# Fertility Rate and Economic Growth

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Abstract
This paper examines the long-run relationship between the economic growth rate and fertility rate in an endogenous growth model with elastic labor supply. The cost of child-rearing in this model is increasing with capital. We find that higher fertility rate impedes economic growth, but higher economic growth boosts the fertility rate.

Keywords: economic growth, endogenous fertility, child-rearing cost
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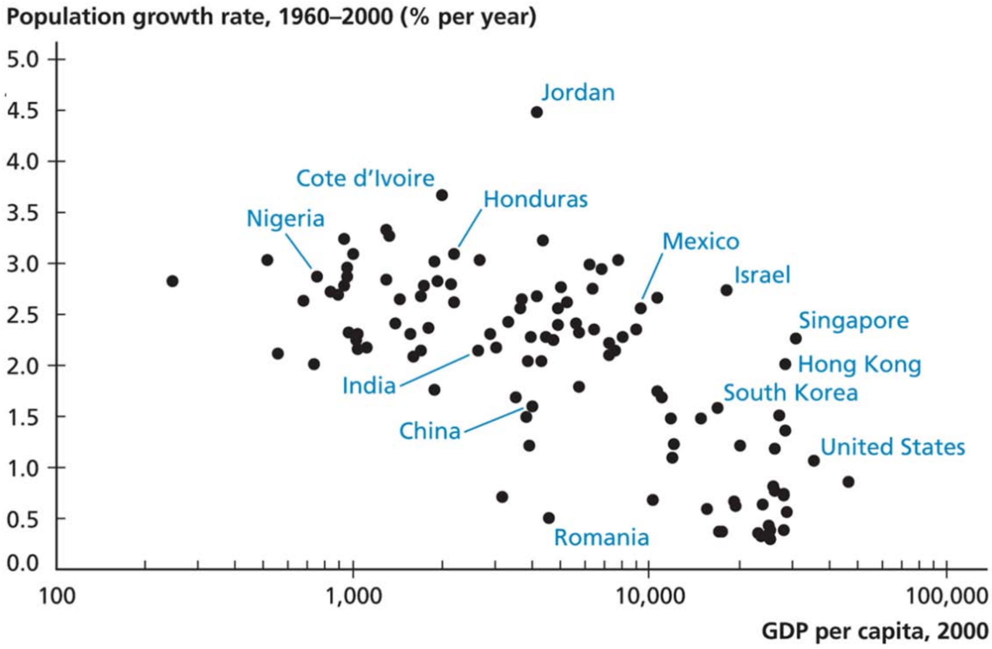
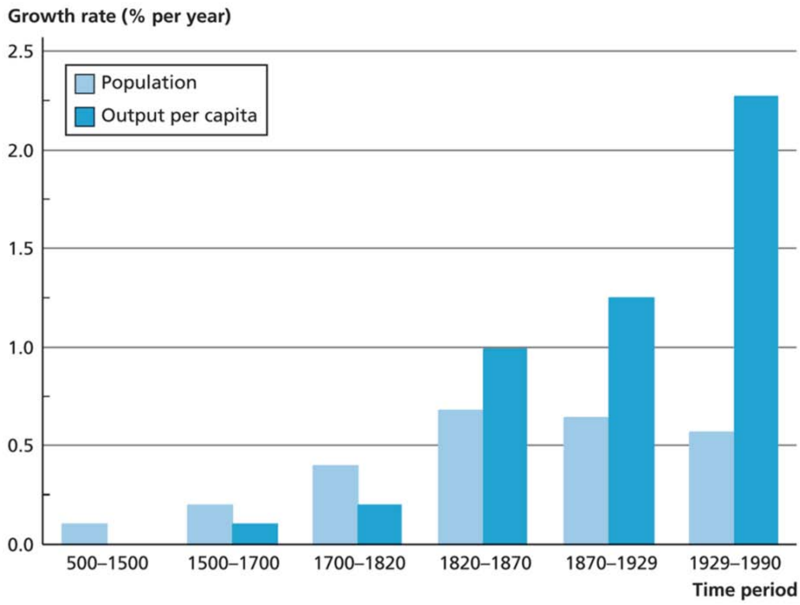
## 1. Introduction

Since Thomas Malthus published “Essay on the Principle of Population” in 1978, the relationship between population and economic growth has received a great deal of attention. From the existing data, this relationship is negative. For example, based on the data of Weil (2009), Figure 1 and Figure 2 show the cross-section and time-series data, respectively. Figure 1 shows that there is a strong negative correlation between GDP per capita and the growth rate of population. Figure 2 shows the growth rates of output per capita and population in Western Europe. It indicates that as the growth of income accelerated, population growth began to fall after 19th century.




