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THE DETERMINATION OF TIMING AND TECHNOLOGY LEVEL OF AN ABATEMENT INVESTMENT

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Abstract: The main contribution of this paper is to reveal WHEN firms, in general, invest in anti-pollution equipment and WHAT technology level they adopt. Their responsive behaviour toward the policy parameters such as observation probability, interest rate or penalty, and endogenous factors such as operating cost and emission volume is analyzed in this paper. The results show that an increase in observation probability leads to an increase in technology level and moves the schedule of abatement investment ahead. However, an increase in interest rate results in a decrease in technology level and delays the timing of abatment investment.

Keywords: Abatement, emission standard, technology level.

1. INTRODUCTION

In modern life, a production technology that can assure efficient production without pollution and assures the local environmental quality is being sought. Unfortunately, it has not yet emerged to date. On the contrary, the rapid rise of economic growth and industrialization in developing countries has spoiled the environment vastly in the past years. Due to the low level of existing emission standards, many plants freely discharged their waste including waste water, waste air, and solid waste into the environment without further treatment. To achieve the target level of environmental quality, a policy maker can choose either to levy an effluent fee (the Pigouvian tax) or set up an emission standard and implement it [2, 9]. Most literature assume that the polluter is technically able to control the emission level and morally likes to be honest and follow the statutory regulations and meet the regulatory emission standard. However, due to difficulty in monitoring each firm's emission, the problem of moral hazard arises. To get rid of the polluter's moral hazard, many articles in the literature have presented various incentive schemes in the form of fines or subsidies which are believed to be efficient ways to achieve the policy planner's goal [e.g. 1, 4, 5, 6, 7, 11, 14, 16, 17]. The sharing rule proposed by Holmstrom [6] is based on the condition of a zero-sum game between the principal and the agent. The outcome (payoff) is shared completely by the two parties. Grossman and Hart [5] already criticize Holmstrom's sharing rule and demonstrate that this kind of approach is generally invalid because Holmstrom's utility is difficult to measure. Baker [1] argues that the optimal incentive scheme depends on the relationship between the performance measure and the principal's objective. In these prior studies, it is assumed that the principal is seeking to maximize profit. However, for nonprofit organizations, profit is not the only objective. These incentives proposed by the above-mentioned literature cannot be applied in our case to prevent violations.

Some authors focus on the effect of the observation schedule on pollution reduction [2, 10, 12, 13]. According to Malik [10], the violation problem is partly caused by the difficulty in continuous monitoring and partly by "the absence of a well-developed mechanism for assessing penalties for noncompliance". He uses a stochastic model to demonstrate the effect of observation frequency on pollution discharge. He asserts that "... pollutant discharge is a function of the frequency of monitoring even when discharge is deterministic". The subjective probability of being detected and punished is proposed as a vector of exogenous parameters set by the inspector. The observation schedule function (termed audit probability function by Malik) is allowed to vary across firms. Each polluter's subjective probability of being observed is given and is the function of policy parameters. Bevis and Walker [2] analyzed the effect of observation frequency on the pollution level and concluded that a firm's pollutant emission in a TDP (transferrable discharge permit) market is not affected by the inspector's monitoring frequency while Malik argues that the pollutant discharge is a function of monitoring frequency.

All these papers assume that the policy maker's enforcement ability is fully supported and assured. In the presence of shirking, the polluter will be caught and punished according to the incentive scheme. As the monitoring cost is not free, removal of moral hazard depends on a sufficient budget to support the regulator to push the firms to follow regulations. However, the enforcement of these environmental policies is not taken well in developing or under-developed countries because of insufficient budgets. Most polluters still look around and wait for other competitors' action in the absence of effective imposition of environmental policy. Some briberies of governmental inspectors for less inspections are expected and attempted. Polluters try to cheat government officials by discharging the waste water or flue gas through hidden ducts into the sea or river or the air. Nobody intends to be a pioneer and start abatement investments to avoid the loss of competitiveness. The installation of environmental protection equipment will raise more costs. As a consequence, it becomes an evil penalty to those who do abatement investment when environmental laws cannot be carried out effectively.

