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A NOTE ON INTERPERSONAL COMPARISONS OF UTILITY

1. INTRODUCTION

Interpersonal comparisons of utility is a controversial topic. Some philosophers believe it is impossible to have interpersonal comparisons of utility, while some others believe it is possible.

In general economists do not believe in interpersonal comparisons of utility. As pointed out by Dan W. Brock, "it has become a generally accepted dogma among economists that interpersonal comparisons of utility are impossible."[1]

However, there are economists who are in favour of cardinal utility and interpersonal comparisons of utility, as represented by John C. Harsanyi.[2] He further points out that "there have been slow but significant changes in the climate of opinion. Several eminent economists, including some who used to be strong advocates of an ordinalist position, have expressed views very close to my own."[3]

The crucial point point is that, in social choices, even if it is impossible to make an objective interpersonal comparison of utility, sometimes one has to make a subjective comparison, in order to be able to make a decision. This is quite similar to the case of decision-making for personal actions under condition of uncertainty, with the objective probabilities of the state of affairs not known. In that case a decision based on subjective probabilities is preferable to a decision based on the maximax or the maximin criterion, as these criteria sometimes lead to very unreasonable or even absurd decisions. Furthermore, the problem of general distribution of income and/or wealth hinges on the use of a social welfare function as an objective function. The impossibility of interpersonal comparisons of utility denies the validity of social welfare functions, because a social welfare function is usually a sum or weighted sum of personal welfare functions, which are, in turn, special personal utility functions.

Therefore, interpersonal comparisons of utility are a fact of life we

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have to face provided that we want to solve particular and general distribution problems, in spite of the extreme difficulty and the subjective nature of the comparison.

Thus the problem is no longer whether or not interpersonal comparisons of utility are possible, but becomes how we shall make interpersonal comparisons of utility, so that the results based on such comparisons may be justifiable or even optimal. In other words, it becomes essential to investigate the question whether or not there are restrictions or conditions to be imposed on interpersonal comparisons of utility, in order to avoid any unreasonable or absurd results.

In this paper I intend to show that, from the viewpoint of distribution, the validity of interpersonal comparisons of utility is not unconditional. The results of such a comparison is considerably affected by the shape of the utility function curves of the two persons whose utilities are under comparison, thus leading to morally absurd conclusions. To avoid this kind of absurdity, a sufficient condition or rectriction on interpersonal comparisons of utility is that the shapes of the utility function curves of the two persons should be the same, or at least similar. In other words, it is not justified to use purely personal utility functions of two persons completely determined from preferences, and we are restricted to use some sort of standard, universal or general utility functions.

2. THE AGGREGATE UTILITY OF TWO PERSONS

To study the problem of interpersonal comparisons of utility, we shall consider only the problem of distributing income, wealth, or a single commodity between two persons, J and K. The distribution between two persons can be readily extended to the distribution among n persons, where $n > 2$. The extension from one commodity to m commodities, $m > 1$, is not so simple. However, the problem of optimizing production and distribution of all commodities, which is known as Pareto optimality and is studied intensively by welfare economists, is to maximize the magnitude of a social welfare function that is given or is already adopted as an objective function for optimization. Now the philosophical problem lies in the choice and justification for the social welfare function, whereas the problem of optimizing production and distribution of commodities is a technical problem rather than a philosophical one. For the present purpose of finding the restriction on interpersonal comparisons of utility, it is sufficient to consider only this particular optimality problem, i.e., the single-commodity two-person distribution problem.

In the following analysis, the concepts of Pareto optimality and Edgeworth box will be used.[5] In this simple case of one commodity, the Edgeworth box degenerates into a line segment, as shown in Figure 1.

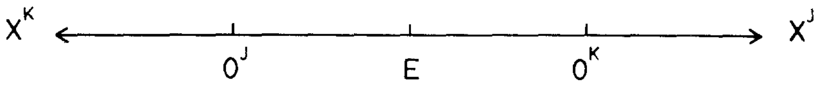

Fig. 1.

Now the Pareto optimality is obtained at a point E on line O^JO^K, such that the sum of utilities for J and K is a maximum.

Let

V^j be value of commodity distributed to J,
V^K be value of commodity distributed to K,
$U^J = F_J(V^J)$ be utility for J,
$U^K = F_K(V^K)$ be utility for K,
$F = F_J(V^J) + F_K(V^K) = U^J + U^K$ be the quantity to be maximized.

