可調式之多質點減振器對於振動機構減振效能之研究

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大綱

一、緒論

二、理論模式推導

三、實驗設計

四、結果與討論

五、結論

緒論-研究動機

- 高深的科技及繁瑣的數學 *v.s.* 傳統的方式
- 主動式減振 *v.s.* 被動式減振

緒論-文獻回顧

(1) Den Hartog, J.P. (1947)

(2) Asustek Computer Inc. (1999)

(3) Heo, J. W. and Chung, Jintai (2002)

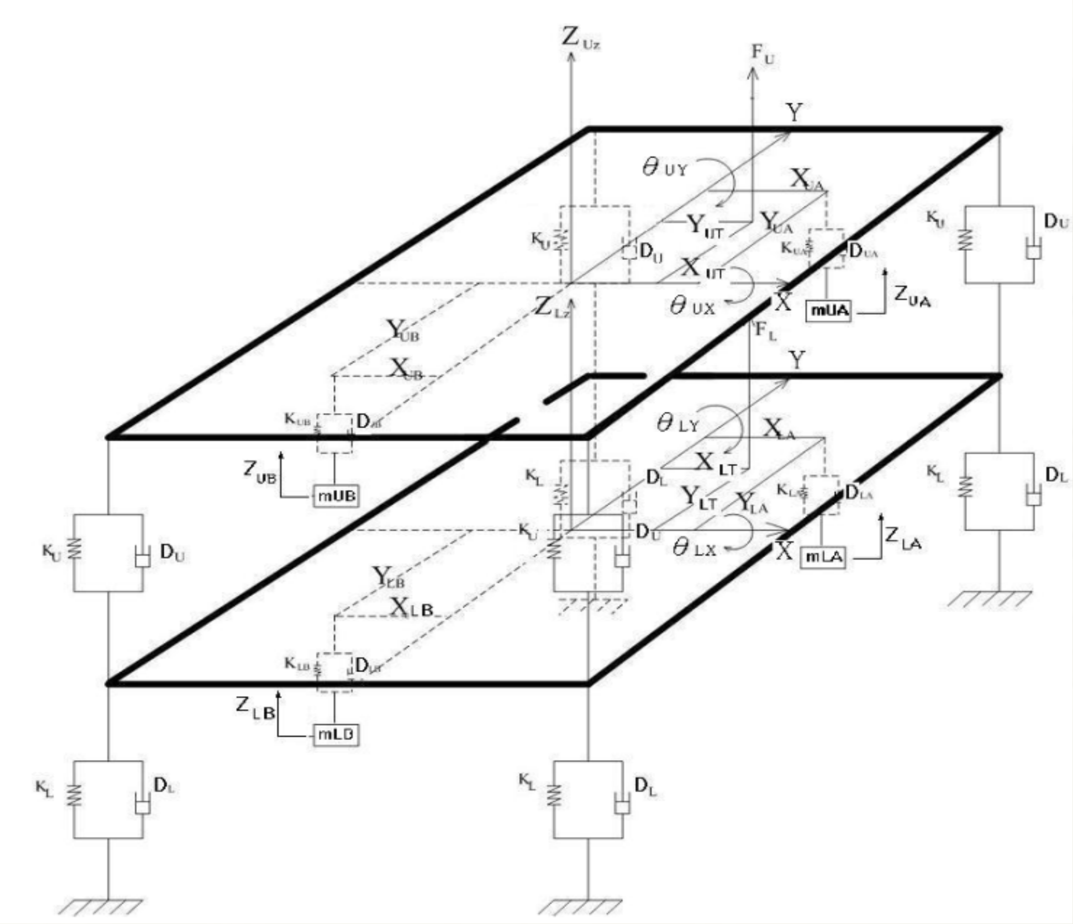
(4)王怡仁和郭蕙蘭(2004)

(5) Wang, Y.-R. and Chen, T.-H. (2008)

(6) Wang, Y.-R. and Chang, H.-L. (2010)

(7) Wang, Y.-R. and Chang, M.-H. (2010)

三維雙層具減振器運動方程式推導



三維雙層具減振器運動方程式推導

- 利用Lagrange’s Equation

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{X}_i}\right)-\frac{\partial T}{\partial X_i}+\frac{\partial R}{\partial \dot{X}_i}+\frac{\partial V}{\partial X_i}=F_i \text{ ， } i=1,2,\ldots n$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{Z}_{UZ}}\right)-\frac{\partial T}{\partial Z_{UZ}}+\frac{\partial R}{\partial \dot{Z}_{UZ}}+\frac{\partial v}{\partial Z_{UZ}}=F_{UZ}(t)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{Z}_{UA}}\right)-\frac{\partial T}{\partial Z_{UA}}+\frac{\partial R}{\partial \dot{Z}_{UA}}+\frac{\partial v}{\partial Z_{UA}}=F_{UA}(t)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{Z}_{UB}}\right)-\frac{\partial T}{\partial Z_{UB}}+\frac{\partial R}{\partial \dot{Z}_{UB}}+\frac{\partial v}{\partial Z_{UB}}=F_{UB}(t)$$

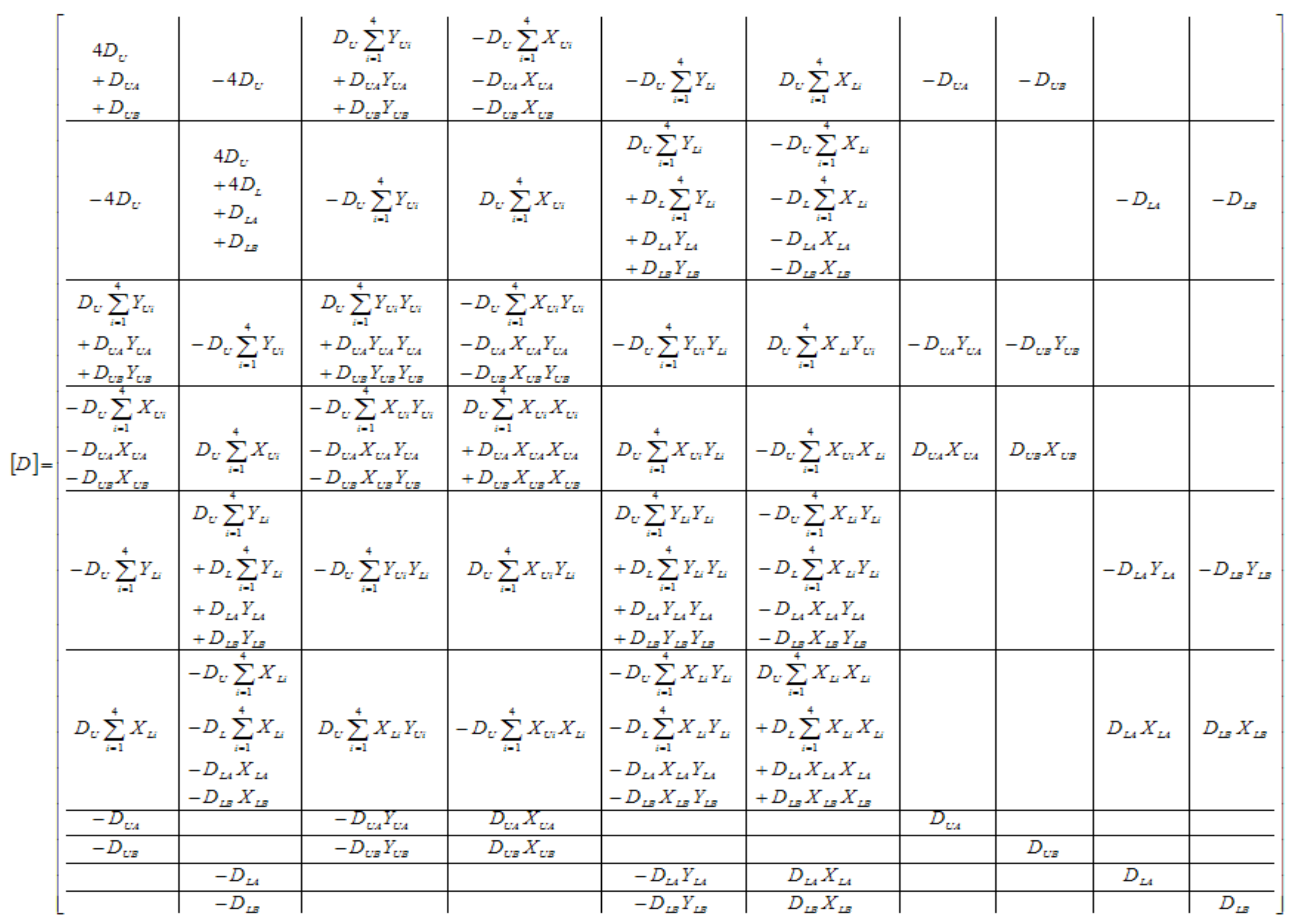
$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_{UX}}\right)-\frac{\partial T}{\partial \theta_{UX}}+\frac{\partial R}{\partial \dot{\theta}_{UX}}+\frac{\partial V}{\partial \theta_{UX}}=F_{UZ}(t)\cdot Y_{UT}$$

