# Prediction Intervals for an Ordered Observation from a Pareto Distribution

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*Key Words* — **Pareto distribution, lifetime data, prediction interval, Monte Carlo simulation.**

*Reader Aids* —
**General purpose: Define bounds on life-test duration**
**Special math needed for explanations: Statistical inference**
**Special math needed to use results: Same**
**Results useful to: Reliability data analysts.**

*Summary & Conclusions* — **The prediction of future ordered observations shows how long a sample of units might run until all fail in life testing. This paper presents the prediction intervals on future ordered-observation $s$ in a sample of size $n$ from a Pareto distribution with known shape parameter where the first $k$ ordered observations have been observed. A useful method is defined for obtaining a bound on life-test duration for sample from a population having Pareto distributed lifetimes.**

## 1. INTRODUCTION

The exponential distribution is widely used as a model of lifetime data. The distribution is characterized by a constant failure rate, say $\lambda$. As noted by McNolty, Doyle, Hansen [9], in a population of components there could be a ubiquitous variation in $\lambda$-values because of small fluctuations in manufacturing tolerances so that a component selected at random can be regarded as belonging to a random subpopulation.[1]

Let the lifetime of a particular component have an exponential distribution with failure rate $\lambda$ and threshold parameter (guarantee time) $\gamma$; and let $\lambda$ follow a Gamma distribution with scale parameter $\sigma$ and shape parameter $\beta$. Then the failure time $X$ of a component selected at random from such a mixed population has a Pareto distribution of the second kind [3]; the Pareto pdf is:

$$f_x(x) = \int_0^\infty \lambda \cdot \exp(-\lambda \cdot (x-\gamma)) \cdot (\sigma^\beta/\Gamma(\beta)) \cdot \lambda^{\beta-1} \cdot \exp(-\sigma \cdot \lambda)\, d\lambda$$

$$= (\beta/\sigma) \cdot [1 + (x-\gamma)/\sigma]^{-(\beta+1)}, \; x \geq \gamma; \; \sigma, \beta > 0. \quad (1\text{-}1)$$

[1]It is important to remember that individual items do not have a failure rate. Only populations have failure rates. Assigning a particular individual to a particular population is both difficult & arbitrary. (While this is obvious to competent statisticians, it is not so obvious to many others.)

Although the Pareto distribution has a simple form, few results in $s$-prediction problems are available in the literature. Likeš [6] gives the $s$-prediction intervals for later order statistics from a Pareto distribution with $\gamma = \sigma$. Nigm & Hamdy [10] give the $s$-prediction bounds using a Bayes approach for later order statistics of a sample from the Pareto distribution with $\gamma = \sigma$. Arnold & Press [1] also discuss the $s$-prediction problems from a Bayes viewpoint. Kaminsky & Nelson [5] give the best linear unbiased predictor for later order statistics of a sample from the Pareto distribution with known shape parameter.

This paper presents the $s$-prediction intervals for later order statistics of a sample from the Pareto distribution with known shape parameter.

*Assumptions*

1. The distribution is known to be Pareto with a known shape parameter.
2. The first $k$ order statistics from a sample of size $n$ are known.
3. We wish to find a $s$-prediction interval of failure time $s$ based on the first $k$ failure times, $1 \leq k < s \leq n$. In particular, if $s = n$, then we wish to predict how long a sample of $n$ components will run until all fail. ◄

Section 2 defines the coefficients that determine probability bounds on future order statistics, and then gives an example of how to calculate a $s$-prediction interval. Section 3 describes how to obtain, by simulation, the percentiles of the statistic in section 2. Section 4 considers another approximate $s$-prediction interval.

*Notation & Acronyms*

| | |
|---|---|
| BLUE | best linear unbiased estimator |
| $\lambda$ | failure rate of the exponential distribution |
| $\gamma, \sigma, \beta$ | parameters of the Pareto distribution |
| $X_{(i)}$ | order statistic of a random sample of size n from Pareto distribution |
| $Y_{(i)}$ | $(X_{(i)} - \gamma)/\sigma$; standardized order statistic |
| $\hat{}$ | implies the BLUE |
| $\Phi_{i,j}$ | $\Gamma(n-i+1) \cdot \Gamma(n+1 - j/\beta)/[\Gamma(n-i+1 - j/\beta) \cdot \Gamma(n+1)]$ |
| $A_i$ | $1/\Phi_{i,1}$ |
| $B_i$ | $\Phi_{i,2}$ |
| $C_k$ | $[(n\cdot\beta-2)((n-1)\cdot\beta-2) - n\cdot\beta \; ((n-k)\cdot\beta-2) \cdot B_k]/[(n\cdot\beta-1)\cdot(\beta+2)]$ |
| $D_k$ | $[(n\beta-2)\cdot(\beta+1) + ((n-k)\cdot\beta-2)\cdot B_k]/(\beta+2)$ |
| $U_1$ | $\hat{\sigma}/\sigma$ |
| $U_2$ | $(X_{(s)} - X_{(k)})/\sigma = Y_{(s)} - Y_{(k)}$ |
| $U$ | $U_2/U_1$ |
| $1-\alpha$ | Pr{$s$-prediction interval includes true value of future order statistics} |

$u(\delta;n,s,k,\beta)$ δ quantile of $U$
$a_j$ $E\{U_j\}$, $j=1,2$
$b_j$ $\mathrm{Var}\{U_j\}$, $j=1,2$
$U_j^*$ $2a_j \cdot U_j/b_j$, $j=1,2$.

Other, standard notation is given in "Information for Readers & Authors" at the rear of each issue.

## 2. PREDICTION INTERVALS

Based on the first $k$ order statistics $X_{(1)} < X_{(2)} < \ldots < X_{(k)}$ of a sample of size n from the Pareto distribution (1−1), Vännman [12] gives $\hat{\gamma}$ & $\hat{\sigma}$ for $k < n+1-2/\beta$:

$$\hat{\gamma} = X_{(1)} - \hat{\sigma}/(n\cdot\beta-1); \quad (2\text{-}1)$$

$$\hat{\sigma} = [(\beta+1)\sum_{i=1}^{k-1} B_i\cdot X_{(i)} + [(n-k+1)\cdot\beta-1]\cdot B_k \cdot X_{(k)} - [1-1/(n\cdot\beta)]\cdot(n\cdot\beta-2-C_k)\cdot X_{(1)}]/C_k$$

$$= [[(\beta+1)\cdot B_1 - (1-1/(n\cdot\beta))\cdot(n\cdot\beta-2-C_k)]\cdot X_{(1)} + (\beta+1)\cdot\sum_{i=2}^{k-1} B_i\cdot X_{(i)} + [(n-k+1)\cdot\beta-1]\cdot B_k\cdot X_{(k)}]/C_k. \quad (2\text{-}2)$$

Balakrishnan & Cohen [2] show that $\hat{\gamma}$ & $\hat{\sigma}$ are location invariant and scale invariant estimators.

