Thus each symbol occupies a symbol interval $\Delta t = T/N$. The signal bandwidth is $1/\Delta t$. Data are transmitted one block at a time (block rate $= 1/T$ and symbol rate $= 1/\Delta t$). Before a block is transmitted, the N symbols $\{X_k\}$ in that block are first passed through an N-point inverse discrete Fourier transformer (IDFT) to produce N parallel complex outputs given as

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{j2\pi nk/N}, \qquad n = 0,1,...,N-1. \tag{1}$$

Thus we are viewing X_k as being in the frequency domain and x_n in the time domain. The parallel $\{x_n\}$ are then converted to a serial sequence over the block by a parallel-to-serial (P/S) converter. Next, the serial sequence is transformed into analog form by digital-to-analog (D/A) conversion. The D/A conversion is equivalent to letting $f_k = k/T$, and $t = n\Delta t = nT/N$. The analog form is

$$x(t) = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{j2\pi f_k t}, \qquad 0 < t < T. \tag{2}$$

Equation (2) only represents the first block. If the ith block is spoken of, we should replace t by $t + iT$ and the range of t should be $iT < t < (i+1)T$. In view of (2), $x(t)$ can be viewed as a sum of complex subcarriers respectively at frequencies { $f_k = k/T$ } with amplitudes X_k over the block, each subcarrier occupies a subband of width $\Delta f = 1/T$. With this subband spacing, it is readily shown that all subcarriers are orthogonal to one another. The total signal bandwidth is $W = N\Delta f = N/T$. Now, the complex signal $x(t)$ is frequency up-shifted to the channel passband for transmission by two quadrature local oscillators (LO) at a center frequency f_c. One LO mixes a cosine carrier with the real part of $x(t)$ and another mixes a sine carrier with the imaginary part of $x(t)$. In other words, the transmitted passband signal is of the form $s(t) = \mathrm{Re}[x(t)e^{j2\pi f_c t}]$, $x(t)$ is simply the equivalent lowpass signal [5].

We shall assume a frequency-selective fading channel whose equivalent lowpass discrete-time channel response can be modeled by a time varying tapped delay line [5]. Thus, the equivalent lowpass discrete frequency response of the channel is given by

$$H_k = \sum_{m=0}^{\nu-1} h_m e^{-j2\pi mk/N}, \quad k = 0,1,...,N-1, \tag{3}$$

where h_m is the discrete-time channel impulse response or tap gain and ν is the channel dispersion length. If $h_m = 0$ for $m \neq 0$, we have a frequency-nonselective or flat fading channel. For most fading media, { h_m } can be assumed spatially uncorrelated [5], i.e., $E[(h_m - \bar{h}_m)(h_{m'} - \bar{h}_{m'})] = 0$, $m \neq m'$, where $E[\cdot]$ denotes expectation and $\bar{h}_m = E[h_m]$. In this work, we will consider two quasi-static fading cases: 1) h_m remains constant over one symbol interval Δt (fast quasi-static fading); 2) h_m remains constant over one block interval T (slow quasi-static fading). Then for the fading process, we assume { h_m } are uncorrelated complex Gaussian random variables (RV). We also assume the tap gain h_0 corresponding to the shortest path delay contains scattered paths plus a line-of-sight (LoS) component or the specular component. Then, h_0 is a non-zero mean Gaussian RV and the amplitude $|h_0|$ is a Rice RV. The rest { h_m, $m \neq 0$ } are zero mean Gaussian RVs and { $|h_m|$, $m \neq 0$ } are Rayleigh RVs. By virtue of (3), we easily find that H_k is a complex Gaussian RV with mean equal to $E[H_k] = \bar{h}_0$ and variance equal to $V[H_k] = \sum_{m=0}^{\nu} \sigma_m^2 - |\bar{h}_0|^2$, where $\sigma_m^2 = V[h_m]$ is the variance of h_m. If the number of subcarriers in $x(t)$ of (2), N is sufficiently large, then each subband width Δf is very small and the magnitude of each kth subband channel frequency response can be approximated by a constant $\approx |H_k|$. Since each subcarrier is of duration T, so the kth subcarrier has a spectrum of the form $\sin c[(f - f_k)T] = \frac{\sin \pi(f - f_k)T}{\pi(f - f_k)T}$ [1]. The sinc function spans the entire frequency spectrum. The kth subband main lobe is of width $2/T$. Thus, even the main lobe will extend over into the two adjacent subbands. Likewise, neighboring subband spectra will also extend into the kth subband. This results in inter-channel or inter-carrier interference (ICI). However, what matters is the sampled discrete-time data. That is, we are only interested in the recovery of X_k at the subband center frequency. If channel fading is slow with { h_m } remaining constant over a block duration T, the recovered discrete-time data will be free of ICI [12]. But when fading is fast with { h_m } remaining constant over only a symbol duration Δt, then ICI will exist in the sampled discrete data. Aside from ICI, channel dispersion (caused by multipath delay spread) will render inter-symbol interference (ISI) within a block. Fortunately, ISI can be completely eliminated by inserting a guard interval or appending a cyclic prefix to each block of data [5]. But guard intervals or cyclic prefixes cannot help get rid of ICI.