It is seen that F is a kind of social welfare function for the society composed of two members J and K.

Assume that both F_J and F_K are risk-averse. Then they will look like the curves shown in Figures 2 and 3, respectively.

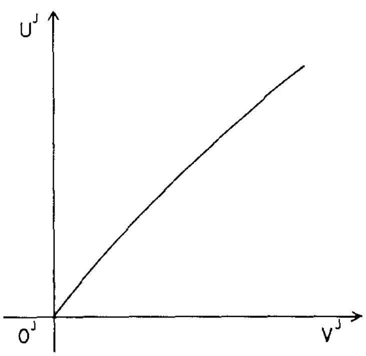

Fig. 2.

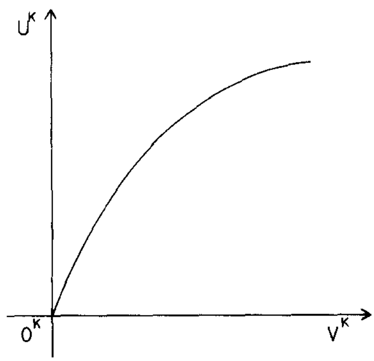

Fig. 3.

U^J and U^K are drawn slightly different, in order to indicate that utility functions depend on personal preference and are inherently different for different members of society.

Let the total value of commodity be V. Then the constraint on the distribution between J and K or on the optimization of F is $V^J + V^K = V$.

Suppose J and K have different degrees of interest in the commodity. Let U be the utility for J corresponding to V, i.e., if all the commodity is distributed to J, and $2U$ be the utility for K corresponding to V, i.e., if all the commodity is distributed to K.

Now the first question that arises, in optimizing F, is whether or not the utility functions of J and K should be normalized. Here by 'normalization' is meant to compress the utility function of K, U^K, so that its maximum value is also U instead of $2U$.

The two cases, i.e., one without normalization and the other with normalization, will be examined separately in the following.

(1) Case 1: With No Normalization

Since $F = U^J + U^K$, we can plot F by adding U^J and U^K with U^K plotted form right to left, on the one-dimensional diagram of Figure 1 as a base or V-axis, as shown in Figure 4.
The point of maximum value of F is indicated by M, corresponding to a point E on the line O^JO^K. It is seen that E is much closer to O^J than to O^K. If the utility function curves are straight lines, then E will coincide with O^J. It means that Pareto optimality is obtained when most or all of the commodity is distributed to K, the person with a higher degree of interest or utility.

This is certainly morally unjustified. It means that, without normalization, to use the sum of two utility functions as a social welfare function is philosophically unreasonable, or the interpersonal comparison of utility is invalid.

(2) Case 2: With Normalization

With normalization, the utility function curve U^K is compressed vertically, so that its maximum value is also U. Then F can be plotted as that shown in Figure 5.

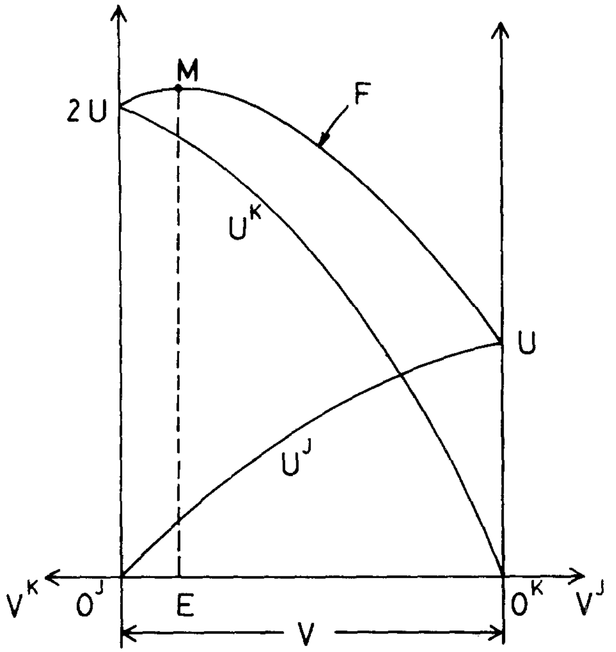

Fig. 4.

