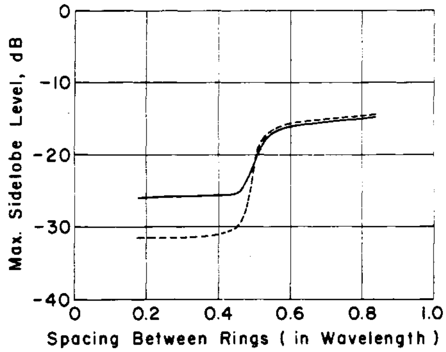

Fig. 2. Synthesized radiation patterns.

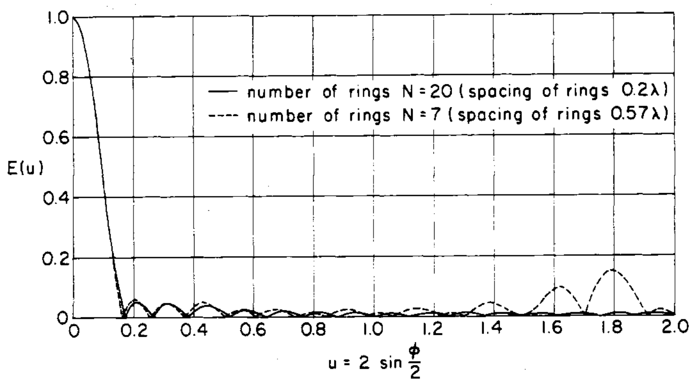

Fig. 3. Maximum sidelobe level of synthesized Taylor patterns. Solid curve: designed sidelobe level = −25 dB. Dashed curve: designed sidelobe level = −30 dB.

when $u \leq 1.3$ or $\phi \leq 40^\circ$, beyond which high sidelobes appear. This is perhaps to be expected because a concentric-ring array has the characteristics of both broadside and endfire arrays and grating lobes will appear in the case of cophasal endfire arrays when element spacings are greater than a half-wavelength. In any case, it is clear that multiple-ring arrays with sampled amplitudes approximate the patterns obtainable from circular aperture distributions closely when the inter-ring spacings are small. Since in practice one does not wish to use more rings or more elements than necessary, it is interesting to examine the effect of inter-ring spacing on the maximum sidelobe level.

The maximum sidelobe level obtained by a concentric-ring array with an outermost radius of 4λ has been computed as a function of the spacing between adjacent rings. Taylor distributions for $\bar{n}=4$ were used for two designed sidelobe levels; namely, −25 dB and −30 dB. The result is shown as the two curves in Fig. 3. It is seen that inter-ring spacing has no significant effect on sidelobe level, but that a drastic rise in the maximum sidelobe level appears in both cases when the spacing increases beyond a half-wavelength. We, therefore, conclude that the radiation pattern of a circular aperture antenna can be closely approximated by a concentric-ring array with sampled amplitudes if the inter-ring spacing is less than about 0.4λ.

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On Detecting Total and Partial Symmetry of Switching Functions

Abstract—A method is presented for identifying total or partial symmetry of switching functions based on the application of the principle of residue test. The invariance of a switching function under a single interchange of two variables can be readily detected from the equality of some of the residues of expansions about these two variables. This procedure of detecting invariance is directly applied for the identification of total or partial symmetry of a switching function whose variables of symmetry are either all unprimed, all primed, or of mixed nature.

Introduction

A switching function of n variables, which are either all unprimed, all primed, or mixed, is said to possess total symmetry in these variables if any permutation of the variables leaves the function invariant [1]. A switching function which remains invariant under any permutation of the variables, either all unprimed, all primed, or mixed, belonging to a subset of the set of n variables is said to exhibit partial symmetry in these variables of the subset. Methods for detecting symmetries of switching functions have been suggested by several authors [2]–[8]. In this letter, based on the application of the principle of residue test by numerical methods [9], we suggest a method that detects all types of symmetries corresponding to a switching function. The invariance of a switching function under a single interchange of two variables is readily detectable from the equality of the definite groups of residues of expansion about these two variables. To detect total symmetry, a comparison of the residues associated with only n expansions is necessary, which also gives all the alternative forms of representations of symmetries with the corresponding a numbers. The partial symmetry of a switching function along with all the alternative representations of symmetries is also similarly detected.

Invariance of a Function Under Single Permutation of Two Variables

Let F be a switching function of n variables $(x_1, x_2, \cdots, x_n)$. Expanding F about any two of these n variables, say x_i and x_j, F can be written as

$$F = \bar{x}_i\bar{x}_j f_0 + \bar{x}_i x_j f_1 + x_i\bar{x}_j f_2 + x_i x_j f_3 \qquad (1)$$

where f_0, f_1, f_2, and f_3 represent residual functions of expansion of $(n-2)$ variables, not including x_i and x_j. From (1) we see that the function F remains invariant under single permutation of the two variables x_i and x_j, if 1) $f_1 = f_2$, when both x_i and x_j are unprimed; 2) $f_0 = f_3$, when both x_i and x_j are primed; 3) $f_0 = f_1 = f_2 = f_3$, when either x_i is primed and x_j is unprimed or vice versa. Thus in order to detect invariance of a function under a single interchange of two variables, where the variables are either both unprimed, both primed, or mixed, we need only expand the function about these two variables and compare their different residues of expansion for equality.

