CAUSES OF THE ALLAIS CHOICE (a1, b2)

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Abstract

Paradox has been an instrument to challenge the traditional expected utility theory. Paradox arises from the inconsistency between the empirical or experimental results and the theoretical deductions. In the expected utility theory field, there are many paradoxes or effects showing behaviors that are contradictory to the "theoretical" ones. Many studies questioned the validity of the expected utility theory by means of these paradoxes; while many others disqualified the expected utility theory as a descriptive model for human decision making behaviors. Among these paradoxes or effects, the Allais Paradox raised by Allais in 1953 is the most famous one.

In this paper, we make a comprehensive study of the causes of the Allais Paradox. From the study, we learn that the Allais Paradox need not necessarily be caused by errors or inadequacy of the traditional expected utility theory. There can certainly be other non-theory-related factors. It can be the same for the other paradoxes or effects, too. Understanding that the traditional expected utility theory is insufficient as a descriptive model, this study is based on a revised expected utility theory, which adds new concepts through logical deduction process. Through the analysis of Allais paradox, this paper presents a revised theory is expected to enhance the descriptive ability of the utility function on human decision making behaviors.

1. Introduction

The "paradox" in decision science field means the inconsistency between the normative interpretation of a decision making process and the empirical results observed from actual decision making activities. In many experiments and observations, the choices made by a so-called "rational decision maker" do not always obey the normative decision theory. It has been an important indicator used by the descriptive schools to challenge the normative schools of the decision science. Certainly, if a decision model cannot describe or predict human decision behaviors effectively, its application value will be hurt greatly. However, does the paradox really exist? Is it only because of the inadequacy of the tradition expected utility theory? Or, is there any other explanation for the paradox?

In the fields of decision science and economics, there is no doubt that the expected utility theory is a widely accepted normative model for decision making under risk conditions. A rational decision maker theoretically "should" choose the alternative with the maximum expected utility. Nevertheless, being a "rational decision making model," the expected utility theory was challenged by many researchers for failing to comprehensively explain the behaviors of the decision makers. As shown by these paradoxes, some axiomatic violations of the expected utility theory have been generated by certain experiment conditions.

According to John Quiggins (1993), the main challenges to the expected utility theory fall into three categories: namely, the inconsistency between the empirical results and the normative axioms, the inability of the theory to predict some market behaviors such as insurance choices, and the elicitation of the utility function using questionnaires. The first category refers to paradoxes. The main purpose of this paper is to explore systematically the reasons for a well-known paradox, based on revised utility functions with added new concepts. The research process can be further applied to other paradoxes in the decision science field.

As the existence of the Allais paradox represents a significant challenge to the traditional expected utility theory, many studies have tried to explore the causes of it from different aspects. In order to revise the traditional theory or to develop a new theory, many psychologists, economists, and decision scientists have approached the issue by human behavior and/or empirical studies. In this thesis, we will focus on the causes of the Allais paradox whether the causes are reasonable or not.

1.1 *The Frame of the Allais' Paradox*

Axiomatic violations of the expected utility theory have been produced by specific experiments or framing procedures. Among them, the paradox shown in Allais' experiment (1953) is the most famous one. The Allais paradox presented us two situations. In situation *A*, a decision maker was offered a choice between receiving $1,000,000 for certain and a lottery that furnished her or him a 0.1 chance of winning $5,000,000, 0.89 chance of winning $1,000,000 and a 0.01 chance of receiving nothing at all. In situation *B*, the choice was between two lotteries. One offered a 0.11 chance of winning $1,000,000 and a 0.89 chance of receiving nothing; the other offered a 0.1 chance of winning $5,000,000 and a 0.9 chance of nothing. Table I presents the decision tables for these two situations.

Table I: An Illustration of Allais' Paradox

	Problem A		
Probability	S_1=0.01	S_2=0.1	S_3=0.89
a_1	1,000,000	1,000,000	1,000,000
a_2	0	5,000,000	1,000,000

	Problem B		
Probability	S_1=0.01	S_2=0.1	S_3=0.89
b_1	1,000,000	1,000,000	0
b_2	0	5,000,000	0

According to the axiom of the "Sure Thing" property, the preference to these two alternatives should not be affected by adding or subtracting a constant factor. Also, following the so-called "Cancellation" axiom, the preference to the two issues in Table I will not be affected by the same change in situation S_3. Per compatibility property,

$u(a1)\text{-}u(a2)$ ....(1)

$=u(1M)\text{-}[0.1u(5M)+0.89u(1M)+0.01u(0)]$

$=0.11u(1M)\text{-}[0.1u(5M)+0.01u(0)]$

$u(b1)\text{-}u(b2)$ ....(2)

$=[0.11u(1M)+.89u(0)[-]0.1u(5M)+0.9u(0)]$

$=0.11u(1M)\text{-}[0.1u(5M)+0.01u(0)]$

Therefore, $u(a_1)$ - $u(a_2)$ should equal to $u(b_1)$ - $u(b_2)$. According to the expected utility theory, the selections in the Table I should be $\{a_1, b_1\}$ or $\{a_2, b_2\}$. But, in Allais' experiments (1953), there were nine out of twenty-four rational testees being trained with knowledge of probability chose $\{a_1, b_2\}$.

1.2 *System Thinking*

One of the main objectives of the expected utility theory is to be an evaluation tool for decision making under risk. Correspondingly, the problem in the Allais experiment is also a risky decision problem. To explain the Allais choice, there can be three different approaches: namely, from the points of view of "theory," "human behavior" and the "experiment/problem itself."