III. Symbol Error Rate Analysis

A. Slow Quasi-Static Fading

For slow quasi-static fading, the DFT outputs are given by

$$R_k = Y_k + Z_k = H_k X_k + Z_k, \quad k = 0,1,...,N-1, \tag{4}$$

where { Y_k } are noiseless DFT outputs and { Z_k } are identical zero-mean Gaussian RVs with variance, say, σ_Z^2. The signal power is the same for all subchannels as given by $\sigma_X^2 = E[|X_k|^2]$. Thus, for a fixed channel realization, the received signal-to-noise ratio (SNR) of the kth subband is

$$\gamma_k = |H_k|^2 \sigma_X^2 / \sigma_Z^2. \tag{5}$$

Consider square M-QAM signals, then using the moment generating function (MGF)-based approach [18], by averaging the conditional M-QAM SER (conditioned on a fixed channel realization) over all channel realizations, we can readily obtain the overall average M-QAM SER for slow quasi-static fading as

$$P_{M,k} = \frac{4(\sqrt{M}-1)}{\pi\sqrt{M}} \int_0^{\pi/2} M_{\gamma_k}\left(-\frac{g}{\sin^2\theta}\right)d\theta - \frac{4(\sqrt{M}-1)^2}{\pi M} \int_0^{\pi/4} M_{\gamma_k}\left(-\frac{g}{\sin^2\theta}\right)d\theta, \tag{6}$$

where $g = \frac{3}{2(M-1)}$ [18] and $M_{\gamma_k}(s) = \int_0^{\infty} p(\gamma_k)e^{s\gamma_k}d\gamma_k$ is the MGF of γ_k with $p(\gamma_k)$ as the probability density function (PDF) of γ_k. Note that $p(\gamma_k)$ are identical for all { γ_k } and { γ_k } are correlated (From (3), all { H_k } are identical complex Gaussian RV's with mean $\bar{h}_0$ and variance

$\sum_{m=0}^{\nu-1} E(|h_m - \bar{h}_m|^2)$. Then from (5), all $\{\gamma_k\}$ are identically distributed. However all $\{\gamma_k\}$ are correlated since they are all related to $\{h_m\}$.). For Ricean fading, the MGF can be readily obtained by use of the general MGF formula for Gaussian gain channels given in [19] as

$$M_{\gamma_k}(s) = \frac{\exp\left[\dfrac{(|\bar{h}_0|\sigma_X/\sigma_Z)^2 s}{1-[\bar{\gamma}_k - (\bar{h}_0\sigma_X/\sigma_Z)^2 s]}\right]}{1-[\bar{\gamma}_k - (|\bar{h}_0|\sigma_X/\sigma_Z)^2 s]}, \tag{7}$$

where

$$\bar{\gamma}_k = E[|H_k|^2 \sigma_X^2/\sigma_Z^2] = \frac{\sigma_X^2}{\sigma_Z^2}\left(|\bar{h}_0|^2 + \sum_{m=0}^{\nu-1}\sigma_m^2\right). \tag{8}$$

Then, replacing s in (7) by $-g/\sin^2\theta$ and substituting the result into (6) (MGF-based approach [18]), we get the desired square M-QAM SER for the OFDM system in slow quasi-static Ricean fading channels. Note that, since all $\{\bar{\gamma}_k\}$ are equal, thus all subbands have the same SER as given by (6). Therefore, the overall averaged system SER is also the same.