The purpose of this paper is to develop a model to analyze the optimal timing for the polluter to start abatement investment and optimal technology level of this abatement investment. It demonstrates the polluter's choice on timing and the technology level of abatement investment under the condition that environmental regulations cannot perform perfectly. Because of the limitation of personnel and measuring technology, continuous monitoring of the polluter's production and operation becomes impossible. Thus, illegal discharge is not easily observed and punished. Many polluters are profitseekers and do not care about the environmental damage. Our model has shown that an increase in monetary fines and reinforcement of inspection in order to increase punishment probability will have a strong impact on entry timing and technology level of the abatement investment.

2. ASSUMPTIONS AND NOTATION

Compared to the productive investment, abatement investment is characterized by:

(1) Negative return: generally, the abatement investment will increase production costs and reduce competitiveness in the market[1]. This characterization makes firms reluctant to install anti-pollution equipment due to cost increases. It becomes a punishment to those firms that have already invested in anti-pollution equipment if the government cannot implement the anti-pollution laws effectively and fairly, because those firms that do not invest in abatement will take advantage of the cost benefits.

(2) Crowded-out effect[2]: due to the given amount of capital in a firm, a portion of capital must be spent on abatement investment. Thus, the portion of productive investments decreases and the production level and operating profit are also reduced.

(3) Difficulty in monitoring: a continuous monitoring system is too costly to install so that the governmental inspector (regulator) cannot observe each firm's emission all the time. In other words, the abatement investment is done and note is taken, but its operation cannot be observed all the time. For example, many polluters in Taiwan install anti-pollution plants and report to the government for tax incentives. After the government's approval, the pollution treatment plant is shut down to save operating costs. Practically speaking, the governmental inspector can only make a sudden visit to the suspected firm and check the emission level randomly. The polluter will be punished by monetary penalty or production shut-down if he is caught violating.

To construct the model, some assumptions are made and described as follows:

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[1] Jorgenson and Wilcoxen [8] present similar viewpoints in their empirical study on environmental regulation and economic growth.

[2] The details are presented in Jorgenson and Wilcoxen's study (p.330) [8].

(1) An expected observation frequency λ is estimated by the firm based on past experience.

(2) Incarceration in becoming a more common tool to implement environmental laws [15], however, civil monetary penalties are assumed to be the only instrument imposed on the violators. Two schemes including constant monetary penalty and variable monetary penalty over time are discussed in this paper.

(3) Firms are seeking for the maximization of profit only. Social norms or moral standards are not chosen as decision making rules of abatement investment. Efforts to reduce emission levels by means of abatement investment are regarded as an outcome of cost-benefit analysis. Tax incentives on abatement are calculated and viewed as a minus term of investment cost.

(4) The regulatory emission standard may be adjusted to be more strict over time continuously and has a linear relationship with time. At time T, a pollution treatment plant with technology level k is invested. Technology level k represents the service life of the plant. The operating of the plant assures emission levels will meet the statutory emission standard at time $T+k$. After time $T+k$, it becomes invalid and must be discarded. The cost of the abatement investment A is a function of emission volume V and regulatory standard m, assumed to be separated into two independent functions with the following properties, i.e.

$$A(m,V) = A(\alpha(T+k),V) = h(T+k)g(V) \quad \text{where } m = \alpha(T+k),$$

$$h'(T+k) > 0, \quad h''(T+k) > 0, \quad h(0) = 0, \quad \lim_{T+k \to 0} h(T+k) > 0$$

(5) During the period of analysis, the emission control technology is assumed to be unchanged and the uncertainty of technology is neglected. In other words, the actual emission is expected to remain constant over the validity period of the pollution treatment plant although the variability of the inputs in quality and quantity may cause fluctuation of the waste emission level.

(6) Continuous running is the only choice for the firm to survive. There is no possibility for the polluter to move his production to other locations or shut down the production to avoid the abatement investment.