三維雙層具減振器運動方程式推導

$$[M]\{\ddot{X}\}+[D]\{\dot{X}\}+[K]\{X\}=\{F\} \qquad (2.11)$$

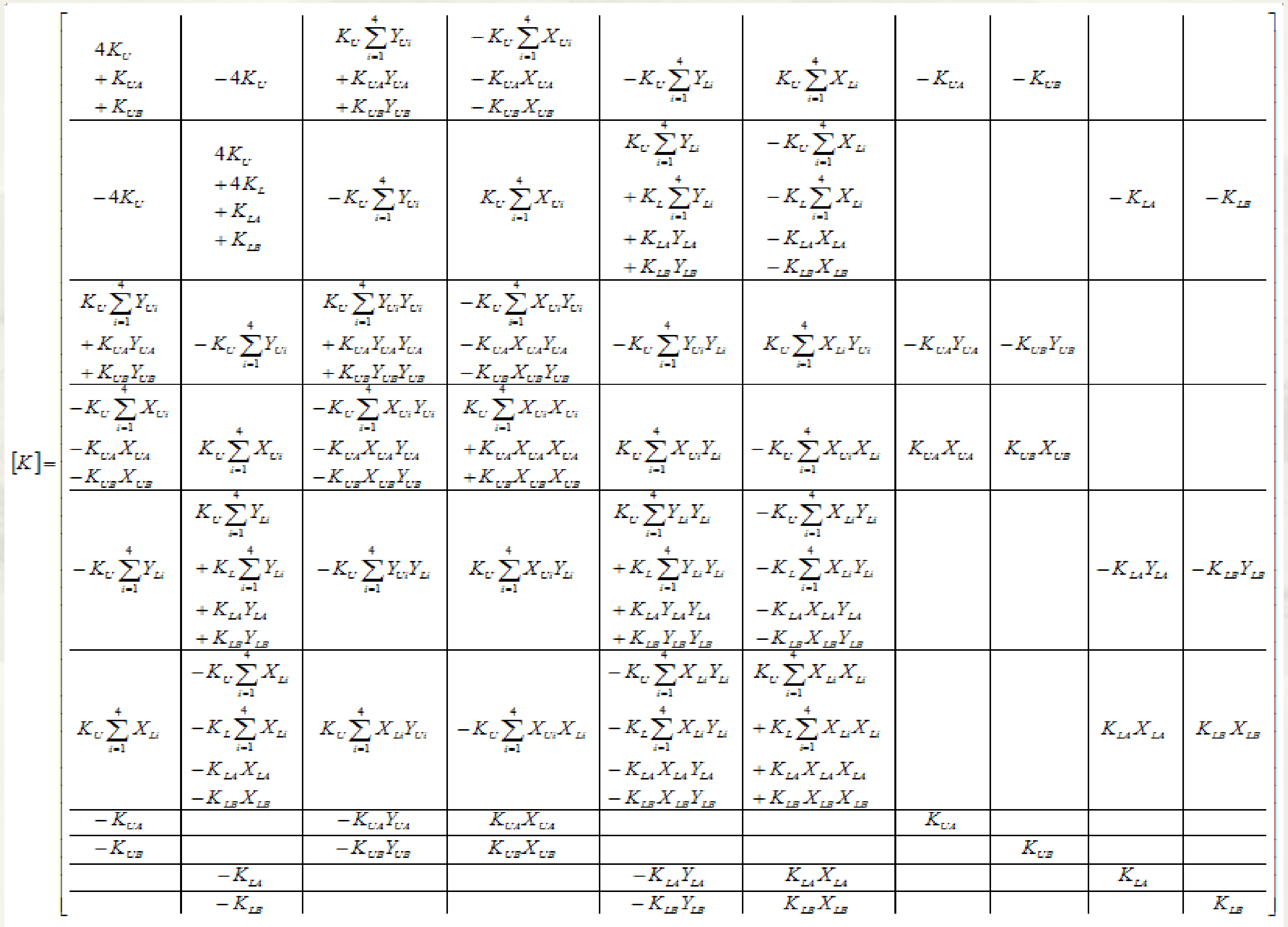
$$[M]=\begin{bmatrix}
M_U & & & & & & & & & \\
 & M_L & & & & & & & & \\
 & & I_{UX} & & & & & & & \\
 & & & I_{UY} & & & & & & \\
 & & & & I_{LX} & & & & & \\
 & & & & & I_{LY} & & & & \\
 & & & & & & M_{UA} & & & \\
 & & & & & & & M_{UB} & & \\
 & & & & & & & & M_{LA} & \\
 & & & & & & & & & M_{LB}
\end{bmatrix}$$

三維雙層具減振器運動方程式推導



$$
[D]=\left[\begin{array}{c|c|c|c|c|c|c|c|c|c}
\begin{array}{l}4D_U\\+D_{UA}\\+D_{UB}\end{array} & -4D_U & \begin{array}{l}D_U\sum_{i=1}^{4}Y_{Ui}\\+D_{UA}Y_{UA}\\+D_{UB}Y_{UB}\end{array} & \begin{array}{l}-D_U\sum_{i=1}^{4}X_{Ui}\\-D_{UA}X_{UA}\\-D_{UB}X_{UB}\end{array} & -D_U\sum_{i=1}^{4}Y_{Li} & D_U\sum_{i=1}^{4}X_{Li} & -D_{UA} & -D_{UB} & & \\
\hline
-4D_U & \begin{array}{l}4D_U\\+4D_L\\+D_{LA}\\+D_{LB}\end{array} & -D_U\sum_{i=1}^{4}Y_{Ui} & D_U\sum_{i=1}^{4}X_{Ui} & \begin{array}{l}D_U\sum_{i=1}^{4}Y_{Li}\\+D_L\sum_{i=1}^{4}Y_{Li}\\+D_{LA}Y_{LA}\\+D_{LB}Y_{LB}\end{array} & \begin{array}{l}-D_U\sum_{i=1}^{4}X_{Li}\\-D_L\sum_{i=1}^{4}X_{Li}\\-D_{LA}X_{LA}\\-D_{LB}X_{LB}\end{array} & & & -D_{LA} & -D_{LB} \\
\hline
\begin{array}{l}D_U\sum_{i=1}^{4}Y_{Ui}\\+D_{UA}Y_{UA}\\+D_{UB}Y_{UB}\end{array} & -D_U\sum_{i=1}^{4}Y_{Ui} & \begin{array}{l}D_U\sum_{i=1}^{4}Y_{Ui}Y_{Ui}\\+D_{UA}Y_{UA}Y_{UA}\\+D_{UB}Y_{UB}Y_{UB}\end{array} & \begin{array}{l}-D_U\sum_{i=1}^{4}X_{Ui}Y_{Ui}\\-D_{UA}X_{UA}Y_{UA}\\-D_{UB}X_{UB}Y_{UB}\end{array} & -D_U\sum_{i=1}^{4}Y_{Ui}Y_{Li} & D_U\sum_{i=1}^{4}X_{Li}Y_{Ui} & -D_{UA}Y_{UA} & -D_{UB}Y_{UB} & & \\
\hline
\begin{array}{l}-D_U\sum_{i=1}^{4}X_{Ui}\\-D_{UA}X_{UA}\\-D_{UB}X_{UB}\end{array} & D_U\sum_{i=1}^{4}X_{Ui} & \begin{array}{l}-D_U\sum_{i=1}^{4}X_{Ui}Y_{Ui}\\-D_{UA}X_{UA}Y_{UA}\\-D_{UB}X_{UB}Y_{UB}\end{array} & \begin{array}{l}D_U\sum_{i=1}^{4}X_{Ui}X_{Ui}\\+D_{UA}X_{UA}X_{UA}\\+D_{UB}X_{UB}X_{UB}\end{array} & D_U\sum_{i=1}^{4}X_{Ui}Y_{Li} & -D_U\sum_{i=1}^{4}X_{Ui}X_{Li} & D_{UA}X_{UA} & D_{UB}X_{UB} & & \\
\hline
-D_U\sum_{i=1}^{4}Y_{Li} & \begin{array}{l}D_U\sum_{i=1}^{4}Y_{Li}\\+D_L\sum_{i=1}^{4}Y_{Li}\\+D_{LA}Y_{LA}\\+D_{LB}Y_{LB}\end{array} & -D_U\sum_{i=1}^{4}Y_{Ui}Y_{Li} & D_U\sum_{i=1}^{4}X_{Ui}Y_{Li} & \begin{array}{l}D_U\sum_{i=1}^{4}Y_{Li}Y_{Li}\\+D_L\sum_{i=1}^{4}Y_{Li}Y_{Li}\\+D_{LA}Y_{LA}Y_{LA}\\+D_{LB}Y_{LB}Y_{LB}\end{array} & \begin{array}{l}-D_U\sum_{i=1}^{4}X_{Li}Y_{Li}\\-D_L\sum_{i=1}^{4}X_{Li}Y_{Li}\\-D_{LA}X_{LA}Y_{LA}\\-D_{LB}X_{LB}Y_{LB}\end{array} & & & -D_{LA}Y_{LA} & -D_{LB}Y_{LB} \\
\hline
D_U\sum_{i=1}^{4}X_{Li} & \begin{array}{l}-D_U\sum_{i=1}^{4}X_{Li}\\-D_L\sum_{i=1}^{4}X_{Li}\\-D_{LA}X_{LA}\\-D_{LB}X_{LB}\end{array} & D_U\sum_{i=1}^{4}X_{Li}Y_{Ui} & -D_U\sum_{i=1}^{4}X_{Ui}X_{Li} & \begin{array}{l}-D_U\sum_{i=1}^{4}X_{Li}Y_{Li}\\-D_L\sum_{i=1}^{4}X_{Li}Y_{Li}\\-D_{LA}X_{LA}Y_{LA}\\-D_{LB}X_{LB}Y_{LB}\end{array} & \begin{array}{l}D_U\sum_{i=1}^{4}X_{Li}X_{Li}\\+D_L\sum_{i=1}^{4}X_{Li}X_{Li}\\+D_{LA}X_{LA}X_{LA}\\+D_{LB}X_{LB}X_{LB}\end{array} & & & D_{LA}X_{LA} & D_{LB}X_{LB} \\
\hline
-D_{UA} & & -D_{UA}Y_{UA} & D_{UA}X_{UA} & & & D_{UA} & & & \\
\hline
-D_{UB} & & -D_{UB}Y_{UB} & D_{UB}X_{UB} & & & & D_{UB} & & \\
\hline
 & -D_{LA} & & & -D_{LA}Y_{LA} & D_{LA}X_{LA} & & & D_{LA} & \\
\hline
 & -D_{LB} & & & -D_{LB}Y_{LB} & D_{LB}X_{LB} & & & & D_{LB}
\end{array}\right]
$$