A problem can always be deconstructed into the problem itself and the person who faces it. The interactions between the two factors (i.e. the decision process) constitute the problematic situation. Theoretically, the existence of a paradox can be attributed to error or inadequacy of the theory or factors related to the problem itself or the decision-maker. If the reason for a paradox lies in the problem itself, it can generally be considered an experimental error or misleading experiment design such as hypothetical experimental question. If the reason for a paradox comes from the decision maker, it may be biased by the decision maker's religious belief, cultural background, habits, knowledge, education, experiences, decision objective, preferences, or the bounded rationality of humanity. Apart from these reasons, there can also be reasons beyond them, for example the degree of deliberation, different decision guidelines, and so on. It is not easy to deconstruct causes clearly. For instance, if a decision maker chooses differently based on different perceptions of the "current wealth," it can be attributed to the decision maker factor or it can also be regarded as failing to clearly state the problem itself. Or, probably, it should be attributed to the insufficiency of the tradition expected utility in defining current wealth.

1.3 *The Causes of General Paradox and the Allais' Paradox*

The contradictions between the theory and empirical study results can be attributed to incompleteness of the theory. According to our analysis, it may also be due to the peculiarity of certain experimental results. For example, a natural gambler tends to be aggressive as to monetary decisions. It may be experimental issues due to semantic ambiguity, false assumption (i.e. different from the actual status of the decision maker), or the inability to reflect the decision maker's true "feeling," etc.

From the research point of view, the causes of paradox can be dealt with by different ways in different situations, such as, to modify the theory, to develop a new theory, or to exclude the peculiar experiment result by certain explanations. Table II lists these possibilities.

Table II: The Causes of the Paradox and Research Purpose

The reasons for Paradox	The research purpose
Theory incompleteness or failure	Revise the theory or develop a new theory
Specific peculiarity of experiment results	Explain and exclude the reasons for the peculiarity
Experiment errors	Find and explain the causes of the errors
Specific situations of the decision maker	Become a constraint of the theory where it will not apply.

We admit that the traditional expected utility theory cannot serve as a comprehensive descriptive model. Probably we should specify conditions under which the expected utility theory is an appropriate prescriptive model. When the situation does not fit, some prescriptive techniques can be developed to help decision-maker conform with the expected utility theory. Table III shows the four classifications of the causes of the Allais choice.

In this paper we use a revised expected utility theory proposed elsewhere (C.L. Sheng and H.Y. Lin, 1996) to explore the causes of the Allais paradox and to improve the expected utility theory as a descriptive model. The revised expected utility theory will make clear why Allais choice happened in some situations and how $u(0)$ will be defined. Moreover, we believe the explanations can be applied to other paradoxes too.

1.4 *New Concepts and Revised Utility Functions*

Even though we do not think that paradoxes can disqualify the normative effectiveness of the traditional expected utility theory, there does exist a need for some improvements in the application of the expected utility theory to individual decision under risk. We proposed a modification of the theory elsewhere, the essences of which are reproduced here.

According to Table III, the Allais' choice could be partly caused by the incompleteness of the traditional expected utility theory itself for its lacking of a clear, operable decision making guideline. At the same time, Allais' Paradox arose because of the weak interpretation ability of the traditional expected utility theory. We think that the Allais choice is possible even for testees with probability training because they can have different risk appetites or tolerance levels. Consequently, we introduce some new concepts and develop them into a revised expected utility theory by logic deduction. The purposes are to make the utility theory operable, to interpret the reason why Allais choice happened, and to better describe decision making behaviors.

R.L. Keeney and H. Raiffa (1976) derived the characteristics of a reasonable utility function from the general investment point of view.

Table III: The Comprehensive Causes of the Allais Choice		
Category	***Reason***	***Description***
Insufficiency of the traditional theory to interpret behaviors (See Section 2)	To avoid desperation (see 2.1)	Testees usually would shy away from possibilities leading to desperation and tended to choose a_1 outright to survive.
	Different interpretations or perceptions of the total wealth (see 2.2)	The certainty effect of a_1 constituted a different reference point to testees who faced both the a_1 and a_2 prospects and might lead to the Allais' choice.
Experimental issues (See Section 3)	Experimental figure beyond normal perception range of the decision makers (see 3.1)	The 1 or 5 millions figures were very exaggerated for folks in 1953 that they probably would not be able to perceive the decision situations clearly.
	The obfuscated definition of u(0) (see 3.2)	Unclear definition of u(0) might lead to the Allais choice.
	Hypothetical question (see 3.3)	Being hypothetical, the Allais' experiment might not be treated by every testee as a serious question to think rationally.
	Natural restrictions of an experiment (see 3.3)	In the Allais experiment, the testees probably were not allowed sufficient time for detailed calculation.
The decision making ability of the testees (See Section 4)	The ability to differentiate two similar prospects (see 4)	If the expected utilities of two prospects were almost the same to the testees, the chance for arbitrary Allais' choice might be high
Other causes (See Section 5)	Different evaluation criteria (see 5.1)	The testees might do the Allais' choice according to the "pessimistic principle" or the "Prospect Theory."

Table III: The Comprehensive Causes of the Allais Choice (continued)		
Category	***Reason***	***Description***
	Other decision goals (see 5.2 and 5.3)	The testees might have different goals during the Allais experiment period; e.g. he might need $1 million dollars for emergency.
	Spiritual Utility (see 5.4)	Some decision makers incline to take risks or pursuit other spiritual value.