Notations:

V	emission volume
t	time
A	amount of abatement investment
k	technology level in terms of validity period
T	timing of abatement investment
f	monetary fine
λ	probability of violation observed and punished

i discount rate
c_1 benefit from waste recycling
c_2 operating cost of abatement investment

3. MODEL CONSTRUCTION

A system analysis describing the benefit and loss of abatement investment by means of cash flow is diagrammed in Fig. 3.1.

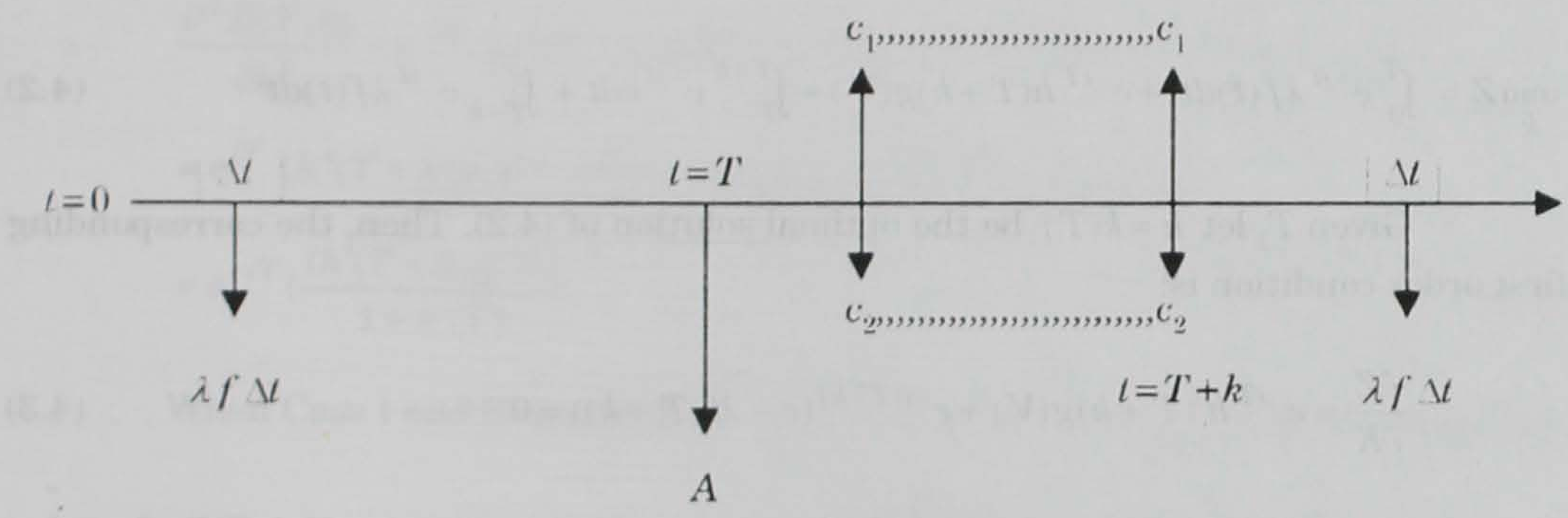

Figure 3.1: Cash flow of a polluter's activities involving abatement investment

Description of the diagram:

Without abatement investment, in time interval $[t.t+\Delta t]$ the polluter will be observed and punished with probability $\lambda\Delta t$. In other words, the probability distribution of the observed and punished event at each time interval is homogeneous and uniform. Before time T, the present value of the monetary fine that the firm is expected to pay amounts to $\int_0^T e^{-it}\lambda f(t)dt$ where f is a given function of t. At time $t=T$, the firm starts its abatement investment, amounting to $A(m.V)=h(T+k)g(V)$. After then, operating cost c_2 for the abatement investment is generated and benefit c_1 on the recycled material or energy is obtained. The operating cost is assumed to be an increasing function of emission volume, i.e. $c_2=c_2(V)$. $c_2'(V)>0$. After time $T+k$, when the validity of the installed plant expireds, a monetary fine is expected again, amounting to $\int_{T+k}^{\infty} e^{-it}\lambda f(t)dt$.