If $\bar{h}_0 = 0$, then $E[H_k] = \sum_{m=0}^{\nu-1}\bar{h}_m e^{-j2\pi mk/N} = \bar{h}_0 = 0$, the Ricean fading reduces to Rayleigh fading. Equations (7) and (8) reduce to

$$M_{\gamma_k}(s) = \frac{1}{(1-\bar{\gamma}_k s)}, \tag{9}$$

$$\bar{\gamma}_k = E[|H_k|^2 \sigma_X^2/\sigma_Z^2] = \frac{\sigma_X^2}{\sigma_Z^2}\left(\sum_{m=0}^{\nu-1}\sigma_m^2\right). \tag{10}$$

Then, the two integrals in (6) can be further evaluated so that a more exact closed form for $P_{M,k}$ can be obtained. Using straightforward calculus on (6), the SER result for slow quasi-static Rayleigh fading can be shown to be

$$P_{M,k} = \frac{1}{M}[M - 1 - \frac{2a(\sqrt{M}-1)}{\sqrt{1+a^2}} - \frac{4a(\sqrt{M}-1)^2}{\pi\sqrt{1+a^2}}\tan^{-1}\frac{a}{\sqrt{1+a^2}}], \tag{11}$$

where $a = \sqrt{\dfrac{3\bar{\gamma}_k}{2(M-1)}}$. Another approach called the pre-averaging method, which is a PDF-based approach, can also be used to obtain exactly the same result [20], [21]. Note here that, for slow quasi-static fading, it is ready shown that (6) and (11) apply to both frequency-selective and flat fading channels. For flat fading, (8) and (10) must be modified to

$$\bar{\gamma}_k = E[|H_k|^2 \sigma_X^2/\sigma_Z^2] = \frac{\sigma_X^2}{\sigma_Z^2}\left(|\bar{h}_0|^2 + \sigma_0^2\right), \tag{12}$$

$$\bar{\gamma}_k = E[|H_k|^2 \sigma_X^2/\sigma_Z^2] = \frac{\sigma_X^2}{\sigma_Z^2}\left(\sigma_0^2\right). \tag{13}$$

From the above analysis, we see that OFDM produces the same performance result for frequency-selective and flat fading channels when fading is slow. This means, OFDM is useful for combating channel frequency selectivity. Obviously, when slow fading is flat, there is no need to use the complicated OFDM system. Without channel dispersion in flat fading, no time domain equalization is required, then a simple single carrier system can be used with diversity combining to combat flat fading and there is no need to worry about ICI caused by frequency offset or symbol timing error.

B. Fast Quasi-Static Fading

We now turn to the more complicated case of fast quasi-static fading. The noiseless DFT outputs can be shown to be

$$\begin{aligned} Y_k &= \sum_{n=0}^{N-1}\sum_{m=0}^{\nu-1} h_m(n) x_{n-m} e^{-j2\pi nk/N} \\ &= \frac{1}{N}\sum_{n=0}^{N-1} H_k(n) X_k + \frac{1}{N}\sum_{n=0}^{N-1}\sum_{\substack{p=0\\p\neq k}}^{N-1} H_p(n) X_p e^{j2\pi n(p-k)/N} \end{aligned} \tag{14}$$