三維雙層具減振器運動方程式推導



$$
[K]=\begin{bmatrix}
\begin{array}{l}4K_U\\+K_{UA}\\+K_{UB}\end{array} & -4K_U & \begin{array}{l}K_U\sum_{i=1}^{4}Y_{Ui}\\+K_{UA}Y_{UA}\\+K_{UB}Y_{UB}\end{array} & \begin{array}{l}-K_U\sum_{i=1}^{4}X_{Ui}\\-K_{UA}X_{UA}\\-K_{UB}X_{UB}\end{array} & -K_U\sum_{i=1}^{4}Y_{Li} & K_U\sum_{i=1}^{4}X_{Li} & -K_{UA} & -K_{UB} & & \\
-4K_U & \begin{array}{l}4K_U\\+4K_L\\+K_{LA}\\+K_{LB}\end{array} & -K_U\sum_{i=1}^{4}Y_{Ui} & K_U\sum_{i=1}^{4}X_{Ui} & \begin{array}{l}K_U\sum_{i=1}^{4}Y_{Li}\\+K_L\sum_{i=1}^{4}Y_{Li}\\+K_{LA}Y_{LA}\\+K_{LB}Y_{LB}\end{array} & \begin{array}{l}-K_U\sum_{i=1}^{4}X_{Li}\\-K_L\sum_{i=1}^{4}X_{Li}\\-K_{LA}X_{LA}\\-K_{LB}X_{LB}\end{array} & & & -K_{LA} & -K_{LB}\\
\begin{array}{l}K_U\sum_{i=1}^{4}Y_{Ui}\\+K_{UA}Y_{UA}\\+K_{UB}Y_{UB}\end{array} & -K_U\sum_{i=1}^{4}Y_{Ui} & \begin{array}{l}K_U\sum_{i=1}^{4}Y_{Ui}Y_{Ui}\\+K_{UA}Y_{UA}Y_{UA}\\+K_{UB}Y_{UB}Y_{UB}\end{array} & \begin{array}{l}-K_U\sum_{i=1}^{4}X_{Ui}Y_{Ui}\\-K_{UA}X_{UA}Y_{UA}\\-K_{UB}X_{UB}Y_{UB}\end{array} & -K_U\sum_{i=1}^{4}Y_{Ui}Y_{Li} & K_U\sum_{i=1}^{4}X_{Li}Y_{Ui} & -K_{UA}Y_{UA} & -K_{UB}Y_{UB} & & \\
\begin{array}{l}-K_U\sum_{i=1}^{4}X_{Ui}\\-K_{UA}X_{UA}\\-K_{UB}X_{UB}\end{array} & K_U\sum_{i=1}^{4}X_{Ui} & \begin{array}{l}-K_U\sum_{i=1}^{4}X_{Ui}Y_{Ui}\\-K_{UA}X_{UA}Y_{UA}\\-K_{UB}X_{UB}Y_{UB}\end{array} & \begin{array}{l}K_U\sum_{i=1}^{4}X_{Ui}X_{Ui}\\+K_{UA}X_{UA}X_{UA}\\+K_{UB}X_{UB}X_{UB}\end{array} & K_U\sum_{i=1}^{4}X_{Ui}Y_{Li} & -K_U\sum_{i=1}^{4}X_{Ui}X_{Li} & K_{UA}X_{UA} & K_{UB}X_{UB} & & \\
-K_U\sum_{i=1}^{4}Y_{Li} & \begin{array}{l}K_U\sum_{i=1}^{4}Y_{Li}\\+K_L\sum_{i=1}^{4}Y_{Li}\\+K_{LA}Y_{LA}\\+K_{LB}Y_{LB}\end{array} & -K_U\sum_{i=1}^{4}Y_{Ui}Y_{Li} & K_U\sum_{i=1}^{4}X_{Ui}Y_{Li} & \begin{array}{l}K_U\sum_{i=1}^{4}Y_{Li}Y_{Li}\\+K_L\sum_{i=1}^{4}Y_{Li}Y_{Li}\\+K_{LA}Y_{LA}Y_{LA}\\+K_{LB}Y_{LB}Y_{LB}\end{array} & \begin{array}{l}-K_U\sum_{i=1}^{4}X_{Li}Y_{Li}\\-K_L\sum_{i=1}^{4}X_{Li}Y_{Li}\\-K_{LA}X_{LA}Y_{LA}\\-K_{LB}X_{LB}Y_{LB}\end{array} & & & -K_{LA}Y_{LA} & -K_{LB}Y_{LB}\\
K_U\sum_{i=1}^{4}X_{Li} & \begin{array}{l}-K_U\sum_{i=1}^{4}X_{Li}\\-K_L\sum_{i=1}^{4}X_{Li}\\-K_{LA}X_{LA}\\-K_{LB}X_{LB}\end{array} & K_U\sum_{i=1}^{4}X_{Li}Y_{Ui} & -K_U\sum_{i=1}^{4}X_{Ui}X_{Li} & \begin{array}{l}-K_U\sum_{i=1}^{4}X_{Li}Y_{Li}\\-K_L\sum_{i=1}^{4}X_{Li}Y_{Li}\\-K_{LA}X_{LA}Y_{LA}\\-K_{LB}X_{LB}Y_{LB}\end{array} & \begin{array}{l}K_U\sum_{i=1}^{4}X_{Li}X_{Li}\\+K_L\sum_{i=1}^{4}X_{Li}X_{Li}\\+K_{LA}X_{LA}X_{LA}\\+K_{LB}X_{LB}X_{LB}\end{array} & & & K_{LA}X_{LA} & K_{LB}X_{LB}\\
-K_{UA} & & -K_{UA}Y_{UA} & K_{UA}X_{UA} & & & K_{UA} & & & \\
-K_{UB} & & -K_{UB}Y_{UB} & K_{UB}X_{UB} & & & & K_{UB} & & \\
 & -K_{LA} & & & -K_{LA}Y_{LA} & K_{LA}X_{LA} & & & K_{LA} & \\
 & -K_{LB} & & & -K_{LB}Y_{LB} & K_{LB}X_{LB} & & & & K_{LB}
\end{bmatrix}
$$

三維雙層具減振器運動方程式推導

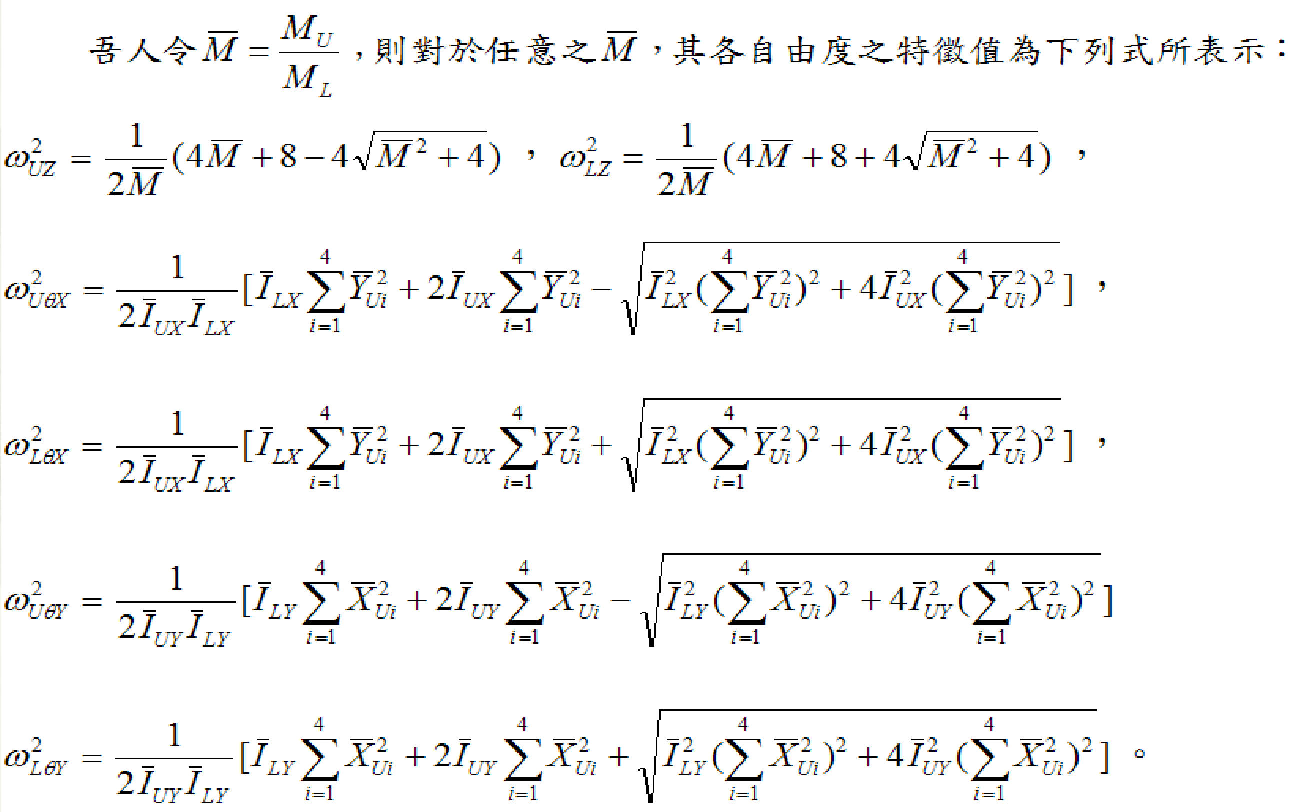
$$[F]=\left[\begin{array}{c}F_{UZ}(t)\\ \hline F_{LZ}(t)\\ \hline F_{UZ}(t)\cdot Y_{UT}\\ \hline -F_{UZ}(t)\cdot X_{UT}\\ \hline F_{LZ}(t)\cdot Y_{LT}\\ \hline F_{LZ}(t)\cdot X_{LT}\\ \hline F_{UA}(t)\\ \hline F_{UB}(t)\\ \hline F_{LA}(t)\\ \hline F_{LB}(t)\end{array}\right]$$