Based on the cash flow, the polluter will determine the optimal timing T and optimal technology level k of the planned investment by minimizing the present value of the net cost Z, i.e.

$$\min_{T,k} Z = \int_0^T e^{-it}\lambda f(t)dt + e^{-iT}h(T+k)g(V) + \int_T^{T+k} e^{-it}c\,dt + \int_{T+k}^{\infty} e^{-it}\lambda f(t)dt \qquad (3.1)$$

where $c=c_2-c_1$ and $c'(V)>0$.

4. THE OPTIMAL SOLUTION WHEN PENALTY IS INCREASING OVER TIME

Problem (3.1) is identical to the following problem:

$$\min_T \min_k Z = \int_0^T e^{-it} \lambda f(t)dt + e^{-iT} h(T+k)g(V) + \int_T^{T+k} e^{-it} c dt + \int_{T+k}^{\omega} e^{-it} \lambda f(t)dt \tag{4.1}$$

First, we start to solve:

$$\min_k Z = \int_0^T e^{-it} \lambda f(t)dt + e^{-iT} h(T+k)g(V) + \int_T^{T+k} e^{-it} c dt + \int_{T+k}^{\omega} e^{-it} \lambda f(t)dt \tag{4.2}$$

Given T, let $k = k(T)$ be the optimal solution of (4.2). Then, the corresponding first order condition is:

$$\therefore \frac{\partial Z}{\partial K} = e^{-iT} h'(T+k)g(V) + e^{-i(T+k)}(c - \lambda f(T+k)) = 0 \tag{4.3}$$

After rearrangement of (4.3), we get

$$h'(T+k)g(V) + e^{ik}(c - \lambda f(T+k)) = 0 \tag{4.4}$$

Rearranging (4.4), we get $e^{ik} = \dfrac{\lambda f(T+k) - c}{h'(T+k)g(V)}$. As RHS (Right Hand Side) of this equation is positive and $h' > 0$, $\lambda f(T+k) - c$ must be greater than zero. The term $\lambda f(T+k) - c$ represents the net benefit of each unit of time when the validity of the invested plant expires. If $\lambda f(T+k) - c$ is negative, no polluter intends to start any efforts on pollution abatement because it is cheaper to pay the monetary penalty than to invest in a pollution treatment plant.

Taking differentiation of (4.4) with respect to T, we get

$$\begin{aligned} k'(T) &= \frac{-h''(T+k)g(V) + e^{-ik} f'(T+k)}{h''(T+k)g(V) + ie^{-ik}(\lambda f(T+k) - c) - e^{-ik}\lambda f'(T+k)} \\ &= \frac{-h''(T+k)g(V) + e^{ik} f'(T+k)}{h''(T+k)g(V) + ih'(T+k)g(V) - e^{-ik}\lambda f'(T+k)} \end{aligned} \tag{4.5}$$

The characteristics of $k(T)$ depend on $h(T+k)$ and $f(T+k)$. As $h(T+k)$ is a monotonically increasing convex function, all cases based on equation (4.5) are discussed below:

Case I: $h''(T+k)g(V) \geq e^{-ik}\lambda f'(T+k) > 0$, then $k'(T) < 0$, and $|k'(T)| < 1$ (C-1)

Case II: $e^{ik}\lambda f'(T+k) > h''(T+k)g(V) \geq 0$ and

$h''(T+k)g(V) + ih'(T+k)g(V) \geq e^{ik}\lambda f'(T+k)$, then $k'(T) > 0$ (C-2

Case III: $h''(T+k)g(V) + ih'(T+k)g(V) < e^{ik}\lambda f'(T+k)$, then $k'(T) < 0$,

and $|k'(T)|>1$ (C-3)

The secondary condition:

$$\frac{\partial^2 Z(T,k)}{\partial k^2} = e^{-iT}\ [h''(T+k)g(V) - ie^{-ik}(c - \lambda f(T+k)) - e^{-ik}f'(T+k)] =$$

$$= e^{iT}\ [h''(T+k)g(V) + ih'(T+k)g(V) - e^{-ik}f'(T+k)] =$$

$$= e^{-iT}\left(\frac{ih'(T+k)g(V)}{1+k'(T)}\right) \tag{4.6}$$

When Case I and II take place, $\frac{\partial^2 Z(T,k)}{\partial k^2} > 0$ and it assures that the minimum value of $Z(T,k(T))$ is attained. For Case III, $\frac{\partial^2 Z(T,k)}{\partial k^2} < 0$. Thus, no optimal point of $k(T)$ exists. In this case, Case III is excluded from our further discussion.