Consider two statistics, $U_1$ & $U_2$. Since $\hat{\sigma}$ is scale invariant and $Y_{(s)}$ & $Y_{(k)}$ are standardized order statistics, the distributions of $U_1$ & $U_2$ do not depend on $\gamma$ & $\sigma$. Hence the distribution of —

$$U = (X_{(s)} - X_{(k)})/\hat{\sigma}, \quad (2\text{-}3)$$

depends only on $n,s,k,\beta$ but not on $\gamma$ or $\sigma$. Thus —

$$1-\alpha = \Pr\{u(½\alpha;n,s,k,\beta) < (X_{(s)}-X_{(k)})/\hat{\sigma} < u(1-½\alpha;n,s,k,\beta)\} = \Pr\{X_{(k)} + u(½\alpha;n,s,k,\beta)\cdot\hat{\sigma} < X_{(s)} < X_{(k)} + u(1-½\alpha;n,s,k,\beta)\cdot\hat{\sigma}\}. \quad (2\text{-}4)$$

From (2-4) it follows that for $k < n+1-2/\beta$,

$$(X_{(k)} + u(½\alpha;n,s,k,\beta)\cdot\hat{\sigma},\ X_{(k)} + u(1-½\alpha;n,s,k,\beta)\cdot\hat{\sigma}), \quad (2\text{-}5)$$

is a 2-sided equal-tail $(1-\alpha)$ $s$-prediction interval for $X_{(s)}$ based on the first $k$ observations. The similar 1-sided $(1-\alpha)$ $s$-prediction interval is:

$$(X_{(k)},\ X_{(k)} + u(1-\alpha;n,s,k,\beta)\cdot\hat{\sigma}). \quad (2\text{-}6)$$

*Example* 1[2]

Let —

- $n=30$
- all items have a common Pareto lifetime distribution with $\beta=5$
- all items be put on test simultaneously
- the life test be terminated at failure 20 ($k=20$). ◄

The 20 observed lifetimes and the coefficients of $X_{(i)}$, $i=1,2,\ldots,20$, in (2-2) are in table 1. '$\beta > 2/3$' so,

$$\hat{\sigma} = (-7.932)\cdot(30.101) + (0.369)\cdot(30.150) + \ldots + (2.219)\cdot(35.274) = 22.595.$$

From the appendix, the 90th percentile of $U$ is $u(0.9;30,30,20,5) = 2.007$. Thus, the 1-sided upper 90% $s$-prediction limit for $X_{(30)}$ is:

$$35.274 + u(0.9;30,30,20,5)\cdot(22.595) = 35.274 + (2.007)\cdot(22.595) = 80.622.$$

The 1-sided 90% $s$-prediction interval is (35.274, 80.622).

TABLE 1
First 20 Failure Observations and Coefficients of $X_{(i)}$
[i = 1,2,...,20; see (2-2)]

| $i$ | hours | coef. of $X_{(i)}$ | $i$ | hours | coef. of $X_{(i)}$ |
|---|---|---|---|---|---|
| 1 | 30.101 | −7.932 | 11 | 32.265 | 0.312 |
| 2 | 30.150 | 0.369 | 12 | 32.517 | 0.310 |
| 3 | 30.374 | 0.364 | 13 | 32.636 | 0.303 |
| 4 | 30.581 | 0.359 | 14 | 33.002 | 0.296 |
| 5 | 30.871 | 0.353 | 15 | 33.552 | 0.289 |
| 6 | 31.086 | 0.347 | 16 | 33.721 | 0.281 |
| 7 | 31.398 | 0.342 | 17 | 34.002 | 0.273 |
| 8 | 31.752 | 0.336 | 18 | 34.023 | 0.265 |
| 9 | 31.792 | 0.330 | 19 | 34.150 | 0.256 |
| 10 | 31.960 | 0.323 | 20 | 35.274 | 2.219 |

## 3. CALCULATIONS FOR THE APPENDIX

The exact distribution of $U$ is too hard to derive algebraically. Instead, we approximate it by Monte Carlo sampling. All the simulations were run on an IBM3090-120E computer with the aid of IMSL [4] software package.

### Simulation Procedure

We consider only $s=n$ and $\beta=5$. For each combination of $n$ & $k$, Pareto random samples are generated using IMSL

[2]The number of significant figures is not intended to imply any accuracy in the estimates, but to illustrate the arithmetic.

subroutine RNEXP and the relation between the exponential distribution and the Pareto distribution with $\gamma=0$ and $\sigma=1$. For each sample, calculate $U = (X_{(n)}-X_{(k)})/\hat{\sigma}$. Because the Monte Carlo sample size is very large and computer time is expensive, we counted the number of $U$ in the compartments of a histogram. Because the distribution of $U$ extends over the positive half of the real line, we used an upper bound for $U$. We generated 1000 Pareto random samples and choose the 5th largest value of $U$ as the upper bound of this statistic, so that any reasonable errors in this approximation would not cause loss of information in the tails of the distribution [8: p 361].

In determining the $\delta$ quantile $u(\delta;n,s,k,\beta)$ of $U$, a Monte Carlo sample size of 20k is used. The 20k values are sorted into a histogram of 5k cells with the lower bound 0 and the upper bound from the previous paragraph. The values in the cell containing the desired sample percentiles are ordered to yield two adjoining sample percentiles; a weighted linear combination of them estimates the percentile. The appendix lists these percentiles for $n=s=3(1)10(5)30$ and $k=2(1)(n-1)$.