三維雙層無減振器之頻率解析解

由 $\sum_{i=1}^{4} Y_{Ui} = \sum_{i=1}^{4} X_{Ui} = \sum_{i=1}^{4} X_{Ui} Y_{Ui} = 0$ $K_U = K_L$ 以及 $X_{Ui} = X_{Li}$，$Y_{Ui} = Y_{Li}$。

$$\begin{bmatrix} 4 & -4 & 0 & 0 & 0 & 0 \\ -4/\overline{M} & 8/\overline{M} & 0 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{\sum_{i=1}^{4} Y_{Ui}^2}{I_{UX}} & 0 & -\dfrac{\sum_{i=1}^{4} Y_{Ui}^2}{I_{UX}} & 0 \\ 0 & 0 & 0 & \dfrac{\sum_{i=1}^{4} X_{Ui}^2}{I_{UY}} & 0 & -\dfrac{\sum_{i=1}^{4} X_{Ui}^2}{I_{UY}} \\ 0 & 0 & -\dfrac{\sum_{i=1}^{4} Y_{Li}^2}{I_{LX}} & 0 & \dfrac{2\sum_{i=1}^{4} Y_{Li}^2}{I_{LX}} & 0 \\ 0 & 0 & 0 & -\dfrac{\sum_{i=1}^{4} X_{Li}^2}{I_{LY}} & 0 & \dfrac{2\sum_{i=1}^{4} X_{Li}^2}{I_{LY}} \end{bmatrix}$$

三維雙層無減振器之頻率解析解



吾人令$\overline{M}=\dfrac{M_U}{M_L}$，則對於任意之$\overline{M}$，其各自由度之特徵值為下列式所表示：

$$\omega_{UZ}^2=\frac{1}{2\overline{M}}(4\overline{M}+8-4\sqrt{\overline{M}^2+4})\text{，}\quad \omega_{LZ}^2=\frac{1}{2\overline{M}}(4\overline{M}+8+4\sqrt{\overline{M}^2+4})\text{，}$$

$$\omega_{U\theta X}^2=\frac{1}{2\bar{I}_{UX}\bar{I}_{LX}}[\bar{I}_{LX}\sum_{i=1}^{4}\overline{Y}_{Ui}^2+2\bar{I}_{UX}\sum_{i=1}^{4}\overline{Y}_{Ui}^2-\sqrt{\bar{I}_{LX}^2(\sum_{i=1}^{4}\overline{Y}_{Ui}^2)^2+4\bar{I}_{UX}^2(\sum_{i=1}^{4}\overline{Y}_{Ui}^2)^2}\,]\text{，}$$

$$\omega_{L\theta X}^2=\frac{1}{2\bar{I}_{UX}\bar{I}_{LX}}[\bar{I}_{LX}\sum_{i=1}^{4}\overline{Y}_{Ui}^2+2\bar{I}_{UX}\sum_{i=1}^{4}\overline{Y}_{Ui}^2+\sqrt{\bar{I}_{LX}^2(\sum_{i=1}^{4}\overline{Y}_{Ui}^2)^2+4\bar{I}_{UX}^2(\sum_{i=1}^{4}\overline{Y}_{Ui}^2)^2}\,]\text{，}$$

$$\omega_{U\theta Y}^2=\frac{1}{2\bar{I}_{UY}\bar{I}_{LY}}[\bar{I}_{LY}\sum_{i=1}^{4}\overline{X}_{Ui}^2+2\bar{I}_{UY}\sum_{i=1}^{4}\overline{X}_{Ui}^2-\sqrt{\bar{I}_{LY}^2(\sum_{i=1}^{4}\overline{X}_{Ui}^2)^2+4\bar{I}_{UY}^2(\sum_{i=1}^{4}\overline{X}_{Ui}^2)^2}\,]$$

$$\omega_{L\theta Y}^2=\frac{1}{2\bar{I}_{UY}\bar{I}_{LY}}[\bar{I}_{LY}\sum_{i=1}^{4}\overline{X}_{Ui}^2+2\bar{I}_{UY}\sum_{i=1}^{4}\overline{X}_{Ui}^2+\sqrt{\bar{I}_{LY}^2(\sum_{i=1}^{4}\overline{X}_{Ui}^2)^2+4\bar{I}_{UY}^2(\sum_{i=1}^{4}\overline{X}_{Ui}^2)^2}\,]\text{。}$$

三維雙層無減振器之頻率解析解

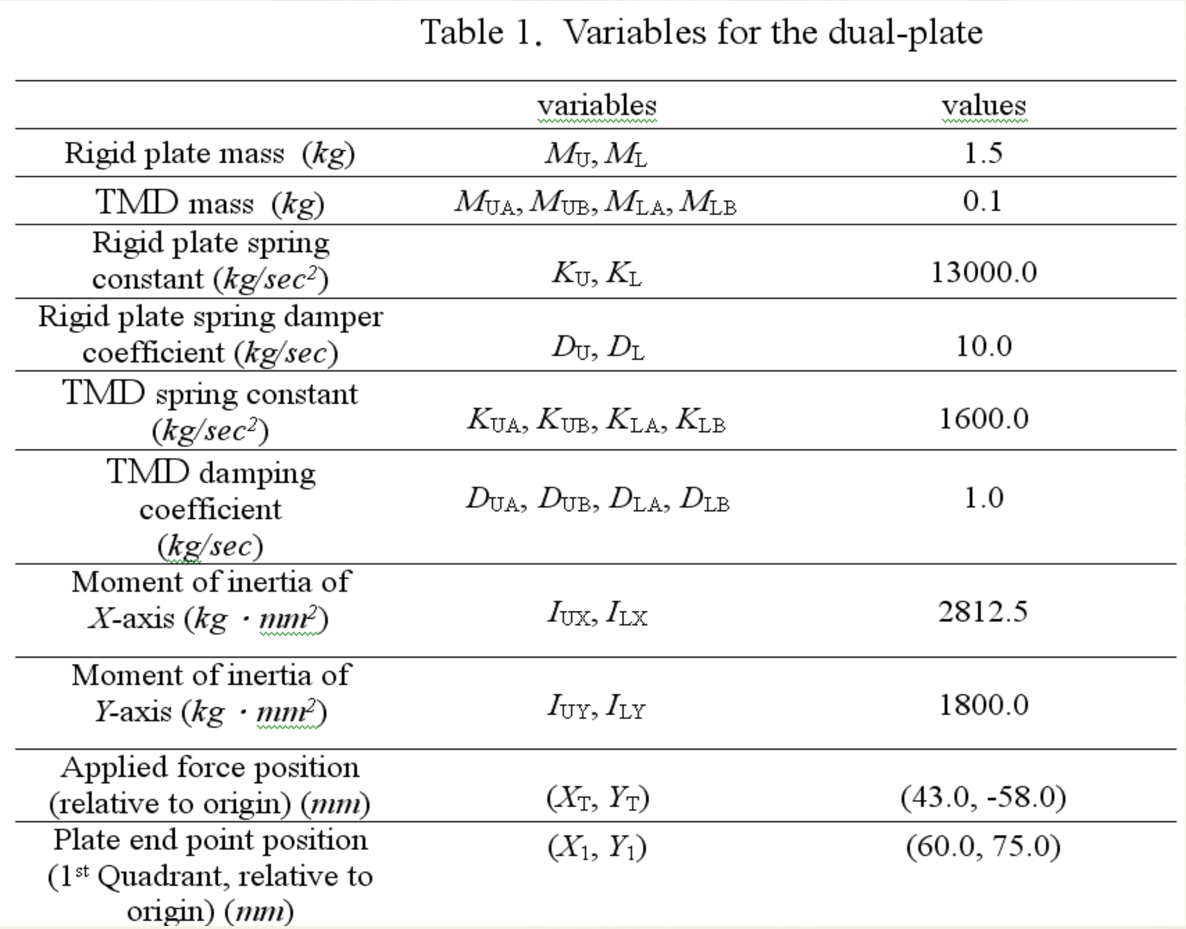


Table 1. Variables for the dual-plate

	variables	values
Rigid plate mass (*kg*)	M_U, M_L	1.5
TMD mass (*kg*)	$M_{UA}, M_{UB}, M_{LA}, M_{LB}$	0.1
Rigid plate spring constant (kg/sec^2)	K_U, K_L	13000.0
Rigid plate spring damper coefficient (kg/sec)	D_U, D_L	10.0
TMD spring constant (kg/sec^2)	$K_{UA}, K_{UB}, K_{LA}, K_{LB}$	1600.0
TMD damping coefficient (kg/sec)	$D_{UA}, D_{UB}, D_{LA}, D_{LB}$	1.0
Moment of inertia of *X*-axis ($kg \cdot mm^2$)	I_{UX}, I_{LX}	2812.5
Moment of inertia of *Y*-axis ($kg \cdot mm^2$)	I_{UY}, I_{LY}	1800.0
Applied force position (relative to origin) (*mm*)	(X_T, Y_T)	(43.0, -58.0)
Plate end point position (1st Quadrant, relative to origin) (*mm*)	(X_1, Y_1)	(60.0, 75.0)