These facts are also confirmed by some empirical studies. For example, Wang, Yip and Scotese (1994) find that the relationship between employment and fertility rate is negative in U.S. and Barlow(1994) also indicate that the growth rate of GDP per capita negatively correlates with the growth rate of population in 144 countries. Moreover, Kelly and Schmidt (1995) collect 89 countries of panel data to test relationship between birth rate and per capita output growth. They find a negative correlation to two variables in the period 1980-1990.




Since these two variables are endogenous, the causality runs in both directions. Separately to discuss this relationship, there are many literatures to indicate that high fertility impedes economic growth. For examples, Avner(2001) finds that one percent decrease in population growth increases GDP per capita growth by more than three percent in 114 countries. Li and Zhang (2007) also find that birth rate has a negative impact on economic growth in China. Bloom et al. (2009) find that the decrease fertility rate raises income per capita in 97 countries. On the contrary, some papers indicate that there is little empirical evidence to prove this causality, such as Becker, Glaeser and Murphy(1999)。

On the other hand, Lee (2005) and Guest and Swift (2008) show that economic growth harms population growth in Korea and U.S data, respectively. In fact, this relationship can easily be uncovered in the advanced-country history of economic development. The idea that economic growth is the best way to reduce fertility was famously summarized at a United Nations conference in 1975 in the phrase “development is the best contraceptive.”

Yip and Zhang (1997) is the first paper to investigate this relationship in an endogenous growth model with endogenous fertility. They find that there exists an inverse relation between population growth and economic growth when all exogenous factors are controlled for. However, if exogenous factors are unchanged, the endogenous variable is impossible to change. Moreover, in the case of multiple equilibria with indeterminacy, low fertility is associated with low economic growth. These results indicate that this relationship cannot be explained very well in this kind model.

Therefore, we extend the model of Yip and Zhang (1997) to introduce endogenous labor supply and Barro and Sala-i-Martin’s (2004) setting of child-rearing cost, which consumes resources and increases with capital. We find that, under a lower elasticity of marginal utility with respect to fertility, fertility rate impedes economic growth rate. The intuition is as follows. Since we assume that the variation of fertility rate derives from child-rearing cost, a higher fertility rate means a lower child-rearing cost. The higher fertility raises the resource consumption and hence reduces the capital accumulation and economic growth. But the lower child-rearing cost reduces the resource consumption and hence enhances economic growth. When the elasticity of marginal utility with respect to fertility is lower, the effect of a lower child-rearing cost on fertility will be greater and hence the former effect dominates the latter effect. Thus, we obtain a negative effect of the fertility rate on economic growth.

However, we cannot find a reverse causality. We find that economic growth rate benefits fertility rate in this model. Since we assume the variation of economic growth rate derives from technology progress, a higher economic growth means a higher technology progress and thus a higher fertility rate.

The remainder paper is organized as follows. In section 2, we introduce the basic model and solve Balanced Growth Path (BGP) in Section 3. In Section 4, we analyze comparative statics and conclusion is in Final section.

## 2. The Model

This section establishes the endogenous growth model where the fertility rate and leisure time are endogenously decided in the economy. The economy is comprised of a representative household and firm.

### 2.1 Representative household

Following the literature on endogenous fertility (Yip and Zhang, 1997), representative household consists of a continuum of identical infinitely-lived households. In the absence of immigration and mortality, there is a one-to-one correspondence between the rate of population growth and the fertility rate. One noticeable character of this specification is that individual derives utility from not only the number of children but also their children's utility, which is an extension of the type of models in which parents care only about the number of their offsprings. Thus, each household derives utility from per capita consumption, *c*, the fertility rate, *n*, and leisure time, *l*.