The reasonable utility function is so-called "*constantly-proportional risk-averse utility functions*." We also regard constantly-proportional risk-averse utility functions as very reasonable utility functions. Nevertheless, we add three new concepts to derive some specific forms of reasonable utility functions. The three new concepts are (1) total current wealth V_N, (2) normal operation point N, and (3) minimum necessity value V_{0+}.

First of all, we think that in evaluating the utility of each prospect to the decision maker, the utility to be considered should be global instead of being local. That is to say, not only the gain or loss of a prospect but also the final status of the decision maker matters. This so-called final status includes the original total current wealth of the decision maker and the gain or loss possibility of each prospect to the decision maker. The value in a utility function should be defined as

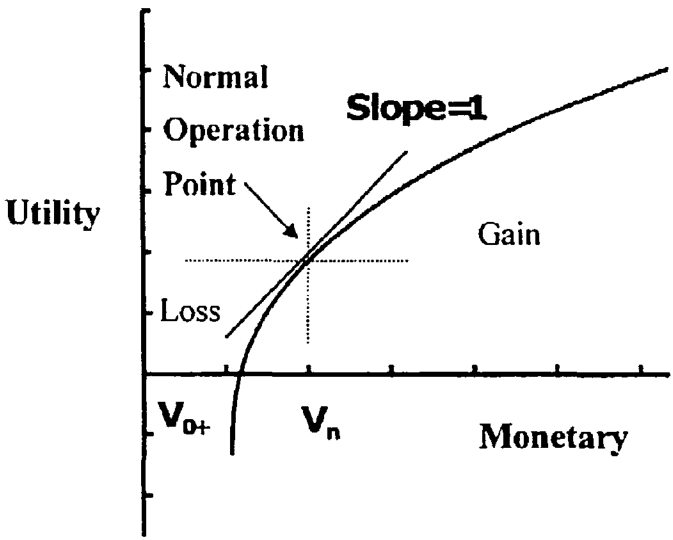
$$V=V_N+x_i \quad ....(3)$$

with x_i representing the possible gain or loss of each prospect.

Generally speaking, the rich and the poor have their own respective perceptions of money and ways of spending. As long as a person, rich or poor, gets used to her or his total current wealth, s/he will consider it as a "stable" status. The "stable" status implies that as a decision-maker is conscious of her or his total current wealth, s/he is supposed to yield fixed utility alternation toward the slight shift of her or his wealth. This is called the "normal operation point" on the utility curve. Conceptually, it reflects the total current wealth V_N of the decision maker; i.e. the total wealth V_N on the horizontal axis of the utility function before decision making.

In order to represent the concept of stable decision situation, the slope of the normal operation point should equal to a constant k. Since the utility theory has a characteristic of being strategically equivalent, we may assume k equals to 1 for calculation convenience.[1] That is, the decision maker will make her or his decision based on a normal operation point and this point has a slope equal to 1 on the utility curve.

Figure 1: The Slope of the Normal Operation Point is Equal to 1



According to previous brief discussion about the normal operating point, we have

$$\frac{du}{dV}\bigg|_{V=V_N} = b(1-c)(V_N)^{-c} = 1 \qquad \text{....(4)}$$

The third new concept is the minimum necessity value, denoted by V_{0+}. According to some researchers' opinions about the traditional expected utility theory, the decision maker gets results out of a closed system. The worst case of utility $u(x_{\text{worst}})$ in a prospect equals to zero. However, from the standpoint of the final asset position, if the worst case will lead total final wealth to less than 0, there is no such thing as $u(x_{\text{worst}})$ because a minimum necessity value V_{0+} must exist to maintain life necessities (please refer to Figure 1). This point also indicates the maximum risk tolerance level. When the current total wealth becomes V_{0+}, it means that the utility at that point is already zero and the wealth can support only the minimum life necessity. That is,

$$U(V_{0+})=0 \qquad \text{....(5)}$$

If the total wealth of a decision-maker is less than V_{0+}, the utility will fall rapidly with the decrease of the total wealth. That is to say, when the total wealth of a decision maker is less than V_{0+} (the utility is negative), the decision situation will become unstable as s/he is facing the threat of survival. Therefore, a decision maker would not undertake the alternative that might cause his or her wealth to be less than the minimum necessity value. According to previous discussion on the three new concepts, we can obtain the following two conditions.

$$\begin{cases} (1) \quad u=0 \quad \text{when} \quad V=V_{0+} \\ (2) \quad u=\dfrac{du}{dV}=1 \quad \text{when} \quad V=V_N \end{cases} \qquad \text{....(6)}$$

There are four types of *constantly-proportional risk-averse utility function,*

$$u(V)\sim\begin{cases} V & \text{for} \quad c=0 \\ V^{1-c} & \text{for} \quad 0<c<1 \\ \log V & \text{for} \quad c=1 \\ -V^{-(c-1)} & \text{for} \quad c>1 \end{cases} \qquad \text{....(7)}$$

When $c = 0$, it basically is a neutral risk situation and the function $u(V)=V$ does not belong to the risk aversion discussion. The other three utility functions in equation 7 meet the constantly proportional risk aversion requirement. Therefore, the reasonable utility functions used in this paper are the three functions in equation 8, where *c* is smaller than, equal to, or greater than 1 respectively.

$$u(V) \sim \begin{cases} V^{1-c} & for \quad 0<c<1 \\ \log V & for \quad c=1 \\ -V^{-(c-1)} & for \quad c>1 \end{cases} \qquad ....(8)$$

The variable *V* represents the final total wealth level, and equation 8 will be regarded as basic utility functions. Finally, by incorporating the two conditions in equation 6 into equation 8, we obtain the three basic utility functions in Table IV.