三維雙層無減振器之頻率解析解

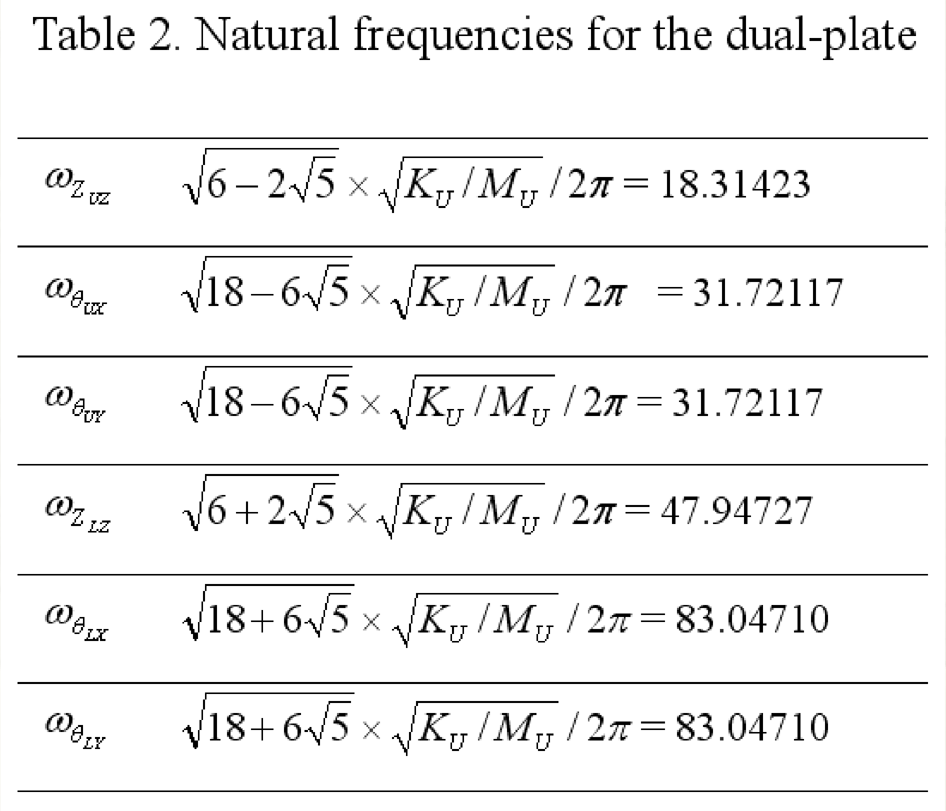


Table 2. Natural frequencies for the dual-plate

$\omega_{Z_{UZ}}$	$\sqrt{6-2\sqrt{5}}\times\sqrt{K_U/M_U}/2\pi=18.31423$
$\omega_{\theta_{UX}}$	$\sqrt{18-6\sqrt{5}}\times\sqrt{K_U/M_U}/2\pi=31.72117$
$\omega_{\theta_{UY}}$	$\sqrt{18-6\sqrt{5}}\times\sqrt{K_U/M_U}/2\pi=31.72117$
$\omega_{Z_{LZ}}$	$\sqrt{6+2\sqrt{5}}\times\sqrt{K_U/M_U}/2\pi=47.94727$
$\omega_{\theta_{LX}}$	$\sqrt{18+6\sqrt{5}}\times\sqrt{K_U/M_U}/2\pi=83.04710$
$\omega_{\theta_{LY}}$	$\sqrt{18+6\sqrt{5}}\times\sqrt{K_U/M_U}/2\pi=83.04710$