## 4. APPROXIMATE PREDICTION INTERVALS

This section defines an approximate value of $u(\delta;n,s,k,\beta)$ in (2-5). Because $\hat{\sigma}$ is scale invariant, the sum of the coefficients of the $X_{(i)}$ in (2-2) must be 0 [2]. Hence, $\hat{\sigma}$ can be represented as the linear combination of $X_{(i)}-X_{(i-1)}$, $i=1,2,\ldots,k$. Thus, by Pyke's results [11: pp 395-407], the $U_1 = \hat{\sigma}/\sigma$ is approximately a weighted sum of $s$-independent $\chi^2$ variables with mean & variance:

$$a_1 = 1, \quad (4\text{-}1a)$$

$$b_1 = [(n\cdot\beta-1)-D_k/(n\cdot\beta-2)]/[(n\cdot\beta-1)\cdot(n\cdot\beta-2)-n\cdot\beta\cdot D_k] \quad (4\text{-}1b)$$

Therefore, by Mann *et al* [8: p 177],

$$U_1^* = 2\hat{\sigma}/(b_1\cdot\sigma), \quad (4\text{-}1c)$$

is approximately $\chi^2$ distributed with $2/b_1$ degrees of freedom. Also,

$$X_{(s)}-X_{(k)} = \sum_{i=k+1}^{s}(X_{(i)}-X_{(i-1)}),\ k<s.$$

Thus, by the asymptotic result of Pyke [11: pp 395-407], $U_2 = (X_{(s)}-X_{(k)})/\sigma$ is approximately a weighted sum of $s$-independent $\chi^2$ variables with mean & variance:

$$a_2 = A_s-A_k, \quad (4\text{-}2a)$$

$$b_2 = B_s^{-1} + B_k^{-1} - 2A_s\cdot(A_k\cdot B_k)^{-1} - (A_k-A_s)^2, \quad (4\text{-}2b)$$

The mean & variance of $U_2$ can also be obtained using Malik's [7] results; thus,

$$U_2^* = 2a_2\cdot(X_{(s)}-X_{(k)})/(b_2\cdot\sigma), \quad (4\text{-}2c)$$

is approximately $\chi^2$ distributed with $2a^2{}_2/b_2$ degrees of freedom. We know that $U_2$ is approximately $s$-independent of $U_1$ for a sufficiently large sample size. Therefore,

$$F = [U_2^*/(2a_2^2/b_2)]/[U_1^*/(2/b_1)] = U/(A_s-A_k), \quad (4\text{-}3)$$

where $U$, defined in (2-3), is approximately the Fisher $F$ distribution with degrees of freedom:

$$\nu_n = 2a_2^2/b_2,\ \nu_d = 2/b_1;$$

$b_1$, $a_2$, $b_2$ are defined in (4-1) & (4-2).

*Notation*

$F(\delta;\nu_n,\nu_d)$ $\delta$ quantile of $F$.

By (4-3),

$$u(\delta;n,s,k,\beta) \approx F(\delta;\nu_n,\nu_d)\cdot(A_s-A_k). \quad (4\text{-}4)$$

Hence, from (2-5) the 2-sided $(1-\alpha)$ approximate $s$-prediction interval for $X_{(s)}$ is:

$$(X_{(k)}+F(\tfrac{1}{2}\alpha;\nu_n,\nu_d)\cdot\hat{\sigma}\cdot(A_s-A_k),\ X_{(k)}+F(1-\tfrac{1}{2}\alpha;\nu_n,\nu_d)\cdot\hat{\sigma}\cdot(A_s-A_k)). \quad (4\text{-}5)$$

The 1-sided $(1-\alpha)$ approximate $s$-prediction interval is:

$$(X_{(k)},\ X_{(k)}+F(1-\alpha;\nu_n,\nu_d)\cdot\hat{\sigma}\cdot(A_s-A_k)). \quad (4\text{-}6)$$

*Example2*[2]

Consider exactly the same situation as in example 1. We list some of the necessary quantities in table 2 (see also table 1).

TABLE 2
Quantities Needed

| Quantity | Value |
|---|---|
| $A_{20}$ | 1.2392 |
| $A_{30}$ | 2.3047 |
| $\nu_n$ | 4.4970 |
| $\nu_d$ | 31.8052 |

$F(0.9;4.4970,31.8052)=2.0803$.

The 1-sided upper 90% approximate $s$-prediction limit for $X_{(30)}$ is:

$$35.274 + (2.0803)\cdot(22.595)\cdot(2.3047-1.2392) = 35.274 + (2.2166)\cdot(22.595) = 85.358.$$

Hence the 1-sided 90% approximate $s$-prediction interval is:

(35.274, 85.358). ◄

[2]The number of significant figures is not intended to imply any accuracy in the estimates, but to illustrate the arithmetic.

## APPENDIX

Percentiles $u(\delta;n,n,k,\beta)$ of $U=(X_{(n)}-X_{(k)})/\hat{\sigma}$ with $\beta=5$

