A Reliability Model for Dependent Failures in Parallel Redundant Systems

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ABSTRACT—The concept of Equivalent time is introduced to find the hazard rates of elements in a parallel redundant systems which is subject to dependent failures. True acceleration is defined as an adjunct to this concept.

Reader aids:
Purpose: Report of a derivation
Special math needed for explanations: Probability theory
Special math needed for results: Same
Results useful to: Theoretician

INTRODUCTION

The often-made assumption of statistical independence for components is sometimes not justified. Interactions do occur in the real world; for example, consider a parallel redundant (1-out-of-2:G) system containing two components A and B, there are two possible states for the system to operate:

State 1. Both A and B are operating.
State 2. Only one of A or B is operating.

Assume that in state 1, the components share the load and have a lower hazard rate than in state 2. In state 2, the good component carries the entire load and has a higher hazard rate than it has in state 1. In this case, the behavior of the elements is not statistically independent.

This problem was discussed by Shooman [1] among others, he used a Markov model and joint pdf approach to solve the dependent failure problem with constant hazard rates. Here, we shall develop a model for general hazard rates.

DEFINITION AND THEORY

Consider a component that may operate in state 1 and state 2 under different requirements. Then, for a period of time T_i that the component operates in state 1, we can find the time T_i^* for the system to have operated in state 2 such that the cumulative hazard function is the same for the component in either case.

This leads to the following definition:

Z_1 the hazard rate of the component operating in state 1.
Z_2 the hazard rate of the component operating in state 2.

Revised manuscript received March 19, 1974.

Then, two operating times, T_i and T_i^*, under different states, are Equivalent if the cumulative hazard functions (or equivalently, the reliabilities) are equal,

$$\int_0^{T_1} Z_1(t)dt = \int_0^{T_1^*} Z_2(t)dt,$$

this defines a relation between T_1 and T_1^*

$$T_1^* = G(T_1).$$

Though we have thus defined Equivalent time, the effect of this equivalent is implicit if there isn't any change of state.

Define:

T_{A1}	operating time of A in state 1.
T_{A1}^*	Equivalent operating time in state 2 for T_{A1}.
T_{B1}, T_{B1}^*	Analogous to T_{A1} and T_{A1}^*.
Z_{Ai}, Z_{Bi}	Hazard rate of A and B in state i, $i = 1,2$.

Consider the system at $t = \tau$; suppose A breaks down first. That means B changes from state 1 to state 2 at τ.

1. At $t \leqslant \tau^-$, B is in state 1, and its hazard rate is $Z_{B1}(t)$.
2. At $t = \tau^+$, B suddenly changes from state 1 to state 2, and the hazard rate now will be $Z_{B2}(G_B(\tau))$ instead of $Z_{B2}(\tau)$ or $Z_{B1}(\tau)$. This is the result of the Equivalent Time effect.
3. At $t > \tau^+$, the hazard rate of B will be $Z_{B2}(t - \tau + G_B(\tau))$.

The third argument can easily be understood with the help of the graph below:

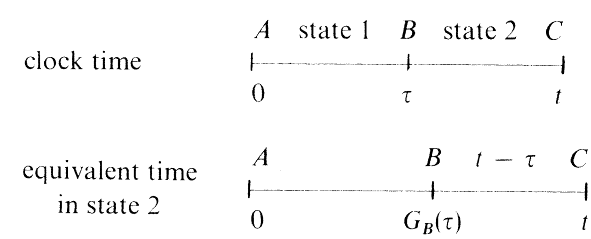

Fig. 1. Equivalent time for different states.

When we define Equivalence, we assume that Equivalent Times in different states have exactly the same influence on the components of a system. This could be rather restrictive

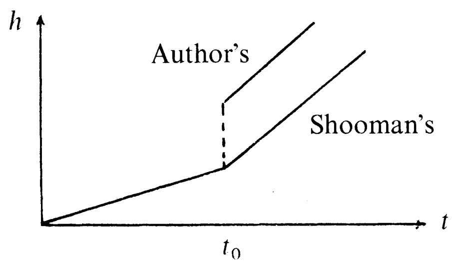

Fig. 2. Comparison of hazard rates.

if, for example, many characteristics of the component are important. The existence of such functions as G_A, G_B is often used to define true acceleration.

SYSTEM FAILURE-PDF CALCULATION

We shall now derive the failure-pdf for the system:

Initially ($t = 0$), both A and B are operating. Consider the following two cases:

Case A: Assume that A breaks down first at τ. The pdf of τ for this case is

$$g_1(\tau) = Z_{A1}(\tau) \exp\left[-\int_0^\tau Z_{A1}(\xi)d\xi\right] \exp\left[-\int_0^\tau Z_{B1}(\xi)d\xi\right].$$

The first two factors are the pdf of failure of A, while the third factor is the probability that B is still operating at τ. After A breaks down at τ, the failure-pdf of B at $t > \tau$ is

$$f_1(t|\tau) = Z_{B2}(t - \tau + G_B(\tau))$$
$$\exp\left[-\int_\tau^t Z_{B2}(\xi - \tau + G_B(\tau))d\xi\right], \qquad 0 < \tau < t$$

since the hazard rate is $Z_{B2}(t - \tau + G_B(\tau))$ at $t > \tau$.

Case B: B breaks down first at τ.

By similar argument, we have

$$g_2(\tau) = Z_{B1}(\tau) \exp\left[-\int_0^\tau Z_{B1}(\xi)d\xi\right] \exp\left[-\int_0^\tau Z_{A1}(\xi)d\xi\right]$$

$$f_2(t|\tau) = Z_{A2}(t - \tau + G_A(\tau))$$
$$\cdot \exp\left[-\int_\tau^t Z_{A2}(\xi - \tau + G_A(\tau))d\xi\right].$$

Since case A and case B are mutually exclusive, the system failure-pdf at t is [2]

$$f(t) = \int_0^t [g_1(\tau)f_1(t|\tau) + g_2(\tau)f(t|\tau)]d\tau.$$

If the hazard rates are constant, the formula becomes simpler, and can easily be seen to coincide with that obtained by the Markov model in [1].

It is straightforward, although more tedious, to apply this method to a system with more than two components.

COMPARISON OF RESULTS

For linearly increasing hazard rates, this model does not agree with that of Shooman's [3, p. 236]. If the system changes from state 1, with hazard rate $k_1 t$ at $t = t_0$, to state 2 with hazard rate $k_2 t$, the actual hazard rate for $t > t_0$ is

$$\begin{aligned} h'(t) &= k_2(t - t_0 + G(t_0)) \\ &= k_2\left(t - t_0 + \sqrt{\frac{k_1}{k_2}}\, t_0\right) \\ &= k_2 t - k_2 t_0 + \sqrt{k_1 k_2}\, t_0 \end{aligned}$$

whereas Shooman has

$$h''(t) = k_1 t_0 + k_2(t - t_0).$$

The difference between them is shown in Fig. 2. The discontinuity of hazard rate at t_0 is a general property of our model.

REFERENCES

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