實驗設計-模型介紹

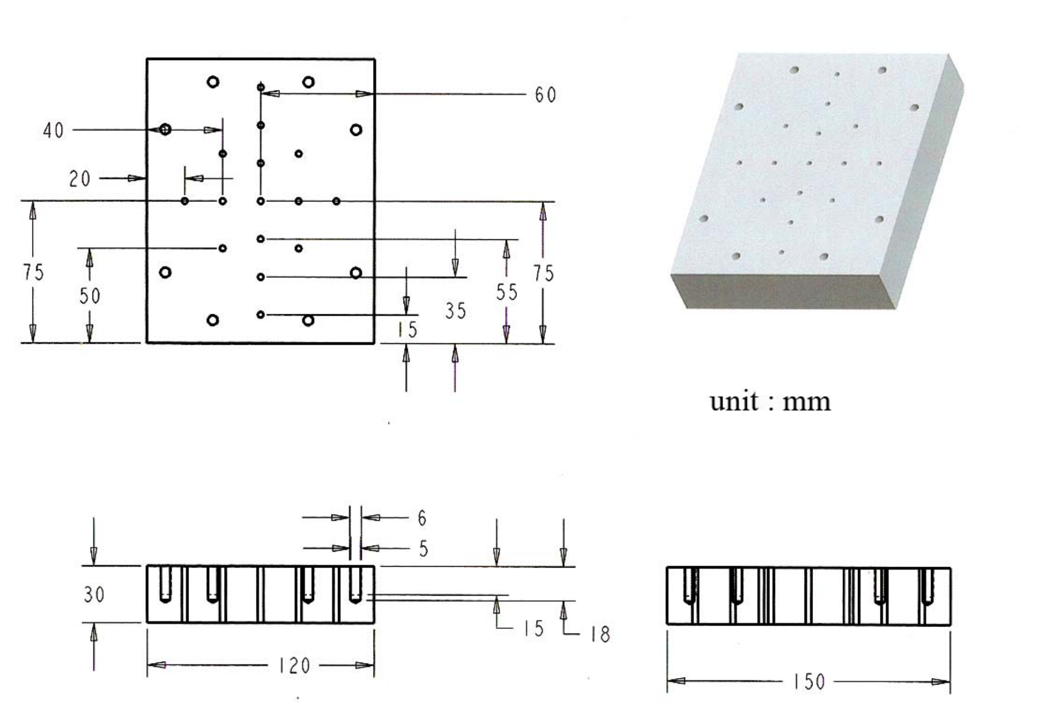


圖1. 振動主體模型設計圖。

實驗設計-模型介紹

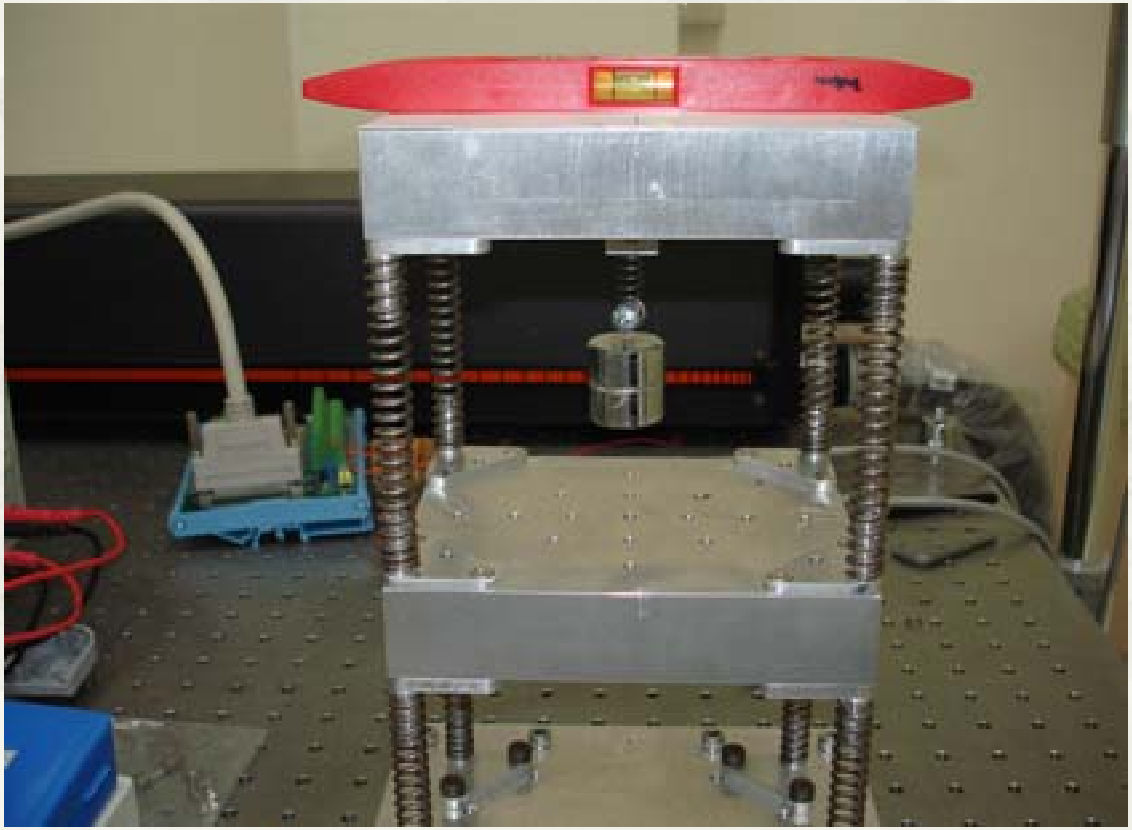
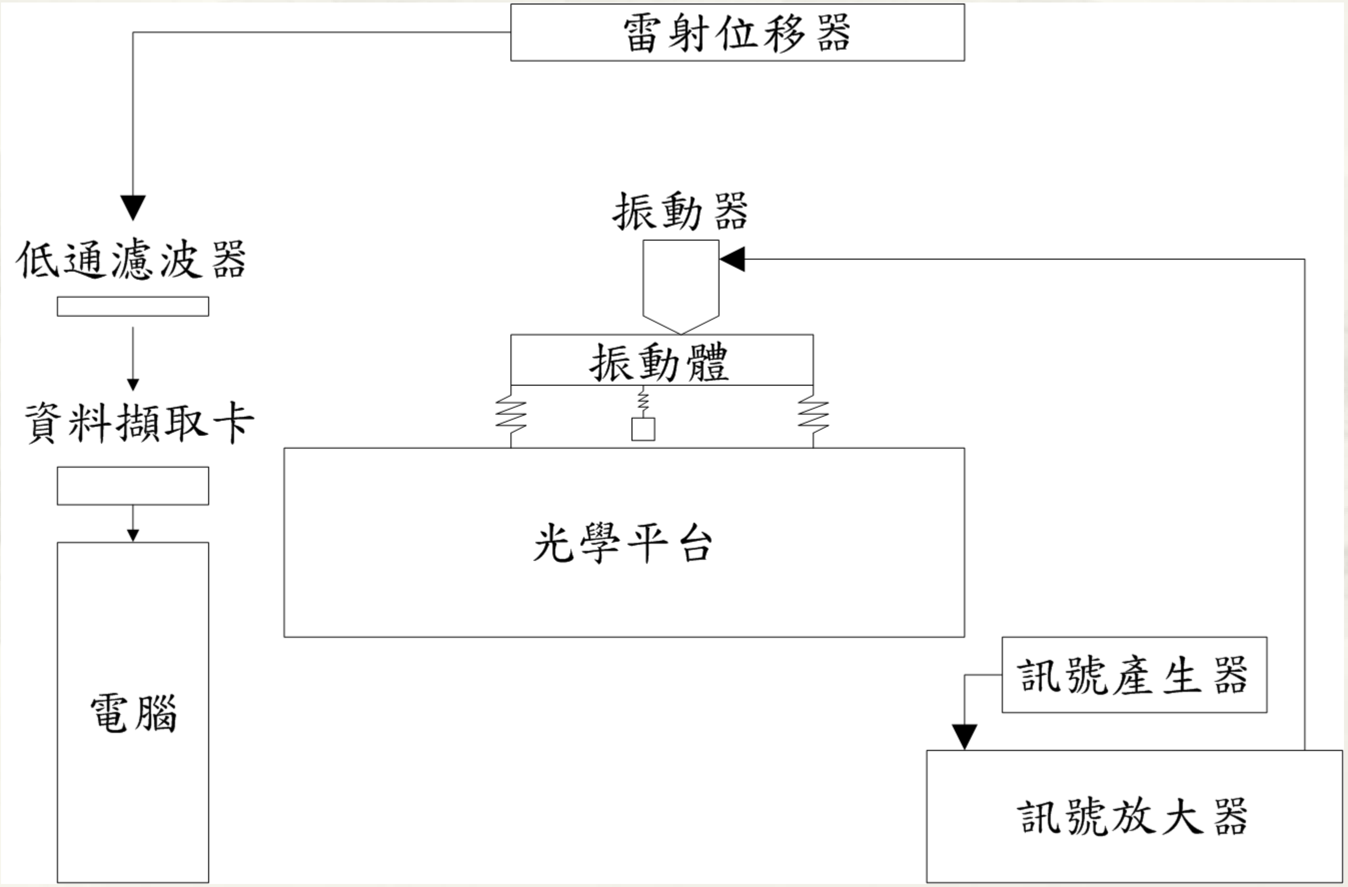


圖2. 雙層振動主體掛載減振器之示意圖。

實驗設計-流程介紹



理論與實驗結果

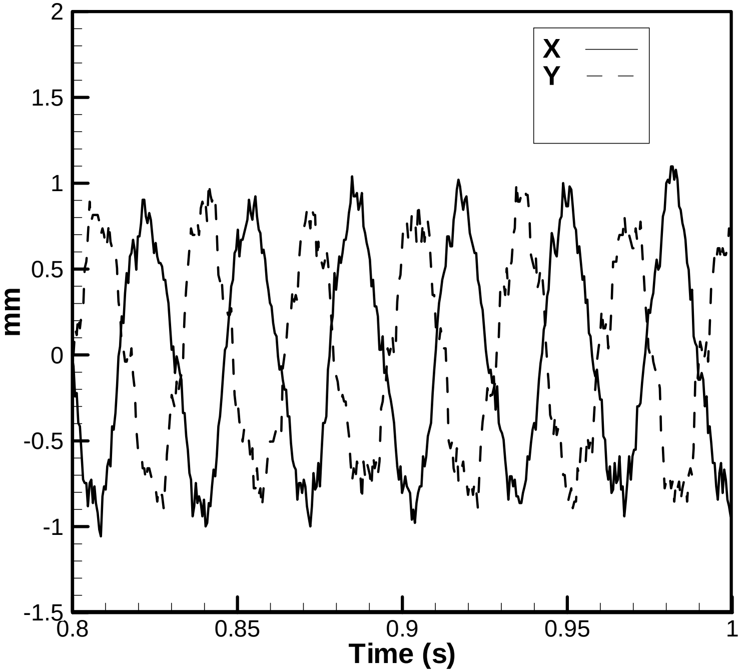


圖6. 雙層振動平板無減振器在31赫茲時之振幅 。

理論與實驗結果

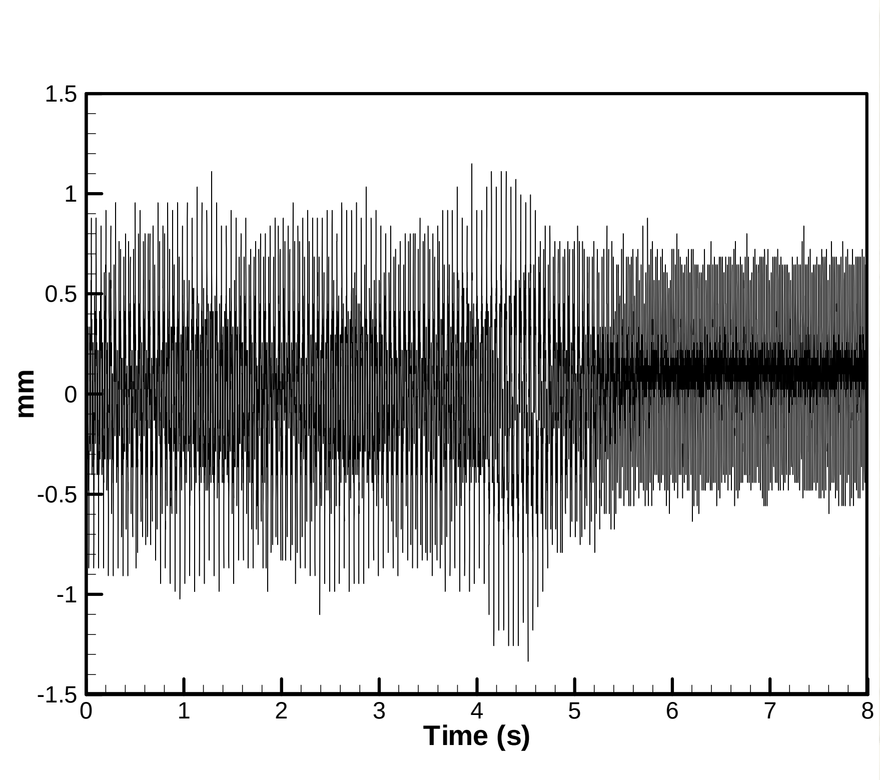


圖5. 減振器抑制振動主體效果示意圖。

理論與實驗結果

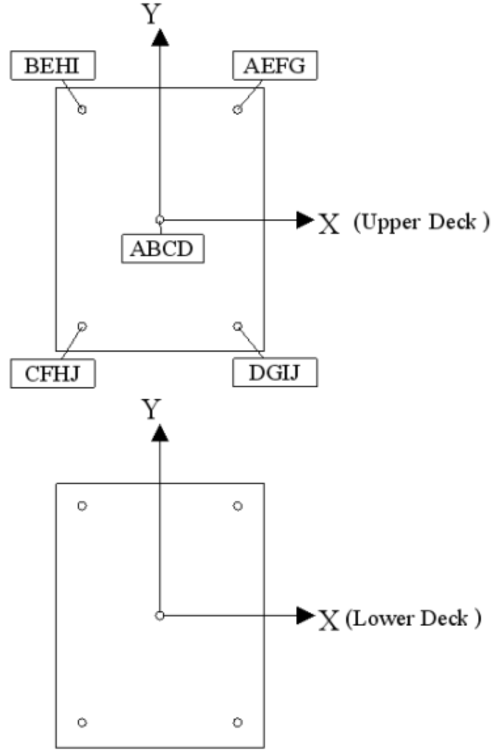


Fig. 7. Various combinations of damper locations (two TMDs attached to the upper deck only).

理論與實驗結果

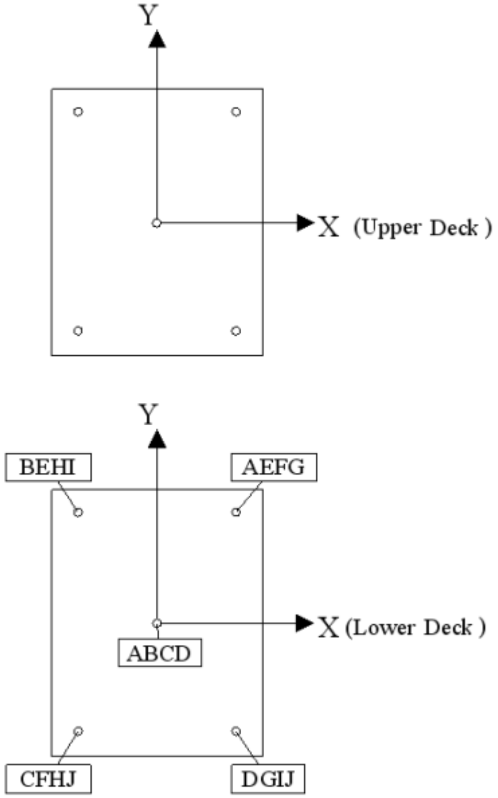


Fig. 9. Various combinations of damper locations (two TMDs attached to the lower deck only).