$\Pr\{U \le u(\delta;n,n,k,5)\} = \delta$

| n | k δ: | 0.005 | 0.01 | 0.025 | 0.05 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 2 | 0.001 | 0.002 | 0.006 | 0.013 | 0.030 | 0.067 | 0.117 | 0.181 | 0.275 | 0.413 | 0.652 | 1.148 | 2.723 | 5.912 | 11.977 | 28.112 | 32.022 |
| 4 | 2 | 0.012 | 0.020 | 0.033 | 0.052 | 0.085 | 0.152 | 0.232 | 0.336 | 0.484 | 0.704 | 1.057 | 1.785 | 3.875 | 8.004 | 16.166 | 39.881 | 53.916 |
| | 3 | 0.001 | 0.002 | 0.006 | 0.014 | 0.029 | 0.066 | 0.110 | 0.164 | 0.236 | 0.335 | 0.486 | 0.736 | 1.299 | 2.169 | 3.534 | 5.816 | 7.421 |
| 5 | 2 | 0.027 | 0.038 | 0.057 | 0.083 | 0.126 | 0.212 | 0.315 | 0.449 | 0.633 | 0.892 | 1.347 | 2.239 | 4.920 | 10.375 | 20.693 | 50.008 | 95.344 |
| | 3 | 0.016 | 0.023 | 0.038 | 0.058 | 0.092 | 0.157 | 0.225 | 0.307 | 0.411 | 0.552 | 0.750 | 1.092 | 1.881 | 3.001 | 4.436 | 7.434 | 11.545 |
| | 4 | 0.001 | 0.003 | 0.007 | 0.014 | 0.030 | 0.066 | 0.109 | 0.161 | 0.230 | 0.326 | 0.454 | 0.665 | 1.100 | 1.676 | 2.432 | 3.743 | 4.748 |
| 6 | 2 | 0.039 | 0.052 | 0.076 | 0.109 | 0.161 | 0.263 | 0.384 | 0.531 | 0.735 | 1.041 | 1.541 | 2.502 | 5.445 | 11.653 | 24.647 | 61.410 | 82.752 |
| | 3 | 0.035 | 0.045 | 0.067 | 0.093 | 0.141 | 0.220 | 0.306 | 0.406 | 0.538 | 0.702 | 0.949 | 1.343 | 2.252 | 3.545 | 5.356 | 8.673 | 12.393 |
| | 4 | 0.016 | 0.024 | 0.041 | 0.061 | 0.096 | 0.161 | 0.228 | 0.306 | 0.400 | 0.523 | 0.698 | 0.963 | 1.511 | 2.240 | 3.168 | 4.526 | 4.999 |
| | 5 | 0.001 | 0.003 | 0.007 | 0.015 | 0.030 | 0.067 | 0.110 | 0.162 | 0.230 | 0.317 | 0.444 | 0.635 | 1.050 | 1.516 | 2.144 | 3.181 | 3.631 |
| 7 | 2 | 0.047 | 0.062 | 0.092 | 0.126 | 0.185 | 0.303 | 0.433 | 0.602 | 0.835 | 1.179 | 1.743 | 2.874 | 6.079 | 12.551 | 25.254 | 70.979 | 136.629 |
| | 3 | 0.048 | 0.064 | 0.090 | 0.125 | 0.177 | 0.272 | 0.371 | 0.484 | 0.622 | 0.809 | 1.075 | 1.537 | 2.553 | 3.986 | 5.923 | 9.549 | 11.040 |
| | 4 | 0.039 | 0.049 | 0.073 | 0.102 | 0.148 | 0.227 | 0.309 | 0.402 | 0.514 | 0.657 | 0.845 | 1.150 | 1.825 | 2.590 | 3.565 | 5.304 | 6.713 |
| | 5 | 0.018 | 0.026 | 0.042 | 0.063 | 0.099 | 0.166 | 0.236 | 0.314 | 0.405 | 0.518 | 0.676 | 0.926 | 1.421 | 2.040 | 2.822 | 4.135 | 5.171 |
| | 6 | 0.001 | 0.003 | 0.007 | 0.015 | 0.031 | 0.068 | 0.111 | 0.164 | 0.229 | 0.320 | 0.436 | 0.621 | 1.013 | 1.476 | 2.054 | 2.910 | 3.809 |
| 8 | 2 | 0.060 | 0.076 | 0.108 | 0.147 | 0.210 | 0.334 | 0.480 | 0.668 | 0.921 | 1.278 | 1.879 | 3.076 | 6.558 | 13.863 | 29.897 | 76.467 | 116.088 |
| | 3 | 0.069 | 0.085 | 0.117 | 0.154 | 0.212 | 0.312 | 0.422 | 0.547 | 0.699 | 0.898 | 1.189 | 1.678 | 2.829 | 4.448 | 6.495 | 11.356 | 13.736 |
| | 4 | 0.058 | 0.076 | 0.103 | 0.137 | 0.191 | 0.282 | 0.371 | 0.477 | 0.598 | 0.753 | 0.971 | 1.305 | 2.014 | 2.877 | 3.993 | 6.021 | 6.758 |
| | 5 | 0.043 | 0.055 | 0.080 | 0.109 | 0.156 | 0.238 | 0.316 | 0.404 | 0.506 | 0.636 | 0.821 | 1.088 | 1.619 | 2.272 | 3.047 | 4.453 | 4.646 |
| | 6 | 0.020 | 0.028 | 0.045 | 0.066 | 0.104 | 0.167 | 0.234 | 0.309 | 0.398 | 0.507 | 0.658 | 0.893 | 1.346 | 1.876 | 2.500 | 3.579 | 4.190 |
| | 7 | 0.001 | 0.003 | 0.008 | 0.016 | 0.031 | 0.068 | 0.112 | 0.165 | 0.231 | 0.316 | 0.439 | 0.627 | 0.987 | 1.452 | 1.933 | 2.832 | 3.697 |
| 9 | 2 | 0.070 | 0.087 | 0.122 | 0.164 | 0.231 | 0.365 | 0.515 | 0.708 | 0.969 | 1.350 | 1.979 | 3.269 | 7.058 | 14.214 | 29.484 | 73.612 | 123.159 |
| | 3 | 0.082 | 0.102 | 0.134 | 0.172 | 0.235 | 0.346 | 0.464 | 0.592 | 0.756 | 0.976 | 1.302 | 1.828 | 3.008 | 4.662 | 6.859 | 11.526 | 14.692 |
| | 4 | 0.076 | 0.095 | 0.128 | 0.167 | 0.225 | 0.321 | 0.421 | 0.533 | 0.664 | 0.831 | 1.059 | 1.439 | 2.191 | 3.170 | 4.469 | 6.369 | 8.068 |
| | 5 | 0.065 | 0.081 | 0.112 | 0.146 | 0.199 | 0.289 | 0.377 | 0.475 | 0.590 | 0.729 | 0.921 | 1.228 | 1.806 | 2.491 | 3.327 | 4.730 | 6.037 |
| | 6 | 0.045 | 0.059 | 0.084 | 0.112 | 0.160 | 0.241 | 0.320 | 0.408 | 0.509 | 0.633 | 0.804 | 1.072 | 1.530 | 2.084 | 2.799 | 3.889 | 4.677 |
| | 7 | 0.020 | 0.028 | 0.045 | 0.069 | 0.105 | 0.172 | 0.240 | 0.316 | 0.404 | 0.513 | 0.660 | 0.889 | 1.315 | 1.786 | 2.377 | 3.314 | 3.832 |