Table IV: The Basic Forms of the General Utility Functions

General Utility Functions	*Basic Forms*
$u=(V)^{1-c}$ *for* $c<1, c\neq 0$	$u=\frac{(V_N)^c}{1-c}(-(V_0)^{1-c}+(V)^{1-c})$
$u=\ln(V)$ *for* $c=1$	$u=V_N(-\ln V_{0+}+\ln V)$
$u=-(V)^{-(1-c)}$ *for* $c>1$	$u=\frac{(V_N)^c}{c-1}((V_{0+})^{-(c-1)}-(V)^{-(c-1)})$

It can be seen clearly from Table IV that there are only three factors to affect the general utility functions; namely, the current total wealth V_N, the minimum life necessity V_{0+}, and a constant *c* which represents the degree of risk aversion of the decision-maker.

2. The Causes of the Allais Choice Derived from Theory Incompleteness

As we previously pointed out, there can be many reasons for paradoxes. One possible reason is the incompleteness or mistake of the theory itself. We do not think that the traditional expected utility theory is wrong. However, as a descriptive model, it is not comprehensive enough to explain all human decision behaviors. Therefore, in this section, we try to use the basic forms of utility functions in Table IV to explain some causes of the Allais choice.

We believe that at least two reasons for the Allais choice can be explained by our revised expected utility theory. They are: (1) the decision-maker will try hard to avoid confronting "survival" situation, and (2) the Allais choice is caused by the dynamic nature of utility and the certainty effect.

First of all, from the point of view of the total wealth, the 0 value in Table I should not be treated as an "absolute zero;" otherwise the prospects of a_2, b_1 and b_2 in Table I may lead the decision-maker to the possibility of fighting for survival. For analysis purpose, we assume that the total current wealth of a testee is $1,000,000 and the minimum life necessity requirement is $300,000. We then add the total current wealth into Table I to come up with the "final total wealth" in Table V.

Table V: Allais' Paradox Table After Adding Current Wealth

	Problem A		
Probability	S_1=0.01	S_2=0.1	S_3=0.89
a_1	2,000,000	2,000,000	2,000,000
a_2	1,000,000	6,000,000	2,000,000

	Problem B		
Probability	S_1=0.01	S_2=0.1	S_3=0.89
b_1	2,000,000	2,000,000	1,000,000
b_2	1,000,000	6,000,000	1,000,000

2.1 *To Avoid Running into Survival Situation*

According to the previous analysis, V_{0+} represents the minimum life necessity. Generally speaking, people have to be able to meet the minimum life necessity to remain in a normal and stable condition and to be considered a "rational" decision maker. Even if the testee does regard the 0 value in Table I as a real or absolute 0, it is quite natural that this 0 is regarded as very small that it may be way below the minimum life necessity value V_{0+}. In that case, this V_{0+} may be considered as a constraint, a situation the testee do not like to get into, even if the probability of it is as small as 0.01. In other words, when the testee anticipates a situation of $V_N<V_{0+}$ (i.e. a negative utility), this choice will simply be excluded from consideration even the total expected utility of this choice, which includes the negative utility corresponding to the entry of 0, may be greater than each of the total expected utilities of all other choices.

Likewise, even if we add the current wealth into the original Allais paradox table to constitute Table V, the testee will still be under an unstable condition during the Allais experiment period if the smallest figure ($1,000,000) in Table V is less than her or his V_{0+}. Even he barely makes survival, it is still possible that he will try hard to avoid running into the "worst" situation. Under this situation, the traditional maximum utility axiom may be violated. But, it does not mean that the traditional expected utility theory is false. It only indicates that the theory is not complete enough to explain some phenomenon. Generally speaking, from the personal decision making point of view, the so-called "feasible alternatives" refer to the potential alternatives already after screening by certain constraints. Accordingly, the prospect should not have been included as a feasible alternative in Table I or Table V if the final total wealth V_N from that prospect should become less than or equal to V_{0+}. However, since the Allais experiment did not make this point clear, the testees might arbitrarily add their own assumptions and distorted the original issue.

Other than the semantic problem of the Allais' experiment, "to avoid running into survival situation" becomes an issue due to the inadequacy and weak interpretation power of the traditional expected utility theory. This weakness may be improved by the introduction of the concepts of normal total wealth V_N and minimum necessity value V_{0+}. That is to say, if a testee in the Allais' experiment is guided by the revised expected utility theory, "to avoid running into survival situation" should not have been a cause of the Allais choice. So, this weakness does not really undermine the expected utility theory.

2.2 *The Allais Choice Happened Naturally*

Another cause or explanation is related to the concept of normal total wealth. According to Kahneman and Tversky's (1979) opinions, the main reason for the Allais choice is *certainty effect.* Because the prospect a_1 in Table I possess certainty effect, the Allais paradox happens. Our opinion about certainty effect is similar to that of Tversky. We think that some testees in the Allais experiment may regard the certainty of a_1 prospect as her or his total current wealth and thus shift their utility curves.