理論與實驗結果

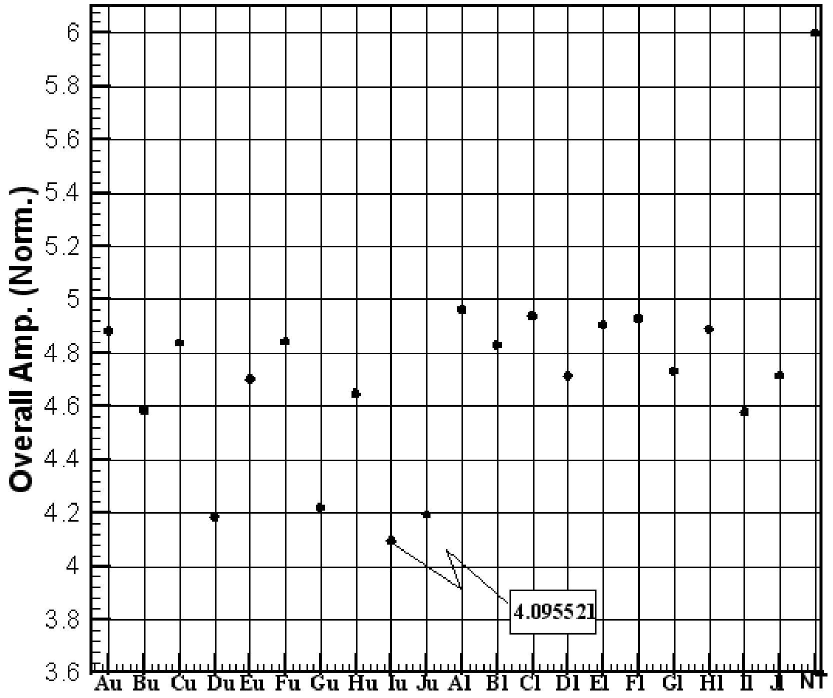


Fig. 8. Overall normalized amplitudes of 2TMDs System.
Force applied on upper plate end point.

理論與實驗結果

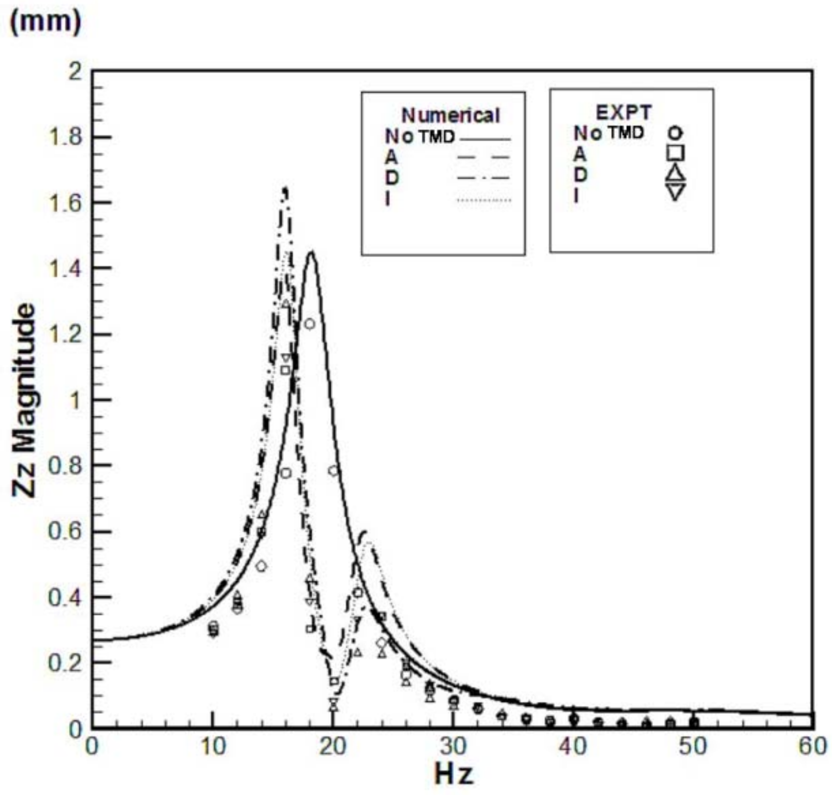


Fig. 13. Frequency response of the Z_Z mode (two TMDs are attached to the upper deck).

理論與實驗結果

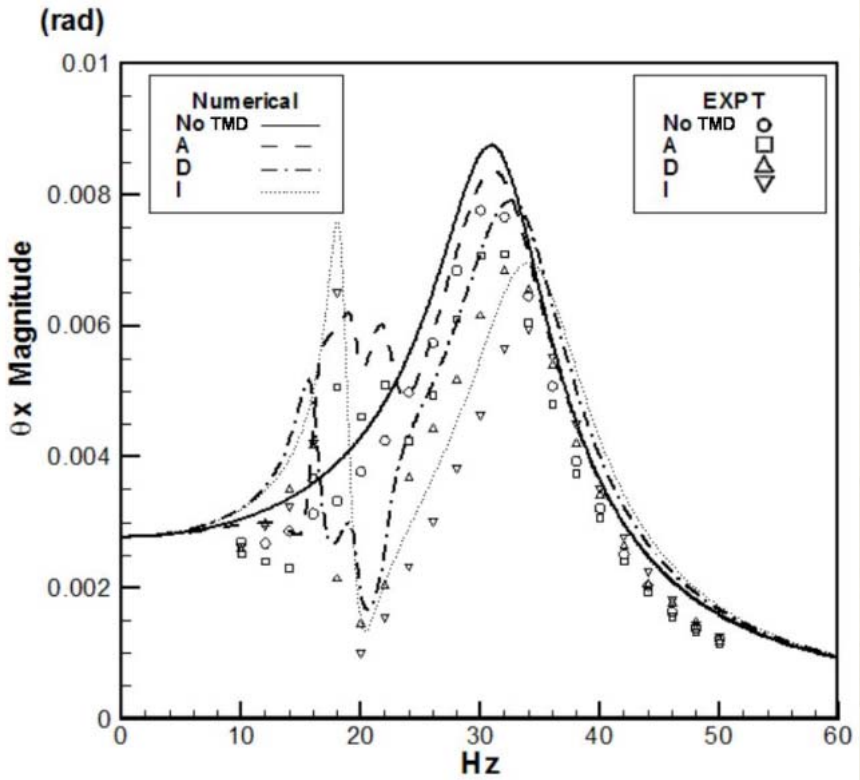


Fig. 14. Frequency response of the θ_X mode (two TMDs are attached to the upper deck).

理論與實驗結果

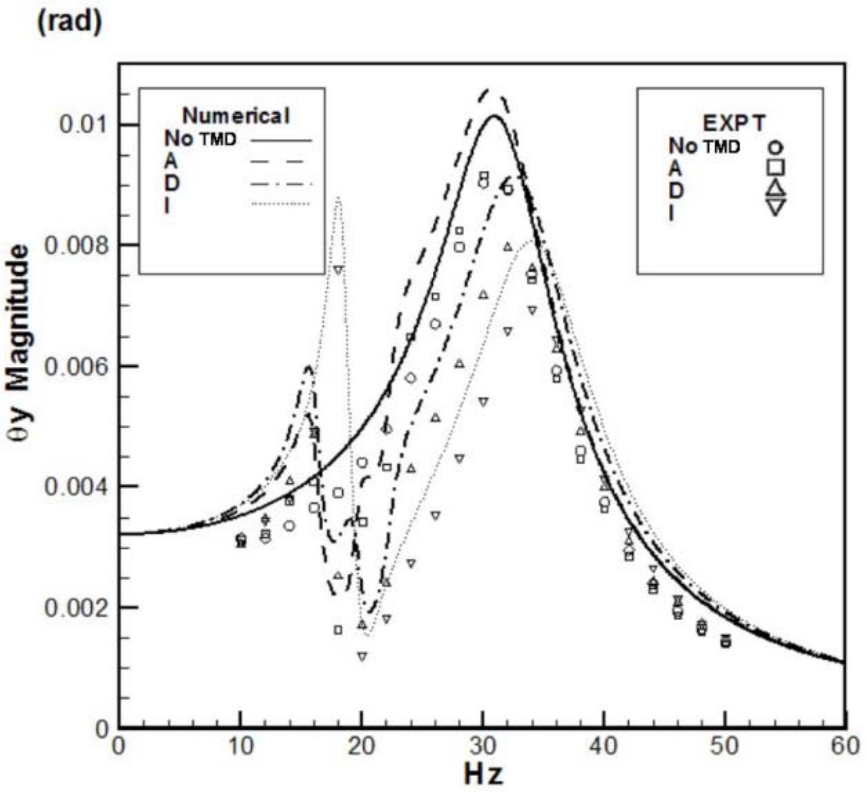


Fig. 15. Frequency response of the θ_Y mode (two TMDs are attached to the upper deck).