| | 8 | 0.001 | 0.003 | 0.008 | 0.016 | 0.032 | 0.070 | 0.116 | 0.170 | 0.237 | 0.324 | 0.445 | 0.635 | 1.006 | 1.417 | 1.931 | 2.730 | 3.451 |
| 10 | 2 | 0.073 | 0.091 | 0.129 | 0.175 | 0.247 | 0.392 | 0.549 | 0.758 | 1.046 | 1.462 | 2.170 | 3.520 | 7.518 | 15.533 | 32.160 | 81.405 | 173.077 |
| | 3 | 0.093 | 0.110 | 0.149 | 0.192 | 0.260 | 0.376 | 0.498 | 0.636 | 0.813 | 1.051 | 1.392 | 1.958 | 3.191 | 4.956 | 7.442 | 12.451 | 18.155 |
| | 4 | 0.090 | 0.112 | 0.150 | 0.189 | 0.252 | 0.360 | 0.461 | 0.581 | 0.723 | 0.900 | 1.149 | 1.551 | 2.342 | 3.313 | 4.614 | 6.890 | 8.116 |
| | 5 | 0.081 | 0.102 | 0.138 | 0.174 | 0.234 | 0.333 | 0.428 | 0.533 | 0.649 | 0.804 | 1.009 | 1.327 | 1.973 | 2.722 | 3.581 | 5.164 | 6.911 |
| | 6 | 0.067 | 0.084 | 0.116 | 0.150 | 0.205 | 0.297 | 0.382 | 0.473 | 0.581 | 0.723 | 0.911 | 1.179 | 1.720 | 2.349 | 3.091 | 4.257 | 4.482 |
| | 7 | 0.042 | 0.059 | 0.087 | 0.119 | 0.166 | 0.246 | 0.323 | 0.408 | 0.504 | 0.625 | 0.794 | 1.047 | 1.514 | 2.066 | 2.705 | 3.737 | 4.176 |
| | 8 | 0.019 | 0.026 | 0.044 | 0.069 | 0.107 | 0.173 | 0.243 | 0.317 | 0.402 | 0.512 | 0.656 | 0.877 | 1.297 | 1.791 | 2.344 | 3.350 | 4.013 |
| | 9 | 0.001 | 0.003 | 0.007 | 0.015 | 0.032 | 0.071 | 0.116 | 0.171 | 0.238 | 0.326 | 0.449 | 0.633 | 0.995 | 1.426 | 1.925 | 2.732 | 2.815 |
| 15 | 2 | 0.104 | 0.129 | 0.174 | 0.233 | 0.323 | 0.498 | 0.696 | 0.943 | 1.284 | 1.776 | 2.616 | 4.296 | 9.223 | 18.925 | 38.000 | 99.131 | 148.068 |
| | 3 | 0.138 | 0.165 | 0.214 | 0.267 | 0.353 | 0.500 | 0.645 | 0.816 | 1.031 | 1.307 | 1.705 | 2.351 | 3.827 | 5.932 | 8.664 | 14.860 | 16.476 |
| | 4 | 0.153 | 0.179 | 0.227 | 0.280 | 0.360 | 0.494 | 0.627 | 0.772 | 0.941 | 1.155 | 1.459 | 1.928 | 2.848 | 4.063 | 5.576 | 8.322 | 9.357 |
| | 5 | 0.159 | 0.183 | 0.229 | 0.281 | 0.357 | 0.487 | 0.608 | 0.737 | 0.886 | 1.075 | 1.326 | 1.707 | 2.489 | 3.408 | 4.434 | 6.105 | 7.838 |
| | 6 | 0.155 | 0.181 | 0.228 | 0.277 | 0.351 | 0.472 | 0.585 | 0.704 | 0.845 | 1.007 | 1.235 | 1.563 | 2.208 | 3.013 | 3.935 | 5.475 | 6.558 |
| | 7 | 0.145 | 0.171 | 0.217 | 0.266 | 0.337 | 0.452 | 0.558 | 0.672 | 0.801 | 0.958 | 1.159 | 1.469 | 2.044 | 2.719 | 3.501 | 4.778 | 5.653 |
| | 8 | 0.139 | 0.158 | 0.201 | 0.250 | 0.321 | 0.430 | 0.531 | 0.638 | 0.759 | 0.908 | 1.103 | 1.383 | 1.923 | 2.561 | 3.302 | 4.319 | 5.314 |
| | 9 | 0.126 | 0.151 | 0.185 | 0.228 | 0.296 | 0.401 | 0.500 | 0.603 | 0.717 | 0.861 | 1.050 | 1.312 | 1.827 | 2.406 | 3.101 | 4.063 | 4.906 |
| | 10 | 0.102 | 0.124 | 0.161 | 0.204 | 0.266 | 0.367 | 0.460 | 0.556 | 0.666 | 0.804 | 0.980 | 1.236 | 1.725 | 2.279 | 2.894 | 3.865 | 4.739 |
| | 11 | 0.074 | 0.094 | 0.133 | 0.170 | 0.227 | 0.324 | 0.411 | 0.501 | 0.608 | 0.735 | 0.909 | 1.155 | 1.603 | 2.131 | 2.740 | 3.657 | 4.424 |
| | 12 | 0.047 | 0.064 | 0.093 | 0.127 | 0.179 | 0.265 | 0.349 | 0.434 | 0.536 | 0.657 | 0.815 | 1.042 | 1.482 | 1.988 | 2.549 | 3.498 | 3.923 |
| | 13 | 0.021 | 0.031 | 0.050 | 0.073 | 0.113 | 0.187 | 0.260 | 0.338 | 0.426 | 0.542 | 0.683 | 0.901 | 1.320 | 1.794 | 2.352 | 3.178 | 3.689 |
| | 14 | 0.001 | 0.002 | 0.007 | 0.016 | 0.034 | 0.074 | 0.124 | 0.182 | 0.254 | 0.344 | 0.469 | 0.665 | 1.042 | 1.471 | 2.027 | 2.714 | 3.143 |
| 20 | 2 | 0.127 | 0.155 | 0.206 | 0.266 | 0.367 | 0.566 | 0.788 | 1.069 | 1.474 | 2.041 | 3.004 | 4.781 | 10.233 | 21.567 | 43.711 | 109.956 | 169.399 |
| | 3 | 0.169 | 0.198 | 0.256 | 0.316 | 0.412 | 0.581 | 0.751 | 0.945 | 1.176 | 1.497 | 1.956 | 2.694 | 4.360 | 6.744 | 10.099 | 16.969 | 20.417 |
| | 4 | 0.190 | 0.219 | 0.283 | 0.343 | 0.431 | 0.583 | 0.734 | 0.899 | 1.086 | 1.326 | 1.658 | 2.164 | 3.236 | 4.639 | 6.474 | 9.466 | 11.016 |
| | 5 | 0.198 | 0.232 | 0.291 | 0.353 | 0.437 | 0.580 | 0.717 | 0.865 | 1.042 | 1.241 | 1.516 | 1.930 | 2.780 | 3.786 | 4.994 | 6.652 | 6.654 |
| | 6 | 0.204 | 0.237 | 0.297 | 0.355 | 0.441 | 0.575 | 0.705 | 0.843 | 0.999 | 1.187 | 1.429 | 1.797 | 2.524 | 3.398 | 4.339 | 6.073 | 6.528 |
| | 7 | 0.211 | 0.240 | 0.295 | 0.353 | 0.440 | 0.566 | 0.691 | 0.817 | 0.966 | 1.141 | 1.366 | 1.704 | 2.372 | 3.133 | 3.956 | 5.291 | 5.717 |