The lifetime utility function *U* of the representative household is described as:

$$U = \int_0^{\infty} e^{-\rho t} \left( \ln c + \frac{n^{1-\theta} - 1}{1-\theta} + \frac{l^{1-\eta} - 1}{1-\eta} \right)$$

(1)

where *c* is per capita consumption, *n* is fertility rate, $\rho \in (0,\infty)$ is a constant rate of time preference, and $l \in (0,1)$ is the leisure time, $\theta$ and $\eta$ are the elasticity of marginal utility with respect to fertility and leisure, respectively.

The household income is earned by wage and the family's assets earn the rate of return. This income is used for expenditure on consumption, child-rearing costs, and accumulation of real wealth. The household budget constraint is expressed as:

$$\dot{k} = rk + w(1-l) - nk - bnk - c\,, \tag{2}$$

where $\dot{k}$ is capital accumulation, *k* is per capita capital stock, *r* represents the family's assets earn the rate of return, *w* is the wage rate, the *b* term represents the cost of child rearing and $b \geq 0$, *bnk* represents the child-rearing cost that increases with the capital intensity (Barro and Sala-i-Martin, 2004).

The optimization problem for the representative household is to maximize the lifetime utility (1), subject to equation (2). The Hamiltonian of the optimization is given by:

$$\mathrm{H} = \left\{ \ln c + \frac{n^{1-\theta}-1}{1-\theta} + \frac{l^{1-\eta}-1}{1-\eta} + \lambda(rk + w(1-l) - nk - bnk - c) \right\}, \tag{3}$$

where $\lambda$ is the co-state variable and it associates with *k*. From equation (3) the first-order necessary conditions with *c*, *n*, *l*, and, *k* are:

$$\frac{1}{c} = \lambda\ , \tag{4}$$

$$n^{-\theta} = \lambda k(1+b)\ , \tag{5}$$

$$\ell^{-\eta} = \lambda w\ , \tag{6}$$

$$\lambda\left(r - n - bn\right) = \rho\lambda - \dot{\lambda}\ , \tag{7}$$

and the transversality condition of *k*：

$$\lim_{t\to\infty} e^{-\rho t}\lambda k = 0\,. \tag{8}$$

Equation (4) is the optimality condition for consumption, implying that the marginal utility of consumption equals the marginal cost of consumption which is equal to the shadow price of capital. Equation (5) is the optimality condition for fertility by equalizing its marginal benefit and cost. The LHS is the marginal utility of an additional unit of child, and the RHS is the marginal cost of an additional unit of child stemming from the opportunity cost plus real capital, which is measured by the shadow price of capital λ. Equation (6) is the optimality condition for leisure time, implying that the marginal utility of leisure time equals per capita wage, which is measured by the shadow price of capital λ. Equation (7) is the Euler equation governing the optimal accumulation for capital.

## 2.2 Representative Firm

We follow Romer (1986) to postulate a Cobb-Douglas production function that depend on the firm-specific inputs-capital stock, work time and average capital stock. The production function exhibits constant returns of scale in private capital and average capital stock. The representative firm's production function is expressed as：

$$y = Ak^{1-\alpha}\left[(1-l)\bar{k}\right]^{\alpha}, \quad A>0 \tag{9}$$

where y is per capita output, k is per capita capital stock, $\bar{k}$ is average per capita capital stock, A is the technology level, $(1-\ell)$ is the work effort time, $0<\alpha<1$. In equilibrium, capital stock is equal to average capital stock. Then, the profit of this representative firm is:

$$\pi = Ak^{1-\alpha}\left[(1-l)\bar{k}\right]^{\alpha} - rk - w(1-l) \tag{10}$$

From (10), firm pursues the max profit and its first-order necessary conditions are：

$$r = (1-\alpha)Ak^{-\alpha}\left[(1-l)\bar{k}\right]^{\alpha}, \tag{11}$$

$$w = \alpha Ak(1-l)^{\alpha-1}, \tag{12}$$

In the long run, k=$\bar{k}$ . Equation (11) is simplified as following:

$$r = (1-\alpha)A(1-l)^{\alpha} \quad . \tag{13}$$

## 2.3 Equilibrium condition

Now we turn to solve the equilibrium condition.

Definition. Given initial capital k(0), a perfect foresight equilibrium (PFE) is a tuple $\{c, k, n, l, \lambda, y, w, r\}$, that satisfies:

(i) production function, (9);

(ii) firm optimization, (11), (12);

(iii) household optimization, (4), (5), (6), (7), together with the one transversality condition (8);

(iv) household budget constraint (2).