理論與實驗結果

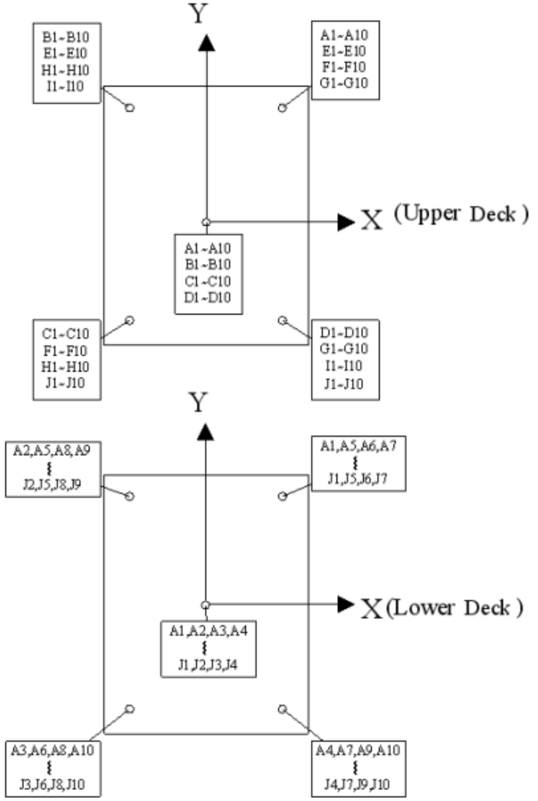


Fig. 10. Various combinations of damper locations
(two TMDs attached to both the upper and lower decks).

理論與實驗結果

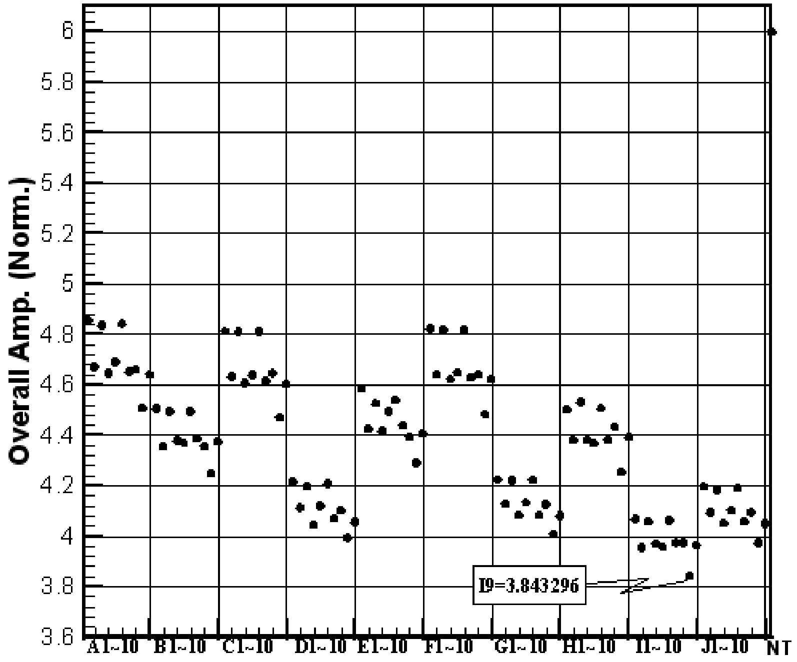


Fig. 11. Overall normalized amplitudes of 4TMDs System.
Force applied on upper plate end point.

理論與實驗結果

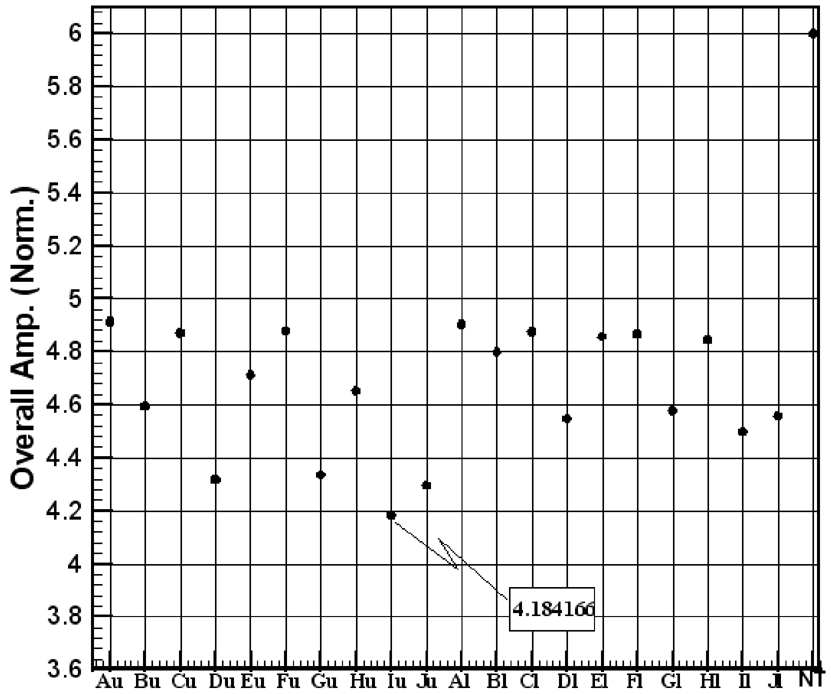


Fig. 16. Overall normalized amplitudes of 2TMDs System.
Force applied on both plates end point.

理論與實驗結果

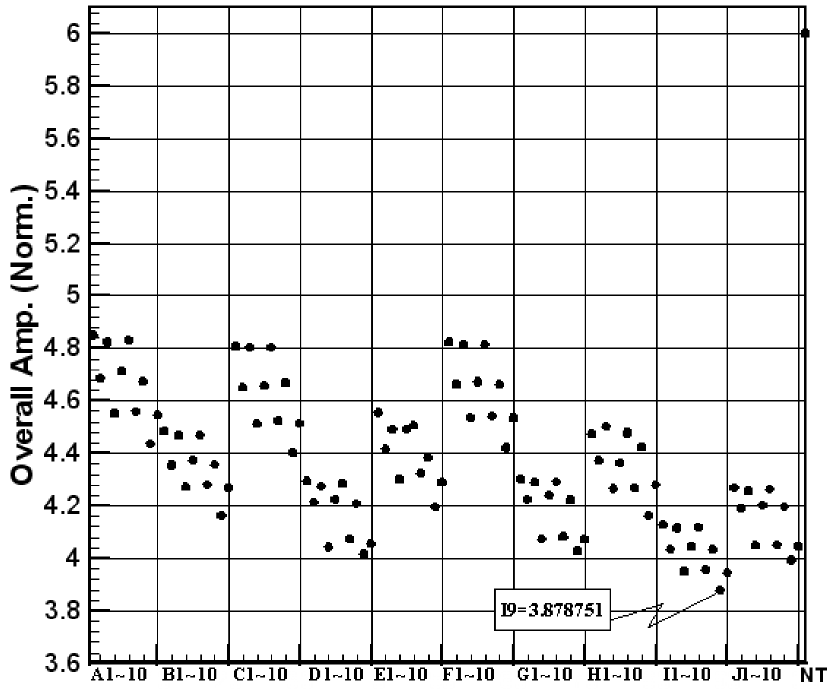


Fig. 17. Overall normalized amplitudes of 4TMDs System.
Force applied on both plates end point.

理論與實驗結果

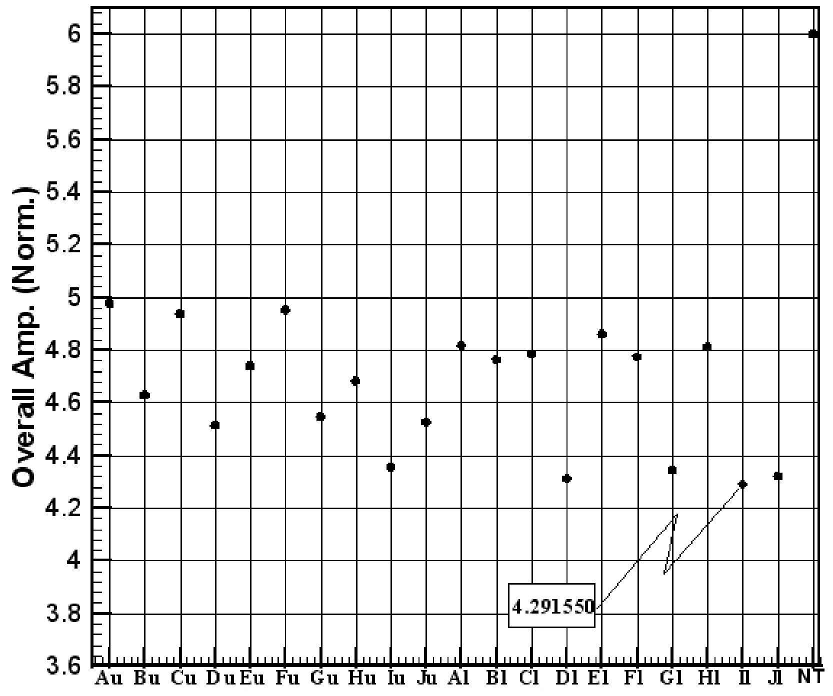


Fig. 18. Overall normalized amplitudes of 2TMDs System.
Force applied on lower plate end point.

結論

1. The optimal position for TMDs in suppressing vertical and rotational vibration is different; therefore, the locations of TMDs should be adjusted accordingly.

2. The overall optimal hanging locations for two TMDs are beneath the point of applied force and at the diagonal quadrant endpoint. In general, when external force is not applied in the center, placing dampers in the center is not preferable.

結論

3. When both of the rigid plates are subjected to external force, attaching four TMDs (two to each plate), provides superior performance in vibration reduction than only using two TMDs.

4. If only two TMDs are allowed and only one of the two plates is subjected to external force, attaching both TMDs to the plate of applied force is optimal.

報告完畢

感謝聆聽