(*Continued Next Page*)

| $n$ | $k$ $\delta$: | 0.005 | 0.01 | 0.025 | 0.05 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 8 | 0.206 | 0.239 | 0.290 | 0.350 | 0.432 | 0.555 | 0.672 | 0.799 | 0.936 | 1.103 | 1.309 | 1.627 | 2.226 | 2.926 | 3.765 | 4.984 | 5.934 |
| | 9 | 0.200 | 0.234 | 0.285 | 0.339 | 0.420 | 0.541 | 0.656 | 0.776 | 0.909 | 1.072 | 1.270 | 1.571 | 2.139 | 2.776 | 3.547 | 4.692 | 5.640 |
| | 10 | 0.196 | 0.224 | 0.275 | 0.328 | 0.406 | 0.527 | 0.635 | 0.751 | 0.878 | 1.033 | 1.235 | 1.515 | 2.055 | 2.668 | 3.377 | 4.460 | 4.927 |
| | 11 | 0.189 | 0.215 | 0.263 | 0.316 | 0.391 | 0.510 | 0.617 | 0.725 | 0.850 | 0.999 | 1.184 | 1.461 | 1.994 | 2.550 | 3.223 | 4.229 | 4.612 |
| | 12 | 0.171 | 0.199 | 0.248 | 0.297 | 0.370 | 0.487 | 0.589 | 0.696 | 0.817 | 0.963 | 1.142 | 1.409 | 1.926 | 2.468 | 3.115 | 4.132 | 4.567 |
| | 13 | 0.159 | 0.185 | 0.228 | 0.276 | 0.347 | 0.457 | 0.560 | 0.665 | 0.780 | 0.918 | 1.097 | 1.356 | 1.858 | 2.385 | 2.992 | 4.017 | 4.822 |
| | 14 | 0.139 | 0.163 | 0.203 | 0.251 | 0.321 | 0.426 | 0.527 | 0.626 | 0.741 | 0.875 | 1.054 | 1.304 | 1.770 | 2.292 | 2.894 | 3.864 | 4.480 |
| | 15 | 0.122 | 0.140 | 0.176 | 0.218 | 0.285 | 0.385 | 0.479 | 0.580 | 0.690 | 0.819 | 0.995 | 1.240 | 1.690 | 2.201 | 2.796 | 3.727 | 4.500 |
| | 16 | 0.087 | 0.107 | 0.143 | 0.184 | 0.244 | 0.339 | 0.428 | 0.524 | 0.630 | 0.759 | 0.925 | 1.164 | 1.596 | 2.100 | 2.676 | 3.604 | 4.176 |
| | 17 | 0.154 | 0.070 | 0.101 | 0.136 | 0.189 | 0.278 | 0.363 | 0.452 | 0.552 | 0.672 | 0.832 | 1.072 | 1.491 | 1.976 | 2.557 | 3.461 | 4.013 |
| | 18 | 0.022 | 0.031 | 0.053 | 0.079 | 0.119 | 0.194 | 0.268 | 0.349 | 0.441 | 0.556 | 0.707 | 0.935 | 1.325 | 1.809 | 2.347 | 3.178 | 3.875 |
| | 19 | 0.001 | 0.003 | 0.008 | 0.017 | 0.036 | 0.078 | 0.128 | 0.187 | 0.266 | 0.355 | 0.487 | 0.682 | 1.066 | 1.498 | 2.024 | 2.745 | 3.132 |
| 25 | 2 | 0.150 | 0.176 | 0.231 | 0.300 | 0.408 | 0.616 | 0.861 | 1.186 | 1.598 | 2.225 | 3.250 | 5.329 | 11.484 | 24.391 | 49.034 | 119.777 | 119.790 |
| | 3 | 0.193 | 0.231 | 0.296 | 0.360 | 0.464 | 0.646 | 0.830 | 1.047 | 1.305 | 1.640 | 2.119 | 2.911 | 4.653 | 7.243 | 11.206 | 18.863 | 23.940 |
| | 4 | 0.216 | 0.257 | 0.318 | 0.389 | 0.490 | 0.654 | 0.811 | 0.989 | 1.197 | 1.426 | 1.837 | 2.370 | 3.511 | 4.948 | 6.880 | 10.122 | 10.124 |
| | 5 | 0.237 | 0.275 | 0.337 | 0.404 | 0.502 | 0.658 | 0.806 | 0.965 | 1.141 | 1.370 | 1.679 | 2.120 | 2.977 | 4.039 | 5.357 | 7.315 | 9.188 |
| | 6 | 0.244 | 0.285 | 0.346 | 0.414 | 0.509 | 0.660 | 0.799 | 0.942 | 1.108 | 1.319 | 1.589 | 1.986 | 2.744 | 3.645 | 4.798 | 6.274 | 7.295 |
| | 7 | 0.254 | 0.290 | 0.348 | 0.416 | 0.509 | 0.656 | 0.789 | 0.926 | 1.086 | 1.270 | 1.525 | 1.894 | 2.582 | 3.374 | 4.345 | 5.819 | 6.905 |
| | 8 | 0.256 | 0.291 | 0.355 | 0.417 | 0.512 | 0.650 | 0.780 | 0.914 | 1.063 | 1.236 | 1.471 | 1.813 | 2.467 | 3.206 | 4.068 | 5.360 | 6.212 |
| | 9 | 0.258 | 0.290 | 0.354 | 0.416 | 0.504 | 0.641 | 0.763 | 0.894 | 1.041 | 1.212 | 1.432 | 1.755 | 2.355 | 3.043 | 3.864 | 5.114 | 5.235 |
| | 10 | 0.255 | 0.290 | 0.346 | 0.413 | 0.498 | 0.628 | 0.751 | 0.877 | 1.018 | 1.180 | 1.394 | 1.708 | 2.277 | 2.919 | 3.640 | 4.791 | 5.880 |
| | 11 | 0.253 | 0.285 | 0.345 | 0.405 | 0.488 | 0.618 | 0.737 | 0.859 | 0.994 | 1.161 | 1.356 | 1.653 | 2.216 | 2.829 | 3.507 | 4.643 | 5.588 |