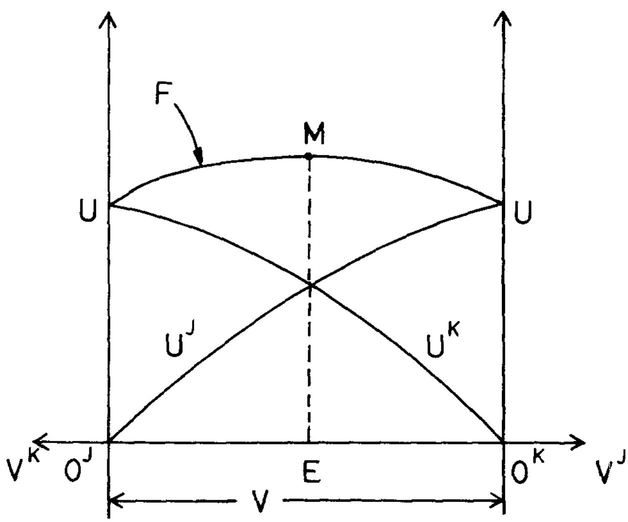

Fig. 5.

Figure 5 looks all right, because point E is somewhat close to the middle point between O^J and O^K. However, the position of E still depends on the shapes of utility function curves of J and K. Let U^J have a shape close to a straight line, as shown in Figure 6, and let U^K have two different shapes, as shown in Figure 7 and Figure 8, respectively.

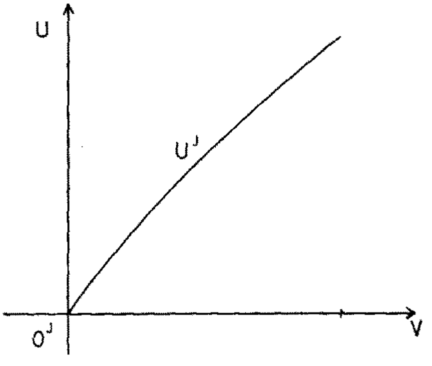

Fig. 6.

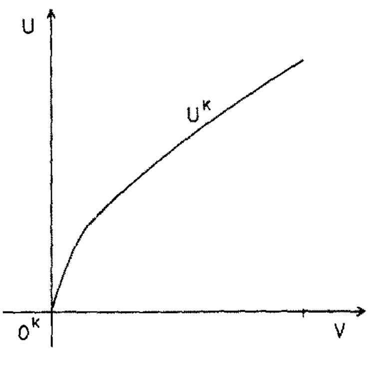

Fig. 7.

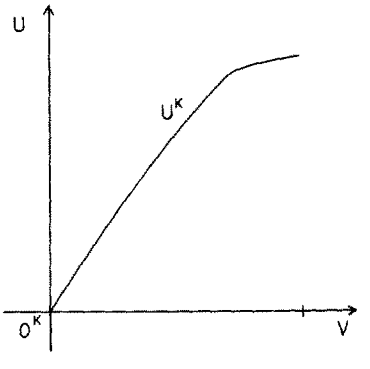

Fig. 8.

Now the function F obtained by adding U^J of Figure 6 and U^K of Figure 7 is shown in Figure 9. It is seen that point E is very close to O^K. The function F obtained by adding U^J of Figure 6 and U^K of Figure 8 is shown in Figure 10. It is seen that point E is very close to O^J.

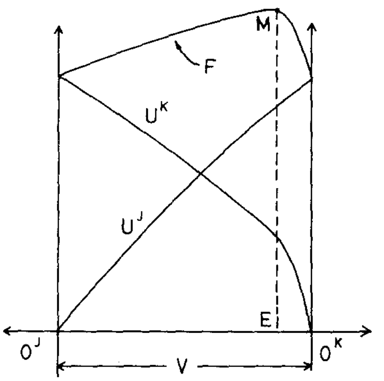

Fig. 9.

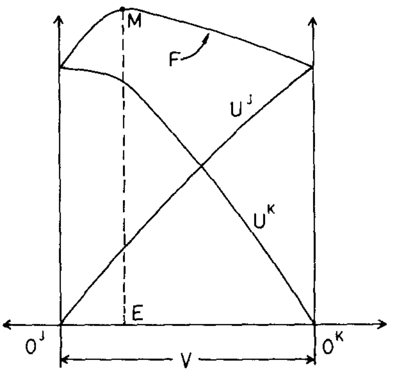

Fig. 10.

By comparing Figures 9 and 10 it can be concluded that the point E corresponding to Pareto optimality depends heavily on the relative shapes of U^J and U^K. In one case E is very close to O^K and in the other case E is very close to O^J. There is certainly no moral justification for this result.