According to the definition, We know equations (4), (5), (6), (7), (8), (9), (11), (12) are the optimality conditions. First, differentiating (4) with respect to time yields

$$\frac{\dot{c}}{c} = -\frac{\dot{\lambda}}{\lambda}, \tag{14}$$

Combining (7) and (14), it follows that

$$\frac{\dot{c}}{c} = r - n - bn - \rho, \tag{15}$$

Equation (2) divides by k, we obtain

$$\frac{\dot{k}}{k} = r + \frac{w}{k}(1-l) - n - bn - \frac{c}{k}. \tag{16}$$

Then, using (4), (5) is simplified as following.

$$\mathrm{n}^{\theta} = \frac{c}{k}\frac{1}{(1+\mathrm{b})}, \tag{17}$$

Then,

$$\mathrm{n} = \left[\frac{c}{k}\frac{1}{(1+\mathrm{b})}\right]^{\frac{1}{\theta}}, \tag{18}$$

The leisure time can be derived from equation (4), (6) and (12) as follows.

$$l^{\eta}=\frac{c}{k}\frac{1}{\alpha A(1-l)^{\alpha-1}} \quad , \tag{19}$$

Equations (15) and (16) can solve *c* and *k*. Then, substituting into (18), (19) yields *n* and *l*. As a result, eight endogenous variables, {*c*, *k*, *n*, *l*, *λ*, *y*, *w*, *r*}, are determined.

## 3. Long-run equilibrium

A balanced- growth-path (BGP) equilibrium is a collection of functions of time {c, k} such that they grow at the same rate, and n and *l* is constant.

Along the balanced growth path, both c and k grow at the same rate. Thus, we follow Barro and Sala-i-Martin (2004) to define the following transformed variable:

$$x \equiv c/k. \tag{20}$$

Then, differentiating (20) and combining equation (15) and (16) obtain

$$\frac{\dot{x}}{x}=\frac{\dot{c}}{c}-\frac{\dot{k}}{k}=x-\rho-\frac{w}{k}(1-l), \tag{21}$$

In the long-run equilibrium, $\dot{x}=0$, which means $\frac{\dot{c}}{c}=\frac{\dot{k}}{k}$.

After replacing by (12), we get

$$x=\rho+\alpha A(1-l)^{\alpha}. \tag{22}$$

Based on (22) and (18), we can get *n* in the long run

$$n=\left\{\left[\rho+\alpha A(1-l)^{\alpha}\right]\frac{1}{1+b}\right\}^{\frac{1}{\theta}}. \tag{23}$$

To ensure the existence of BGP, we substitute (22) into (19) to get

$$(1-l)^{\alpha-1}\left[l^{\eta}-(1-l)\right]=\frac{\rho}{\alpha A} \quad , \tag{24}$$

First, we interpret equation (24). Denote the left-hand side of (24) as *LHS*.

When $l$=0, LHS＝－1. When $l$=1, LHS$\rightarrow\infty$. It is easy to prove that this locus is convex as Figure 1. Alternately, the right-hand side of (24) is a constant which is represented by a horizontal line in Figure 1. It is obvious that the long-run equilibrium exists uniquely. Then, substituting the value of $l$ into other equations yields $x$ and $n$. In sum, we get the following proposition.

**Proposition 1.** *There exists a unique long-run equilibrium.*

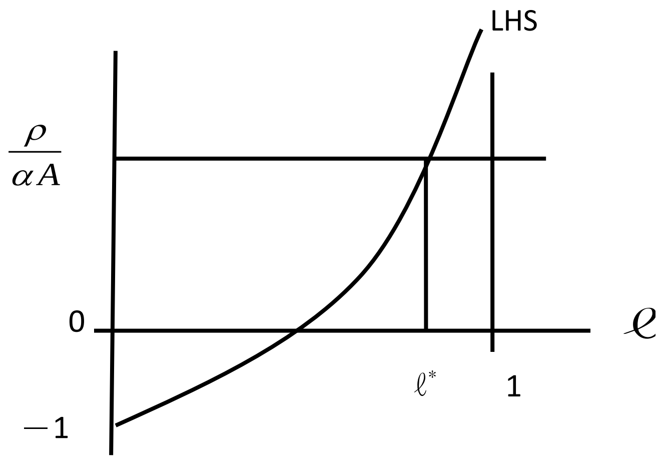



Figure 1: the existence of long-run equilibrium

Moreover, combining (15), (18) and (22), we obtain long-run economic growth rate.

$$\gamma = (1-\alpha)A(1-l)^{\alpha} - (1+b)n - \rho\,. \tag{25}$$

## 4. Comparative static analysis

In this section, we examine the relationship between fertility rate and economic growth along the BGP. First, we examine the effect of fertility rate on economic growth and, second, we do the reverse.