Table V supposes that a testee has a normal total wealth of one million dollars before the test. Here comes the crucial point. What should the correct normal total wealth be? There are two views. The first one is that the normal total wealth is two million dollars because choice a_1 corresponds to a situation of "certainty," as called by Kahneman and Tversky (1979), and choice a_2 is a lottery or game to bet one million dollars, with a probability of 0.01 to lose it, for a probability of 0.10 to win five million dollars. The second view is that the normal total wealth is one million dollars with a windfall of either one million for sure or a windfall of a combined affairs: 0 million with a probability of 0.01, five million with a probability of 0.10, and one million with a probability of 0.89.

We think either view may be said to be correct. Sheng (1994) has discussed the dynamic nature of utility functions. According to Sheng, the normal total wealth of a person changes with her or his wealth and income; the utility function or curve changes with such changes, too. However, it takes some time for adaptation and transition. If a person obtains a big lottery prize suddenly, at first s/he may appear to still have the original function. But the operating point will move along the utility curve to a point corresponding to the new total wealth. After some time when s/he has adapted to the new situation, there will be a new utility curve with a new normal total wealth. In the above example, the old normal total wealth is one million dollars and the new normal total wealth is two million dollars.

Hence, because of the certainty effect of a_1, there may be two interpretations of the decision between a_1 and a_2. First, the decision between a_1 and a_2 in Table V can be interpreted as that the decision maker has included the certainty of $1,000,000 in her or his current total wealth and then to decide either to participate in a_2 prospect or not to. Second, a_1 and a_2 prospects are considered separately. That is to say, the total wealth of the testee includes the certainty one million dollars in prospect a_1, while does not include the one million dollars in prospect a_2.

According to the expected utility theory, there should be one normal total wealth for a specific decision situation. But, for some empirical results, a person may have the feeling of two different normal total wealth though not necessarily explicitly. For choice a_1, one tends to accept two million dollars as the normal total wealth because of the certainty effect. However, for choice a_2, because of some small uncertainty, one may accept one million dollars as the normal total wealth.

For example, someone's current total wealth is one million dollars before decision. Based on the first interpretation, s/he may consider prospect a_1 as a certain gain, changes her or his total wealth to two million dollars and regards the new total wealth as total current wealth in the new utility. On the other hand, based on the second interpretation, prospects a_1 and a_2 will be considered separately. The current total wealth of prospect a_1 will change to two million dollars due to certainty effect; but, in prospect a_2, it will still be one million dollars.

As long as the recognition of a testee about decision between a_1 and a_2 is similar to the latter in the previous discussion, the selection combination $\{a_1,b_2\}$ in the Allais' Paradox will naturally happen. That is to say, if someone regards the certainty prospect a_1 in Table V as her or his total wealth in consideration of prospect a_1 and applies the conclusion in Table IV and the relative concepts to the Allais' paradox illustration in Table IV, the Allais choice will happen naturally.

2.3 *Mathematical simulation*

We run a computer simulation on the revised utility function derived from Table V to prove that as long as the certainty effect exists and the testee considers a_1 and a_2 separately, the Allais choice will happen naturally. Suppose the total current wealth of a testee is one million dollars and the minimum life necessity requirement is 300,000 dollars, the analysis procedure is as follows.

In Table V, for prospect a_1, the current total wealth V_N is two million dollars and the minimum life necessity V_{0+} is 300,000 dollars. We can then get the specific utility function from Table IV as: (assume $c<1$, and set the unit in million dollars)

$$u=\frac{(2)^c}{1-c}C-(0.3)^{1-c}+(2)^{1-c}) \quad ....(9)$$

When a decision maker considers prospects a_1 and a_2 separately, the total current total wealth of a_2 will still be one million dollars, being the same as in the b_1 and b_2 situations. That is to say, for prospects a_2, b_1, and b_2 the current total wealth V_N are all one million dollars and the minimum life necessity V_{0+} are still 300,000 dollars.[2] We can then get the utility function from Table V as: (assume $c<1$, and set the unit in million dollars)

$$u=\frac{(1)^c}{1-c}(-(0.3)^{1-c}+(V)^{1-c}) \quad ....(10)$$

where V represents respective final total wealth status. Assuming c is 0.5, we can calculate the utility of each prospect in each situation by equations 10 and get the expected utility value as Table VI.

From Table VI, we can find that $u(a1) > u(a2)$ and $u(b2) >) u(b1)$. That is to say, if the testee considers prospect a_1 and a_2 separately, the minimum life necessity requirement is 300,000 dollars, the total current wealth equals one million dollars before decision, and c is 0.5, Allais' choice will appear naturally.

Furthermore, by using computerized electronic spreadsheet[3] to analyze and simulate different total current wealth levels and different

Table VI: The Modified Decision Table of Allais' Paradox

	Problem A			
Probability	S_1=0.01	S_2=0.1	S_3=0.89	EU
a_1	2.451	2.451	2.451	2.451
a_2	0.905	3.804	1.733	1.932

	Problem B			
Probability	S_1=0.01	S_2=0.1	S_3=0.89	EU
b_1	1.733	1.733	0.905	0.996
b_2	0.905	3.804	0.905	1.194

c, we can find that Allais choice exists in different V_{0+}. For instance, setting the V_{0+} equal to 50% of total current wealth V_N, we can get the simulated results as Table VII: (⊙represent the Allais choice).

We also prosecute another computerized simulation using different variable and get some results like Table VII. From the computer simulation results, we have the following observations:

1. For a given fixed risk-aversion value *c*, when V_{0+} is set to be a fix proportion of the current total wealth, the behavior tends to be more conservative with the increase of total current wealth V_N.

2. The Allais' choice will happen at lower current total wealth level if the risk-aversion value c is small: e.g. *c*=0.1 or 0.2. And the Allais' choice will happen at the higher current total wealth level when the risk-aversion value *c* is large, e.g. *c*=4 or 6.