The first term in (14) is the desired term and second term is the ICI. For a fixed channel realization (all subband responses are held constant), as N is usually large so that the ICI is a sum of many independent RVs ($X_p, p = 0,\ldots,k-1,k+1,\ldots,N$). We may invoke the central limit theory to assume that the ICI term can be approximated by a Guassian RV [12]. For a fixed channel realization, the desired term and the random ICI term of (14) respectively have the power given as

$$P_S = \frac{\sigma_X^2}{N^2}\sum_{n=0}^{N-1}\sum_{l=0}^{N-1} H_k(n) H_k^*(l), \tag{15}$$

$$P_{ICI} = \frac{\sigma_X^2}{N^2}\sum_{\substack{p=0\\p\neq k}}^{N-1}\sum_{n=0}^{N-1}\sum_{l=0}^{N-1} H_p(n) H_p^*(l) e^{j2\pi(n-l)(p-k)/N}. \tag{16}$$

Then, with noise taken into account, the sum of ICI and noise is still Gaussian having the variance $P_{ICI} + \sigma_Z^2$. Thus, for a fixed channel realization, the received signal-to- interference plus noise ratio (SINR) is

$$\gamma_k = \frac{P_S}{P_{ICI} + \sigma_Z^2}. \tag{17}$$

For this SINR, we can use the well-known M-QAM SER expression under AWGN [5], [12], [18] for the given channel realization. Then, we remove the condition of fixed channel realization and consider subband responses $H_k(n)$ and $H_p(n)$ in (15) and (16) as RVs. Thus now, the γ_k of (17) is treated as a RV. For fast quasi-static fading, even with flat fading, this γ_k has very complicated PDF not just because of the quotient form given by (17) but also because of the correlations between subband responses. In the numerator, $\{H_k(n),\ n = 0,1,\ldots,N-1\}$ are temporally correlated, while in the denominator $\{H_p(n),\ n = 0,1,\ldots,N-1,\ p = 0,1,\ldots$

$k-1, k+1, \ldots, N-1$ } are both temporally and spatially correlated. As a result, All $\{\gamma_k\}$ are correlated but identically distributed and are no more Ricean, which means the MGF of (7) can no longer be applied thus making it difficult to use (6). However, we can resort to numerical calculations to obtain the $P_{M,k}$. We take the AWGN M-QAM SER conditioned on γ_k for a given channel realization and average it over sufficient number of γ_k realizations, each realization of γ_k is obtained from correlated random calls of the Gaussian RVs $\{h_m(n)\}$ (for a given m, $h_m(n)$ and $h_m(l)$ are correlated) and by substituting (3) into (15)-(17). Such a simulation approach has also been adopted by [22] for single carrier equalizers over slow fading channels and by [23], [24] for frequency-domain decision feedback equalizers for single carrier OFDM systems over fading channels. In simulations, we use modified Jakes model [25] for $h_0'(n)$ and $\{h_m(n),\ m \neq 0\}$ and (12) and (13) are used for channel correlations. For the fast fading case, in view of (34)-(36), $\bar{\gamma}_k$ will be different for different k. Thus $P_{M,k}$ will be different for different k. The system SER is obtained by averaging over all subchannel SER's. Hence

$$P_M = \frac{1}{N}\sum_{k=0}^{N-1} P_{M,k}\,. \tag{18}$$

IV. NumericaL Examples

We now present some simulation results of SER performance for slow and fast quasi-static Ricean/Rayleifg fading channels. Figure 2 shows the SER vs. SNR performance curves for OFDM in slow quasi-static Ricean/Rayleigh fading channels. As noted earlier, for this case, flat and frequency-selective channels yield the same results. In simulations, we have assumed a frequency-selective channel model with channel taps corresponding to scattered paths (i.e., $h_0', h_1, \ldots h_v$) having exponentially decayed tap powers. The dispersion length is $v = 4$. And we use square 16-QAM signaling with $N = 16$. Three Ricean factors $K_R = -\infty$ (Rayleigh), 5dB, and 10dB are considered. Both theoretical results of (26) and simulation results (marked by circles) are given and are seen in excellent agreement.