| | 12 | 0.242 | 0.275 | 0.336 | 0.397 | 0.480 | 0.602 | 0.720 | 0.838 | 0.973 | 1.129 | 1.330 | 1.618 | 2.152 | 2.742 | 3.455 | 4.509 | 5.036 |
| | 13 | 0.234 | 0.268 | 0.326 | 0.386 | 0.468 | 0.591 | 0.702 | 0.822 | 0.950 | 1.104 | 1.299 | 1.581 | 2.105 | 2.668 | 3.304 | 4.421 | 4.992 |
| | 14 | 0.226 | 0.257 | 0.312 | 0.373 | 0.452 | 0.572 | 0.684 | 0.797 | 0.928 | 1.076 | 1.269 | 1.544 | 2.057 | 2.622 | 3.223 | 4.210 | 5.141 |
| | 15 | 0.213 | 0.246 | 0.299 | 0.358 | 0.436 | 0.556 | 0.662 | 0.772 | 0.902 | 1.050 | 1.236 | 1.509 | 2.003 | 2.544 | 3.156 | 4.224 | 4.951 |
| | 16 | 0.200 | 0.229 | 0.283 | 0.340 | 0.416 | 0.535 | 0.642 | 0.748 | 0.872 | 1.021 | 1.202 | 1.473 | 1.958 | 2.492 | 3.083 | 4.126 | 4.966 |
| | 17 | 0.179 | 0.203 | 0.264 | 0.318 | 0.394 | 0.511 | 0.613 | 0.718 | 0.838 | 0.982 | 1.165 | 1.426 | 1.907 | 2.441 | 3.004 | 4.073 | 4.817 |
| | 18 | 0.164 | 0.192 | 0.244 | 0.294 | 0.370 | 0.482 | 0.584 | 0.684 | 0.804 | 0.943 | 1.124 | 1.379 | 1.847 | 2.323 | 2.962 | 4.027 | 4.692 |
| | 19 | 0.143 | 0.169 | 0.216 | 0.268 | 0.338 | 0.448 | 0.548 | 0.647 | 0.762 | 0.905 | 1.077 | 1.329 | 1.787 | 2.278 | 2.892 | 3.929 | 4.517 |
| | 20 | 0.112 | 0.140 | 0.184 | 0.233 | 0.302 | 0.406 | 0.502 | 0.600 | 0.710 | 0.850 | 1.021 | 1.262 | 1.707 | 2.217 | 2.824 | 3.769 | 4.407 |
| | 21 | 0.084 | 0.109 | 0.149 | 0.191 | 0.255 | 0.358 | 0.448 | 0.544 | 0.650 | 0.787 | 0.951 | 1.188 | 1.629 | 2.135 | 2.720 | 3.711 | 4.380 |
| | 22 | 0.054 | 0.070 | 0.104 | 0.144 | 0.212 | 0.291 | 0.378 | 0.467 | 0.571 | 0.694 | 0.864 | 1.101 | 1.522 | 2.004 | 2.578 | 3.561 | 4.202 |
| | 23 | 0.023 | 0.033 | 0.054 | 0.082 | 0.127 | 0.208 | 0.283 | 0.334 | 0.364 | 0.460 | 0.575 | 0.734 | 0.958 | 1.375 | 1.845 | 2.424 | 3.330 |
| | 24 | 0.001 | 0.003 | 0.009 | 0.018 | 0.038 | 0.083 | 0.136 | 0.198 | 0.275 | 0.373 | 0.507 | 0.713 | 1.090 | 1.538 | 2.109 | 2.937 | 3.486 |
| 30 | 2 | 0.155 | 0.188 | 0.250 | 0.325 | 0.447 | 0.673 | 0.946 | 1.269 | 1.733 | 2.411 | 3.499 | 5.723 | 12.561 | 25.776 | 55.862 | 154.233 | 169.878 |
| | 3 | 0.216 | 0.250 | 0.312 | 0.384 | 0.501 | 0.693 | 0.898 | 1.123 | 1.403 | 1.769 | 2.288 | 3.161 | 5.099 | 7.996 | 12.342 | 20.155 | 26.145 |
| | 4 | 0.247 | 0.284 | 0.352 | 0.423 | 0.532 | 0.714 | 0.883 | 1.074 | 1.301 | 1.583 | 1.977 | 2.597 | 3.807 | 5.337 | 7.260 | 10.512 | 12.549 |
| | 5 | 0.269 | 0.304 | 0.374 | 0.446 | 0.549 | 0.718 | 0.879 | 1.052 | 1.253 | 1.499 | 1.822 | 2.313 | 3.279 | 4.386 | 5.777 | 7.941 | 9.887 |
| | 6 | 0.280 | 0.318 | 0.385 | 0.461 | 0.563 | 0.718 | 0.868 | 1.030 | 1.213 | 1.434 | 1.729 | 2.141 | 2.995 | 3.917 | 4.978 | 6.892 | 7.510 |
| | 7 | 0.288 | 0.329 | 0.396 | 0.463 | 0.566 | 0.717 | 0.865 | 1.015 | 1.186 | 1.389 | 1.660 | 2.048 | 2.794 | 3.669 | 4.639 | 6.134 | 6.237 |
| | 8 | 0.293 | 0.331 | 0.401 | 0.471 | 0.569 | 0.718 | 0.854 | 1.003 | 1.159 | 1.348 | 1.608 | 1.969 | 2.667 | 3.446 | 4.301 | 5.733 | 6.681 |
| | 9 | 0.302 | 0.336 | 0.406 | 0.473 | 0.569 | 0.710 | 0.845 | 0.986 | 1.142 | 1.321 | 1.558 | 1.904 | 2.556 | 3.305 | 4.090 | 5.447 | 6.718 |
| | 10 | 0.297 | 0.338 | 0.404 | 0.476 | 0.568 | 0.704 | 0.839 | 0.975 | 1.124 | 1.300 | 1.531 | 1.855 | 2.483 | 3.200 | 4.007 | 5.195 | 5.532 |
| | 11 | 0.300 | 0.338 | 0.404 | 0.473 | 0.562 | 0.696 | 0.824 | 0.959 | 1.107 | 1.281 | 1.503 | 1.814 | 2.417 | 3.114 | 3.887 | 3.960 | 5.639 |