3. The choices will depend on the risk-aversion value *c*, as shown in Table VII, when the minimum life necessity V_{0+} is set at a fixed proportion of the current total wealth V_N. The Allais' choice will occur at the lower *c* value when V_N is

Table VII: The Simulated Results of V_{t+1}=50% of V_n												
Total Current Wealth Level	0.1	0.2	0.3	0.5	0.5	0.8	1	1.5	2	3	4	6
20,000	⊙	⊙	⊙	⊙	⊙	⊙	⊙	(a1,b1)	(a1,b1)	(a1,b1)	(a1,b1)	(a1,b1)
50,000	(a2,b2)	⊙	⊙	⊙	⊙	⊙	⊙	⊙	(a1,b1)	(a1,b1)	(a1,b1)	(a1,b1)
100,000	(a2,b2)	⊙	⊙	⊙	⊙	⊙	⊙	⊙	(a1,b1)	(a1,b1)	(a1,b1)	(a1,b1)
200,000	(a2,b2)	⊙	⊙	⊙	⊙	⊙	⊙	⊙	⊙	(a1,b1)	(a1,b1)	(a1,b1)
500,000	(a2,b2)	(a2,b2)	⊙	⊙	⊙	⊙	⊙	⊙	⊙	⊙	(a1,b1)	(a1,b1)
1,000,000	(a2,b2)	(a2,b2)	⊙	⊙	⊙	⊙	⊙	⊙	⊙	⊙	⊙	(a1,b1)
2,000,000	(a2,b2)	(a2,b2)	(a2,b2)	⊙	⊙	⊙	⊙	⊙	⊙	⊙	⊙	⊙
3,000,000	(a2,b2)	(a2,b2)	(a2,b2)	⊙	⊙	⊙	⊙	⊙	⊙	⊙	⊙	⊙
4,000,000	(a2,b2)	(a2,b2)	(a2,b2)	⊙	⊙	⊙	⊙	⊙	⊙	⊙	⊙	⊙
5,000,000	(a2,b2)	(a2,b2)	(a2,b2)	(a2,b2)	⊙	⊙	⊙	⊙	⊙	⊙	⊙	⊙
6,000,000	(a2,b2)	(a2,b2)	(a2,b2)	(a2,b2)	⊙	⊙	⊙	⊙	⊙	⊙	⊙	⊙

⊙: (a_1,b_2) Allais Choice

small (comparing to $1,000,000 of possible gain) and at the *higher c value when V_N is large.*

4. When the current total wealth is fixed, Allais' choice will happen at the smaller V_{0+}. When V_{0+} is large (comparing to the current total wealth), Allais' choice seems not likely to happen. But other than the Allais' choice, another inconsistent pair (a_2,b_1) occurs at the higher risk-aversion value c.

From the viewpoint of normative utility theory, it is desirable to have various prospects taken into account under the same basis (including the current total wealth V_N and the minimum life necessity V_{0+}). It means that all prospects should be considered simultaneously rather than separately to truly reflect their individual utility toward a decision maker. On the other hand, when a decision maker considers every single prospect alone, theoretically, her or his decision behavior should not violate utility theory axioms no matter whether the certainty effect is included or not. Therefore, if certainty effect is included in a decision, the Allais' choice is expected to occur naturally as long as prospects a_1 and a_2 are considered separately.

Consequently, we can draw a conclusion that even a well trained decision maker or probability and utility concept recognizes and interprets prospects by a variety of ways. The Allais' choice may occur even though s/he is regarded as a rational decision-maker and applies the expected utility theory (the revised one in this paper) in the Allais' experiments. Different recognitions lead to Allais' choice; however, it can not be definitely proved that the decision-maker is irrational, nor can it be said that the expected utility theory is inaccurate. The "irrationality" of the testee should be explained by the dynamic or adaptive nature of utility functions and curves.

To sum up, we show that the Allais paradox does expose some inadequacy of the von Neumann-Morgenstern expected utility theory, which, however, does not undermine the theory because the Allais paradox may be well explained by our new interpretation and modification of the theory.

3. Problem Statement and Experiment Situation

Besides the inadequacy and lacking of interpretation power of the traditional expected utility theory, the Allais choice can result from other reasons. We can further categorize the reasons for the paradox into problem statement related and decision maker related. Among factors related to the experiment problem statement itself, there can be two major possibilities: (1) the experiment amount is too big for the decision maker to perceive normally; and (2) u(0) has some definition problem.

3.1 *Huge experiment amount*

Table I shows that one million dollars is a sure thing in prospect a_1. We believe that one million dollars in 1953 was an extraordinarily huge number compared to the current total wealth of the general folks. Theoretically speaking (according to Figure 1), when the decision maker is asked to evaluate any value on the utility curve to the right of the normal operation point, the farther the point is, the more difficult it is for the decision maker to distinguish the utility change because the figure is too huge to be perceived correctly. We may also say that it goes beyond the normal range that the utility function can best describe.

Simply using 6% p.a. as the discount rate, one million dollars in 1953 equals to 12.25 million dollars in 1996. Nowadays, the average GNP of the developed countries is roughly US$20,000, which means a ratio of 612.5 between the two figures. As the ratio may be too big for the decision-maker to perceive correctly, there can naturally be arbitrary choices. The Allais' choice might happen under this situation. And, it actually has nothing to do with any deficiency of the expected utility theory.