3. VARIOUS CRITERIA FOR DISTRIBUTION

It may be argued that the reason for point E in Figure 9 to be close to point O^K and for point E in Figure 10 to be close to point O^J is due to

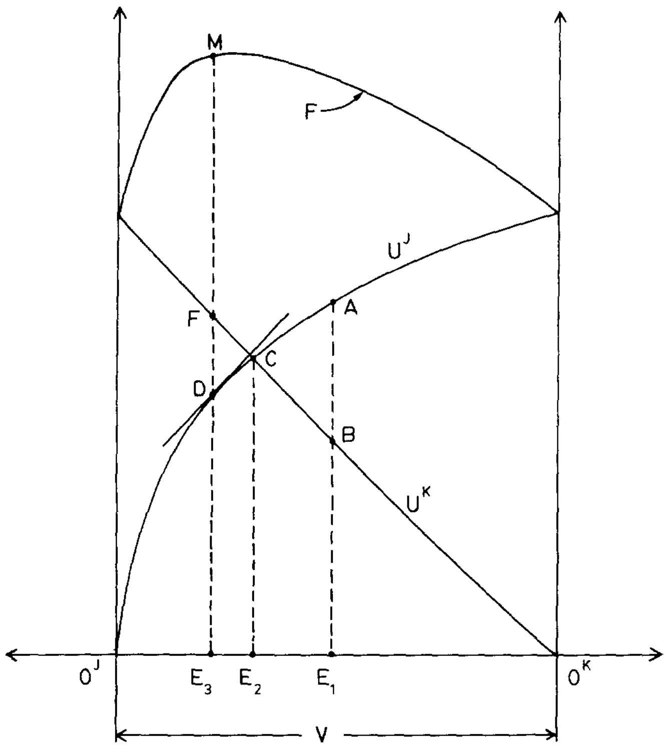

Fig. 11.

the special shapes of the utility function curves U^K in both figures, which are rather irrational.[6] So let us consider a more realistic example. Suppose person J is very conservative and risk-averse so that his utility function U^J has a high curvature, as shown in Figure 11, and person K is very aggressive and non-risk-averse so that his utility function curve is close to a straight line. His utility function curve U^K is drawn from right to left, as shown in Figure 11. These two utility functions are realistic and rational. Suppose J and K belong to the same social class and are similar in need, ability, effort and contribution to society.

We can now see that there exist several criteria for the distribution of the total value V. First, V may be distributed to J and K according to an *egalitarian criterion*, i.e., equal amounts of value will be distributed to

J and K. This is justified because of the assumption that J and K are similar in ability, contribution, etc. Second, V may be distributed to J and K according to a *criterion of equality of interest*, i.e., V will be distributed to J and K in such a way that their utilities resulted from distribution will be equal. Third, V may be distributed to J and K according to *Pareto optimality*, i.e., the resulting aggregate utility will be a maximum.

These three cases will be explained and illustrated diagrammatically using Figure 11, as follows.

(1) Egalitarian or Equal-Value Criterion

In this case the distribution is indicated by the mid-point E_1 on line O^JO^K. Then we have

$$V^J = \text{length } (O^JE_1) = \text{length } (O^KE_1) = V^K = \frac{1}{2}V$$

A vertical line drawn from E_1 intersects curve U^J at point A and curve U^K at point B. It is seen that

$$U^J = \text{length } (E_1A) > \text{length } (E_1B) = U^K$$

(2) Equal-Interest or Equal-Utility Criterion

In this case the distribution is indicated by point E_2 on line O^JO^K, corresponding to equal utility for J and K. A vertical line drawn from E_2 intersects both curve U^J and curve U^K at joint C. Then we have

$$V^J = \text{length } (O^JE_2) < \text{length } (O^KE_2) = V^K$$
$$U^J = \text{length } (E_2C) = U^K$$

(3) Pareto-Optimality or Equal-Slope Criterion

In this case the distribution is indicated by point E_3 on line O^JO^K. A vertical line drawn from E_3 intersects curve U^J at point D and curve U^K at point F. It is readily seen that the slope of curve U^J at point D is equal to that of curve U^K. (In Figure 11 curve U^K is drawn from right to left so that it has a negative slope.) The we have

$$V^J = \text{length } (O^J E_3) < \text{length } (O^K E_3) = V^K$$
$$U^J = \text{length } (E_3 D) < \text{length } (E_3 F) = U^K$$

From the above analysis it is seen that, for the case of two persons J and k, the results from three different criteria, viz., equal-value, equal-utility, and equal-slope criteria, are all different.