4.1 A change in goods cost of child-rearing

This subsection examines the long-run effect of fertility rate on economic growth. And we assume the cause of fertility-rate variation derives from the alteration of child-rearing cost, *b*.

Equation (24) implies

$$\frac{dl}{db} = 0, \tag{26}$$

which indicates that leisure has nothing to do with child-rearing cost. Next, based on (23), we find that fertility rate negatively correlates with child-rearing cost as following.

$$\frac{dn}{db} = -\frac{n}{\theta(1+b)} < 0. \tag{27}$$

Now, the effect of fertility rate on economic growth can be derived from (25) as follows.

$$\frac{d\gamma}{dn} = \frac{d\gamma}{db}\frac{db}{dn} = -(1-\theta)(1+b) \tag{28}$$

**Proposition 2.** *Under* $\theta$<1, *the effect of the fertility rate on economic growth rate is negative.*

The intuition is as follows. Since the variation of fertility rate derives from child-rearing cost, a higher fertility rate means a lower child-rearing cost. The higher fertility raises the resource consumption and hence reduces the capital accumulation and economic growth. But the lower child-rearing cost reduces the resource consumption and hence enhances economic growth. When the elasticity of marginal utility with respect to fertility, $\theta$, is lower, the effect of a lower child-rearing cost on fertility will be greater and hence the former effect dominates the latter effect. Thus, we obtain a negative effect of the fertility rate on economic growth.

4.2 A change in technology progress

This subsection examines the long-run effect of economic growth on fertility. And we assume the cause of economic-growth-rate variation derives from the alteration of production technology, *A*.

First, the effect of technology progress on leisure can be derived from (24) as follows.

$$\frac{dl}{dA} = -\frac{\rho}{A[\alpha^2 A(1-l)^{\alpha-1} + x(\frac{\eta}{l} + \frac{1-\alpha}{1-l})]} < 0. \tag{29}$$

Technology progress enhances the wage rate and thus reduces leisure. Next, the effect of technology progress on fertility can be derived from (23) as follows.

$$\frac{dn}{dA} = \frac{n[\alpha(1-l)^{\alpha} - \alpha^2 A(1-l)^{\alpha-1}\frac{dl}{dA}]}{\theta x} > 0. \tag{30}$$

It implies that technology progress advances the fertility rate. Moreover, the effect of technology progress on economic growth rate can be derived from (25) as follows.

$$\frac{d\gamma}{dA} = (1-l)^{\alpha} - \alpha(1-\alpha)A(1-l)^{\alpha-1}\frac{dl}{dA} - (1+b)\frac{dn}{dA} > 0. \tag{31}$$

Technology progress benefits economic growth, but pushes forward fertility rate which consumes more resource and then impedes economic growth. In general, the former effect dominates the latter effect.

As a result, the effect of economic growth rate on fertility rate is

$$\frac{dn}{d\gamma} = \frac{dn}{dA}\frac{dA}{d\gamma} > 0 \tag{32}$$

In sum, we obtain the following proposition.

**Proposition 3.** *The effect of economic growth rate on the fertility rate is positive.*

The intuition can be explained as follows. Since the variation of economic growth rate derives from technology progress, a higher economic growth induces a higher technology progress and thus a higher fertility rate.

## 5. Conclusion

This paper extends Yip and Zhang (1997) with endogenous labor supply to investigate the long-run relationship between economic growth and fertility. Moreover, following Barro and Sala-i-Martin (2004), we assume the behavior of child-rearing will consume resource and the cost is increasing with capital.

We find that, under a lower elasticity of marginal utility with respect to fertility, fertility rate impedes economic growth rate while economic growth rate benefits fertility rate. Though this model can explain the relationship from fertility to economic growth but cannot explain the reverse. This gap is our future aim to extend the model.

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