3.2 *The obfuscated definition of u(0)*

Another cause of Allais choice yields from the problem statement category is *u(0)*. Intuitively, equation 1 and equation 2 should be equal. But, there exists one controversial point in the calculation process of equation 2; i.e. the *u(0)*. In the deduction process, we make an *assumption as follows:*

$$0.01u(0)=0.9u(0)-0.89u(0) \quad ....(11)$$

What is the meaning of "$u(0)$"? According to our discussions of u(0) in the previous sections, when someone's total wealth suddenly changes to zero, her or his utility will become negative infinity because s/he can not maintain the minimum life necessity. As a result, the formula $0.9u(0)-0.89u(0)$ in equations 11 ends up being meaningless.

To explain it furthermore, the probability of value 0 is 0.9 for problem b. Nobody will regard 0 as the normal total wealth V_N because with V_N equaling to 0, one is unable to live. So, most people will regard the entries in the matrix as windfalls or extra gain in addition to an unspecified normal total wealth V_N on which they depend for their living. Therefore, to calculate the expected utilities, it is more reasonable to add V_N to every entry in the matrix. In that case, the decision becomes the choice of a windfall of lottery of either one million dollars with a probability of 0.11 or 0 with a probability of 0.01, and five million dollars with a probability of 0.10. Except for those who are extremely risk-averse, most people will naturally choose b_2.

So, the trouble is with problem a. In some situations where the choices were $\{a_2, b_2\}$, the testees seem to be over-risk-averse to choose a_1 instead of a_2.

Since the value of choice a_1 is one million dollars for sure or with a probability of 1, a_1 seems to be a normal stable situation. The testee naturally may regard the entry 0 of choice a_2 with a probability of 0.01 as a real or absolute 0, meaning that, if this state of affairs occurs, the testee's wealth, which includes the normal total wealth, will be 0. For all risk-aversion constants $c \geqq 1$, the utility will go to negative infinity when $V = 0$.

Since negative infinity multiplying by 0.01 is still negative infinity, the testee will not choose a_1 at all. Thus, the problem statement can cause Allais choice due to semantic ambiguity, particularly the meaning of $u(0)$. And, it is not a weakness of the expected utility theory at all.

3.3 *General experiment limitation*

Except for the previous factors, Allais choice can have been caused by carelessness of the testees. The carelessness in turn can be a result of the inability of the testee, careless attitude toward a "hypothetical" problem, or time pressure in the experiment to think thoroughly.

4. Deliberative ability of humanity

Bounded rationality of humanity can also be a reason for the Allais' choice. While the previous "beyond-normal-perception" can be said to be an example of bounded rationality of humanity, we proceed with another obvious limitation that can lead to the Allais' choice; i.e. the deliberative ability of humanity. This limitation is not affected by different values perceived by the testees. Moreover, if a testee in the Allais experiment is under an unstable decision situation, Allais' choice is also a possible outcome.

Generally speaking, people can tell which one out of two visible objects they like better by the comparative value, though they do not necessarily know the exact value of them. Even the utility difference to the decision-maker may be very small, there can still be certain preference. However, for situations with undecided probabilities, it will be very difficult for the decision-maker to distinguish two prospects with trivial difference of expected utility value.

From our revised expected utility theory, we know that three factors affect the final utility to the decision-maker; namely, the total current wealth, minimum life necessity requirement and measure of risk aversion. In our simulation in this paper, moreover, as long as s/he has certain risk preference e.g. measure of risk aversion $c>0$, then he will not be able to precisely judge prospect a_1 from a_2, no matter what the total current wealth V_N can be.

To prove it, we define a measurement M for the difference of total expected utilities between prospects a_1 and a_2 as:

$$M = \frac{|Eu(a_1) - Eu(a_2)|}{(Eu(a_1) + Eu(a_2))/2} \text{x}100\% \qquad ...(12)$$

By computer simulation, we can get the following results:

Table VIII shows that the smaller the current wealth is or the more risk averse the decision maker (i.e. the bigger the *c* is), the smaller the difference between prospect a_1 and a_2 is. When the *c* of a certain decision-maker exceeds six, he will not be able to tell the difference between a_1 and a_2 no matter what V_N may be. In such a situation, arbitrary choices very likely will happen, which also include the Allais' choice.

Table VIII: m value - The Simulation Result with V_{0+} Fixed to be equal to 0.2 V_N

N \ c	0.3	0.6	0.99	1.01	1.5	2	5	6
100	18.067%	7.848%	0.721%	0.551%	0.421%	0.200%	0.002%	0.000%
500	18.082%	7.960%	1.006%	0.832%	0.387%	0.199%	0.002%	0.000%
1,000	18.094%	8.035%	1.162%	0.985%	0.361%	0.198%	0.002%	0.000%
5,000	18.139%	8.312%	1.636%	1.454%	0.250%	0.191%	0.002%	0.000%
10,000	18.162%	8.493%	1.909%	1.723%	0.165%	0.183%	0.002%	0.000%
50,000	18.106%	9.086%	2.782%	2.588%	0.201%	0.166%	0.002%	0.000%
100,000	17.882%	9.372%	3.277%	3.081%	0.473%	0.040%	0.002%	0.000%
500,000	15.516%	9.311%	4.387%	4.210%	1.398%	0.376%	0.001%	0.000%
1,000,000	13.068%	8.398%	4.448%	4.298%	1.748%	0.628%	0.001%	0.000%
2,000,000	9.870%	6.782%	3.982%	3.869%	1.840%	0.799%	0.002%	0.000%
4,000,000	6.616%	4.818%	3.076%	3.002%	1.606%	0.796%	0.005%	0.001%
8,000,000	3.991%	3.037%	2.061%	2.019%	1.173%	0.636%	0.007%	0.001%

(the shaded area means K<1)

5. Other causes

Except for the previous three types of causes, there can still be some extra reasons not related to theory, decision-maker, or problem statement.