For fast quasi-static fading, the OFDM SER will not be the same for flat and frequency-selective channels. However, as mentioned earlier, use of OFDM for flat fading channels is a waste. Thus, we will only present results for frequency-selective channels. Using 16-QAM in OFDM with $N = 16$ and with SNR=σ_X^2/σ_n^2 fixed at 32dB, Fig. 3 shows the simulation results of SER vs. SINR performance of the first OFDM subband ($k = 0$) in fast quasi-static frequency-selective Ricean/Rayleigh fading channels. The same frequency-selective channel model with the same three Ricean factors as for Fig. 1 is used. Simulations for other subbands show SER results slightly different but very close to that of Fig. 3. Thus the average SER of (37) will closely resemble that of Fig. 3. Over 50,000 channel realizations have been used for averaging (In [24], 20,000 channel realizations are used). As mentioned earlier, we have used modified Jakes model in simulations for both Fig. 2 and 3. Although theoretical closed-form SER expressions are not available for the fast quasi-static fading case of Fig. 3 to compare with simulation results, the close agreement between the theoretical and simulated results for slow quasi-static fading of Fig. 2 as well as the fact that the same simulation technique has been used by [22]-[24] should pretty much support the accuracy of the simulations of Fig. 2. Comparing Fig. 1 and 2, it is seen that slow fading channels perform better than fast fading channels. This is within expectations.

V. Conclusions

For Ricean/Rayleigh fading, we carry out detailed analysis on M-QAM SER performance in uncoded OFDM system. We consider slow and fast quasi-static fading as well as frequency-selective and -nonselective channels. In the cases of fast quasi-static fading, exact ICI expressions are used without any approximations. For slow quasi-static fading, exact closed-form expression for SER is derived. Simulation results are found in excellent agreement with the theoretical results. For fast quasi-static fading, the SER performances for frequency-selective channels are presented by simulations. Slow fading channels outperform fast fading channels as expected.

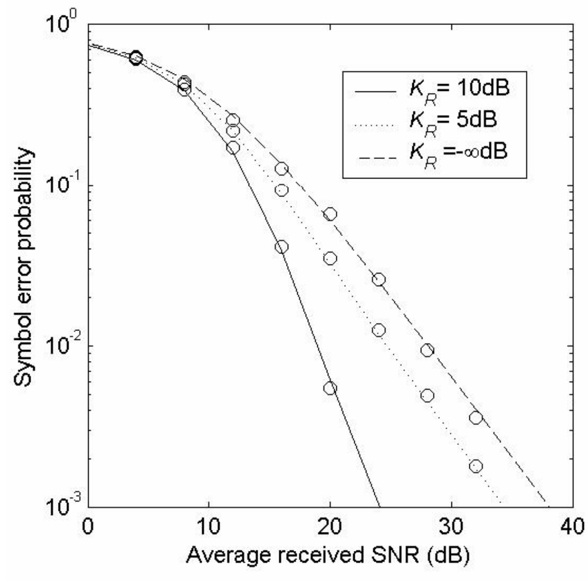

Figure 1. Theoretical and simulated (marked with circles) SER performance for 16 QAM in 16 point OFDM over slow Ricean and Rayleigh fading channels.

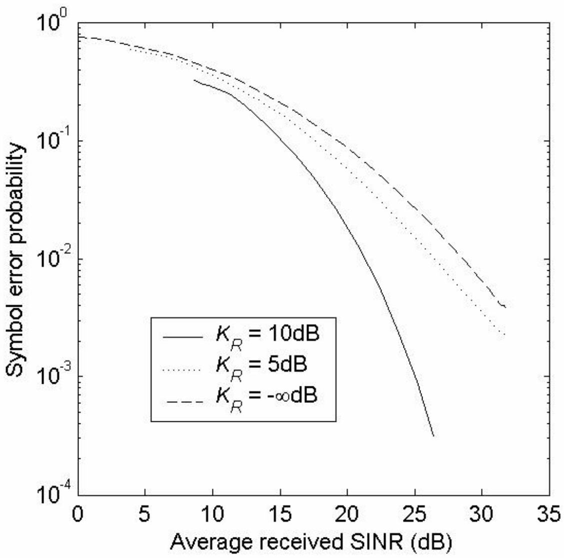

Figure 2. Simulated first subband SER performance of 16 QAM in 16 point OFDM over frequency-selective fast Ricean and Rayleigh fading channels.

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