In order to evaluate directly the residues of expansion about any variable for a function expressed in decimal mode, consider a binary number 111101, of which the decimal equivalent is 61. Suppose we want to know whether the third digit from the left in this binary number is a 0 or 1, without actually writing the same, but from a knowledge of its decimal counterpart only. Put a binary point in 111101 after the third digit from the left, i.e., to the right of the digit whose identity is to be disclosed. We get 111.101 whose decimal equivalent is $7\frac{5}{8}$. Thus by putting the binary point at the aforementioned position we have actually divided the original binary number by 2^3, or its equivalent decimal value 61 by 8. Now discard all the binary digits to the right of the binary point. We obtain 111, which is equivalent to the decimal number 7, an integer which can be obtained independently from the number $7\frac{5}{8}$ by discarding the fractional part $\frac{5}{8}$. The integral part 7 of the decimal quotient $7\frac{5}{8}$ is an odd number, which implies that the right-hand digit of its binary equivalent 111 is 1. If the whole number part had been even, the digit at the extreme right would have been 0 [9].

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Example: $F(x_1, x_2, x_3, x_4) = \sum(4, 6, 8, 12)$

To apply the residue test to the x_2 variable we see that in the binary number representation of the function, the binary point should be shifted two digit positions to the left, which requires that its decimal counterpart be divided by 4. Thus dividing by 4, the results obtained for different terms of the function are $4/4 = 1$, odd; $6/4 = 1+$, odd; $8/4 = 2$, even; $12/4 = 3$, odd. Hence the residues can be written by grouping the decimal numbers as $F = x_2(4, 6, 12) + \bar{x}_2(8)$. Consider now the decimal numbers 12 and 8. We note that x_2 and $\bar{x}_2$ can be factored out from the binary representations of 12 and 8, respectively, resulting in x_2(1-00) and $\bar{x}_2$(1-00), which shows that the x_2 and $\bar{x}_2$ residues are identical. This equality can be shown directly, if we replace 0 in the x_2 position of the binary representation 1000 of 8 by 1 and then expand it about x_2. This replacement of 0 by 1 in the x_2 position of 1000 is simply equivalent to adding 2^2 to the binary number 1000 or 4 to its decimal equivalent. So in the above expansion, by adding 4 to 8, we get $\bar{x}_2(12)$ in place of $\bar{x}_2(8)$, which gives an easy way of identifying whether the $\bar{x}_2$ residue is equal to, contains, or is contained in the x_2 residue.

Consider expansion of F as given in (1). We see that when $f_1 = f_2$, then not only F is invariant for $x_i \sim x_j$ ($\sim$ means "permuted with"), but also all the functions derived from F by applying $(n-2)!2^{n-2}$ transformations (negation and permutation operations) to the remaining $(n-2)$ variables are invariant. The same is the case for $\bar{x}_i \sim \bar{x}_j$ when $f_0 = f_3$. Also, when $f_1 = f_2$, we see that the function derived from F by priming both x_i and x_j remains invariant when $\bar{x}_i \sim \bar{x}_j$, and this invariance is again unaffected by the application of any transformation to the set of $(n-2)$ residual variables. Similarly, when $f_0 = f_3$, the function obtained from F by priming either x_i or x_j remains invariant when $\bar{x}_i \sim x_j$ or $x_i \sim \bar{x}_j$, and this invariance is again unaffected by applying any transformation to the set of $(n-2)$ residual variables [5], [8].

Detection of Total Symmetry

In detecting total symmetry of switching functions by the application of the aforementioned concepts, the following theorem is important.

Theorem 1: A switching function F of n variables $(x_1, x_2, x_3, \cdots, x_{n-1}, x_n)$ is totally symmetric if it is invariant under only n permutations $x_1 \sim x_2$, $x_2 \sim x_3, \cdots, x_{n-1} \sim x_n, x_n \sim x_1$.

The aggregate of n expansions about the pairs of variables (x_1, x_2), $(x_2, x_3), \cdots, (x_{n-1}, x_n), (x_n, x_1)$ is called the set of cyclic expansions. As illustration, consider the following example.