| | 12 | 0.297 | 0.333 | 0.404 | 0.467 | 0.556 | 0.685 | 0.813 | 0.945 | 1.089 | 1.258 | 1.465 | 1.781 | 2.351 | 3.038 | 3.750 | 4.844 | 5.655 |
| | 13 | 0.294 | 0.330 | 0.396 | 0.465 | 0.546 | 0.678 | 0.800 | 0.931 | 1.075 | 1.236 | 1.448 | 1.745 | 2.294 | 2.949 | 3.660 | 4.743 | 5.126 |
| | 14 | 0.292 | 0.325 | 0.391 | 0.454 | 0.534 | 0.668 | 0.788 | 0.914 | 1.057 | 1.216 | 1.416 | 1.707 | 2.252 | 2.883 | 3.611 | 4.662 | 5.194 |
| | 15 | 0.288 | 0.320 | 0.383 | 0.443 | 0.528 | 0.657 | 0.775 | 0.900 | 1.037 | 1.192 | 1.394 | 1.683 | 2.210 | 2.821 | 3.522 | 4.576 | 4.897 |
| | 16 | 0.273 | 0.310 | 0.375 | 0.431 | 0.515 | 0.646 | 0.759 | 0.880 | 1.018 | 1.170 | 1.367 | 1.646 | 2.177 | 2.748 | 3.465 | 4.405 | 4.911 |
| | 17 | 0.262 | 0.299 | 0.360 | 0.421 | 0.504 | 0.630 | 0.741 | 0.863 | 0.999 | 1.153 | 1.348 | 1.618 | 2.129 | 2.706 | 3.396 | 4.356 | 4.805 |
| | 18 | 0.253 | 0.289 | 0.346 | 0.410 | 0.490 | 0.612 | 0.725 | 0.844 | 0.981 | 1.132 | 1.319 | 1.589 | 2.098 | 2.547 | 3.328 | 4.266 | 4.697 |
| | 19 | 0.230 | 0.272 | 0.333 | 0.393 | 0.473 | 0.597 | 0.706 | 0.821 | 0.955 | 1.106 | 1.289 | 1.556 | 2.058 | 2.616 | 3.253 | 4.231 | 4.777 |
| | 20 | 0.230 | 0.259 | 0.317 | 0.374 | 0.455 | 0.577 | 0.685 | 0.801 | 0.930 | 1.080 | 1.257 | 1.522 | 2.007 | 2.581 | 3.180 | 4.149 | 4.902 |
| | 21 | 0.211 | 0.247 | 0.302 | 0.357 | 0.435 | 0.552 | 0.661 | 0.775 | 0.902 | 1.049 | 1.224 | 1.482 | 1.975 | 2.529 | 3.126 | 4.059 | 4.725 |
| | 22 | 0.194 | 0.226 | 0.283 | 0.337 | 0.411 | 0.528 | 0.635 | 0.746 | 0.871 | 1.014 | 1.187 | 1.444 | 1.924 | 2.464 | 3.070 | 4.000 | 4.550 |
| | 23 | 0.173 | 0.202 | 0.253 | 0.311 | 0.384 | 0.496 | 0.602 | 0.712 | 0.838 | 0.976 | 1.144 | 1.403 | 1.881 | 2.409 | 2.979 | 3.936 | 4.392 |
| | 24 | 0.154 | 0.181 | 0.227 | 0.280 | 0.353 | 0.463 | 0.563 | 0.672 | 0.790 | 0.932 | 1.103 | 1.349 | 1.825 | 2.333 | 2.930 | 3.894 | 4.224 |
| | 25 | 0.124 | 0.152 | 0.194 | 0.243 | 0.314 | 0.420 | 0.518 | 0.622 | 0.743 | 0.883 | 1.048 | 1.293 | 1.762 | 2.247 | 2.858 | 3.776 | 4.287 |
| | 26 | 0.092 | 0.115 | 0.156 | 0.200 | 0.266 | 0.369 | 0.465 | 0.562 | 0.674 | 0.813 | 0.979 | 1.217 | 1.686 | 2.169 | 2.760 | 3.664 | 4.177 |
| | 27 | 0.060 | 0.078 | 0.111 | 0.149 | 0.207 | 0.303 | 0.392 | 0.481 | 0.589 | 0.723 | 0.828 | 1.122 | 1.567 | 2.074 | 2.613 | 3.498 | 3.923 |
| | 28 | 0.023 | 0.034 | 0.058 | 0.086 | 0.133 | 0.217 | 0.293 | 0.376 | 0.473 | 0.593 | 0.758 | 0.982 | 1.421 | 1.908 | 2.455 | 3.291 | 3.720 |
| | 29 | 0.002 | 0.003 | 0.009 | 0.019 | 0.040 | 0.086 | 0.142 | 0.207 | 0.284 | 0.383 | 0.518 | 0.732 | 1.125 | 1.608 | 2.120 | 2.945 | 3.084 |

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# Comment on: On Cut-Set Analysis of Networks Using Minimal Paths and Network Decomposition

The following changes/corrections are needed in [1].

p 59, col 1, bottom 2 lines + footnote

The sample network* is shown in figure 1. ...

* The figures are NOT reproduced from [2]; rather they are created for this *Comment on*.

p 59, col 2, figures 1 & 2

Replace those figures with the ones shown here.

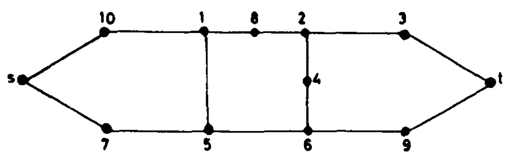
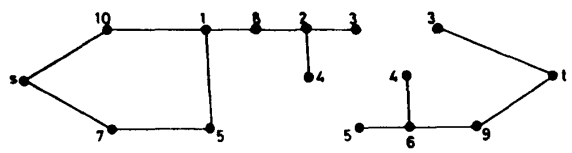



Figure 1. A Sample Network




a. Subnetwork $S_1$ b. Subnetwork $S_2$

Figure 2. Subnetworks of Figure 1 When Decomposed at Vertices 3, 4, 5

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Corrections received 1994 June 4 ◄TR►