4. RESTRICTION ON OPTIMALITY

Now let us have another look at Pareto optimality. Suppose the value to be distributed is not a constant V, but varies from O upwards. Then Pareto optimality dictates that, as long as the total value is less than lentgh $(O^J E_3)$, it is completely distributed to J, because the slope of curve U^J between points O^J and D is greater than the slope of line U^K, and when the total value is greater than length $(O^J E_3)$, the amount of value over length $(O^J E_3)$ is completely distributed to K, no matter how much it is, because the slope of curve U^J above point D is less than the slope of line U^K. This is certainly morally unjustified and is an absurd conclusion.

Furthermore, possible criteria for distribution are not limited to these three. There might exist some other criteria as well. It is seen that, even for these three criteria only, it is in general impossible to have a unanimous result. Therefore, if different utility functions are used for different persons, then it will be inevitable to have this insurmountable difficulty in interpersonal comparisons of utility. With this difficulty it would certainly be unjustified to take the sum of utility functions or personal welfare functions as a social welfare function, or it would be unjustified to adopt the social welfare function as an objective function for the optimization of distribution. In other words, in that case utilitarianism would be unable to take care of the general distribution problem, and this would certainly be a strong reason for objection to utilitarianism.

5. RESOLUTION OF THE DIFFICULTY

To resolve this difficulty, we are forced to place a restriction or condition on the use of interpersonal comparisons of utility. It is readily seen that, when U^J is the same as U^K, the three points E_1, E_2, and E_3 coincide. This means that a distribution according to the criterion of equal value results in equal utility and equal slope or Pareto optimality as well. This seems to be a good way out, and we may conclude that interpersonal comparisons of utility is possible, permissible and/or justified under the condition that the same utility function is used for every member of society. Since interpersonal comparisons of utility are essential to the making of decisions in social choices and in the determination of social welfare functions, it becomes desirable to adopt a general utility function.[7]

If the same utility function is used for every member of society, then the ideal distribution reduces to a uniform distribution, which is the ideal according to the strict egalitarian criterion. Unfortunately the egalitarian criterion is neither reasonable nor practicable.[8] Therefore it is justified and unavoidable to have some sort of equitable inequality. This inequality, however, cannot be taken care of by the utility function itself, but has to be taken care of by some other means. This is a separate problem and will not be covered here.

NOTES

[1] Dan W. Brock: 1973, 'Recent Work in Utilitarianism', *American Philosophical Quarterly* **10**, (October), pp. 241–276.

[2] John C. Harsanyi: 1955, 'Cardinal Utility, Individualistic Ethics, and Interpersonal Comparisons of Utility', *The Journal of Political Economy* **43**, pp. 309–321.

[3] John C. Harsanyi: 1976, 'Introduction', in *Essays on Ethics, Social Behaviour, and Scientific Explanation* (D. Reidel, Dordrecht, Holland), pp. ix–xiii.

[4] C. L. Sheng: 1984, 'A General Utility Function for Decision', *Mathematical Modelling* **5**, pp. 265–274.

[5] See, for example, Yew-Kwang Ng: *Welfare Economics* (John Wiley and Sons, New York), pp. 30–51.

[6] It is now more or less generally accepted that according to human psychology a rational utility function is risk-averse; and that proportional risk-averse utility functions are considered the most rational class of utility functions. For a discussion of risk-averse and proportional risk-averse utility functions, see, for example, Ralph L. Keeney and Howard Raiffa: 1976, *Decisions with Multiple Objectives: Preferences and Value Tradeoffs* (John Wiley and Sons, New York), pp. 162–179.

[7] See Note 4.

[8] John A. Ryan and Nicholas Rescher have pointed out clearly the unreasonableness and impracticability of strict egalitarianism. See John A. Ryan: 1942, *Distributive Justice*, 3rd ed. (Macmillan, New York), p. 180. Nicholas Rescher: 1966, *Distributive Justice: A*

Constructive Critique of the Utilitarian Theory of Distribution (The Bobbs-Merrill Company, Indianapolis, In.), p. 75.
[9] See Note 5, p. 147.

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