5.1 *Using other criteria*

While being a widely accepted criterion, the maximum of expected utility is not the only criterion for decision making under risks. When a testee•does not follow the maximum of expected utility principle, Allais' choice tends to happen, too. For example, if an extremely conservative person adopts "criterion of pessimism" to conduct the Allais experiment, he will definitely choose a_1 between the two prospects. Also, if following the prospect theory raised by Kahneman and Tversky (1979), a decision maker may convert the objective probabilities by certain procedure and enlarges the effect of the probability 0.01.

5.2 *Unstable Decision Situation*

In addition to the points discussed above, there is another necessary condition for the expected utility theory to be a valid rational normative decision making model; i.e. the expected utility theory has to be used under a "stable" situation. For instance, if a decision-maker has just encountered major wealth changes, it is very difficult for her or him to gauge different degrees of satisfaction from different alternatives because her or his monetary perception is still very unstable.

Likewise, if a decision maker is facing severe financial threats because of insufficient income or negative net worth, say, if a testee has one million dollars liability when he undergoes the experiment, then her or his primary decision objective may be "survival" instead of "utility maximization." Facing prospects a_1 and a_2, he certainly will choose a_1 to repay the debt immediately. Allais' choice can then happen.

5.3 *Progressive Objectives*

Moreover, the degree of risk-aversion of a decision-maker may change from one situation to another. People have progressive objectives during the process of wealth accumulation. In a certain situation, taking a risk may become necessary to reach a higher status. If the decision-maker gets more satisfaction out of the pursuit of higher status, s/he may become less risk-averse under such situations or may even become risk-prone.

For instance, suppose a decision-maker desperately needs one million dollars for emergency, his priority goal will be to relieve the emergency. To avoid any prospect that can lead to less than one million dollars' gain, he will certainly choose a_1.

5.4 *Spiritual Utility*

Generally speaking, utility in the expected utility theory normally refers to monetary utility only but not other kinds of utility, e.g. spiritual utility.

Some decision-makers incline to take risks. Sometimes the sensuous encouragement from pursuing risk brings them more satisfaction than the monetary reward does. This satisfaction is a kind of spiritual utility. The decision-maker should not be regarded as irrational even though s/he chooses against the maximum expected utility in the monetary sense in exchange of the maximum spiritual utilities.

On the contrary, some people will not "gamble" because of moral or religious reasons. She or he will certainly choose a_1 instead of a_2 between a_1 and a_2, and cause Allais' choice naturally.

6. Conclusion

To conlcude, according to our analysis, subjective recognition is one of the major causes of the Allais choice. Recognition differences lead to different interpretations of a certain problem and cause various decision-making behaviors. No matter whether it is because of the lack of interpretation power of the traditional expected utility theory that the testees do not understand the minimum life necessity requirement V_{0+} and total current wealth V_N well, or it is because of the semantic ambiguity of the Allais experiment that the testees do not understand the problem itself well, it is all related to recognition issues. So, not all the paradoxes are attributed to the incompleteness of the traditional expected utility theory. Also, another factor is the natural limitation of mankind. Under such situation, they cannot meet the requirements of the expected utility theory; but, their choice cannot be said to be irrational.

The paradoxes that arise from the deficiency of the traditional expected utility theory in some empirical study results have been the argument focal points in the utility theory field. Many studies tried different aspects to revise the model and to explain all the phenomena. In utility and its relative fields, some modified or even brand new utility theories have been developed. As Keller (1992) pointed out, most of these revised theories started from the incompatibility between the traditional utility theory and empirical study results and tried to compensate by mathematical models.

In addition to Allais' paradox, there are some experimental results that violate axiomatic basis of the expected utility theory, e.g. the common ratio effect. We will try to address the empirical results and simulations of these different paradoxes in our other studies. Nevertheless, in this paper, we focus our analysis on the causes of the Allais' choice from three main classifications as Table III shows.

By this study, we found that the traditional expected utility theory is inadequate in interpreting all human behaviors. To enhance it, we developed a revised expected utility theory besides digging into the causes of the Allias choice. This revised expected utility theory can explain why Allais' choice can result from the lack of interpretation power of the traditional expected utility theory. It helps to clearly define the meaning of $u(0)$, too. From another point of view, the revised expected utility theory not only can solve the inadequacy problem of the traditional theory, but also provide a prescriptive function. The introduction of the concepts of "total wealth" and "minimum life necessity requirement" improves the interpretation power of the expected utility theory as well as upgrades it into a prescriptive model for decision making under risks.

Endnotes

1. If k is not set to be 1, then we just need to multiply all figures in the basic form of Table IV by k. No matter whether k exists or not, it does not affect the cardinal utility function as an evaluation tool.

2. According to our discussions in the previous sections, the minimum life necessity V_{0+} is adjustable at different total wealth levels. But, we assume all V_{0+} are the same in Allais Paradox. Allais' choice will still exist even if V_{0+} are not the same in Allais Paradox.

3. The computerized spreadsheet used here is Microsoft Excel 7.0 for Windows 95 and the analysis is done by a self defined function and the two-variable analysis table function of the software.

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