Example: $F(x_1, x_2, x_3, x_4) = \sum(0, 1, 3, 4, 6, 7, 8, 10, 11, 14)$

Expanding F cyclically and checking the residues of expansion for equality, we detect invariance under the following permutation and negation operations: 1) $x_1 \sim x_2$, 2) $\bar{x}_1 \sim \bar{x}_2$, 3) $x_2 \sim \bar{x}_3$, 4) $\bar{x}_2 \sim x_3$, 5) $x_3 \sim \bar{x}_4$, 6) $\bar{x}_3 \sim x_4$, 7) $x_4 \sim x_1$, 8) $\bar{x}_4 \sim \bar{x}_1$. Combining now 2), 4), 5), and 8), we have $F(\bar{x}_1, \bar{x}_2, x_3, \bar{x}_4) \equiv \bar{x}_1 \sim \bar{x}_2, \bar{x}_2 \sim x_3, x_3 \sim \bar{x}_4, \bar{x}_4 \sim \bar{x}_1$, where the sign $\equiv$ denotes invariance under transformation. So F is totally symmetric, and the variables of symmetry are $(\bar{x}_1, \bar{x}_2, x_3, \bar{x}_4)$. The a numbers of the function can be found by writing the function in the truth table form and double negating the columns under x_1, x_2, and x_4. Thus F can be written as $S_{2,3}(\bar{x}_1, \bar{x}_2, x_3, \bar{x}_4)$. Similarly, combining 1), 3), 6), and 7), F can also be identified as $S_{1,2}(x_1, x_2, \bar{x}_3, x_4)$. To find the variables of symmetry, we consider any literal, preferably one with the lowest value of the subscript i, both primed and unprimed in the set of cyclic permutations, and then associate it with literals with which it is connected by $\sim$ signs. We continue in this way until ending on the literal with which we started. The literals that occur in any closed path give a set of variables of symmetry.

Detection of Partial Symmetry

In order to detect partial symmetry of a switching function, we are to search for the invariance of the function under every interchange of the variables belonging to a subset of the set of n variables. The chain of cyclic permutations has to be complete with these variables of the subset, the variables being either all unprimed, all primed, or of mixed type. In this case, depending on the nature of the problem, a complete set of expansions about possible variable pairs may be necessary. Consider the following example as illustration.

Example: $F(x_1, x_2, x_3, x_4) = \sum(0, 1, 3, 4, 6, 7, 9, 10, 12, 15)$

Expanding F about different variable pairs, we detect invariance under the following permutation and negation operations: 1) $\bar{x}_2 \sim x_3$, 2) $x_2 \sim \bar{x}_3$, 3) $\bar{x}_3 \sim x_4$, 4) $x_3 \sim \bar{x}_4$, 5) $x_4 \sim x_2$, 6) $\bar{x}_4 \sim \bar{x}_2$. From this set of expansions we see that a closed chain of cyclic permutations comprises the variables $(x_2, \bar{x}_3, x_4)$ and $(\bar{x}_2, x_3, \bar{x}_4)$ so that the function is partially symmetric with respect to these variables. By writing the function in the truth table form and double negating appropriate columns, the function can be written as $x_1 S_0(x_2, \bar{x}_3, x_4) + \bar{x}_1 S_1(x_2, \bar{x}_3, x_4) + (x_1 + \bar{x}_1) S_2(x_2, \bar{x}_3, x_4)$, or $\bar{x}_1 S_2(\bar{x}_2, x_3, \bar{x}_4) + x_1 S_3(\bar{x}_2, x_3, \bar{x}_4) + (x_1 + \bar{x}_1) S_1(\bar{x}_2, x_3, \bar{x}_4)$. The function is also partially symmetric with respect to the subsets of variables of the sets $(x_2, \bar{x}_3, x_4)$ and $(\bar{x}_2, x_3, \bar{x}_4)$.

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On the Validity of the Gradual-Channel Approximation for Field-Effect Transistors

Abstract—The valid range of the gradual-channel approximation is found in terms of the width of the conductive channel. Limitations of the approximation for the description of field-effect devices are clarified by considering the external characteristics and the internal conduction mechanism separately.

The most widely accepted method of analysis for field-effect devices is the gradual-channel approximation first introduced by Shockley.[1] Although the approximation has the inherent shortcoming that it cannot explain the internal current conduction mechanism beyond pinchoff, the external drain characteristics predicted by it have been found to agree reasonably well with the experimental observation for long devices.[2]

In this letter, a qualitative estimate for the valid range of the approximation is found in terms of the conductive channel width with the length-to-width ratio and the drain current as two parameters. Useful device design information can be obtained by this relatively simple method. From this estimate, furthermore, one can find the reason why the approximation gives reasonable external characteristics despite its inability to explain the internal operation mechanism.

From the assumptions of 1) the one-dimensional character of the electric fields in the space-charge region and in the conductive channel, and 2) the complete depletion of free carriers in the space-charge region and the neutrality of the conductive channel, the gradual-channel approximation gives the following results for the potential inside the conductive channel and the drain current of an n-channel device.

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