In the second step, we put the optimal value $k(T)$, existing as an implicit form in (4.4), into (4.1) and solve:

$$\min_{T} Z(T,k(T)) \tag{4.7}$$

The corresponding first order condition becomes:

$$0 = \frac{dZ(T,k(T))}{dT} = \frac{\partial Z(T,k(T))}{\partial T} + \frac{\partial Z(T,k(T))}{\partial k}k'(T) =$$

$$= e^{-iT^*}\ [\lambda f(T^*) - c]\ - ie^{-iT^*}h(T^* + k(T^*))g(V) + e^{-iT}h'(T^* + k(T^*))g(V) +$$

$$+ e^{i(T^*+k(T^*))}(c - \lambda f(T^* + k(T^*)) \tag{4.8}$$

Let $k^* = k(T^*)$. Putting (4.4) into the above equations, we get

$$h(T^* + k^*) = \frac{\lambda f(T^*) - c}{ig(V)} \tag{4.9}$$

Equation (4.9) represents the relationship between the optimal point T^* and k^*, which is interpreted as: the interest rate of the abatement investment must be equal to the benefit, equivalent to the savings of the monetary penalty plus recycling profit minus operating cost.

Combining (4.9) and (4.4), we get

$$\exp[i(h^{-1}(\frac{\lambda f(T^*)-c}{ig(V)})-T^*)] = \frac{\lambda f(h^{-1}(\frac{\lambda f(T^*)-c}{ig(V)})-c}{g(V)h'(h^{-1}(\frac{\lambda f(T^*)-c}{ig(V)}))} \quad (4.10)$$

T^* is obtained by equation (4.10) and then k^* is obtained by equation (4.9). By (4.8), the secondary condition of problem (4.7) is given by:

$$\frac{d^2 Z(T,k(T))}{dT^2} =$$
$$= e^{-iT}\ [-i(\lambda f(T) - c - i^2 h(T+k)g(V) + \lambda f'(T) - ih'(T+k)g(V)(1+k'(T)]$$

Putting equation (4.9) into the above equation, we get

$$\frac{d^2 Z(T,k(T))}{dT^2} = e^{-iT}\ [\lambda f'(T) - ih'(T+k)g(V)(1+k'(T)] \quad (4.11)$$

Case A: On condition C-I, if $\lambda f'(T^*) < ih'(T^*+k^*)g(V)$, then

$$\frac{d^2 Z(T^*,k^*)}{dT^2} = e^{-iT^*}[\lambda f'(T^*) - ih'(T^*+k^*)g(V)(1+k'(T^*)] <$$
$$< e^{-iT^*} ih'(T^*+k^*)g(V)k'(T^*) > 0$$

This condition cannot identify the optimality of the solution to be an optimal point or a saddle point.

Case B: On condition C-I, if $\lambda f'(T^*) > ih'(T^*+k^*)g(V)$, then

$$\frac{d^2 Z(T^*,k^*)}{dT^2} = e^{-iT^*}\ [\lambda f'(T^*) - ih'(T^*+k^*)g(V)(1+k'(T^*)] >$$
$$> -e^{-iT^*}\ ih'(T+k^*)g(V)k'(T) > 0,$$

therefore, the optimal point is detected.

Case C: On condition C-II if $\lambda f'(T^*) < ih'(T^*+k^*)g(V)$, then

$$\frac{d^2 Z(T,k^*)}{dT^2} = e^{-iT^*}\ [\lambda f'(T^*) - ih'(T^*+k^*)g(V)(1+k'(T^*)] <$$
$$< -e^{-iT^*}\ ih'(T^*+k^*)g(V)k'(T^*) < 0,$$

therefore, no minimum value can be obtained. (T^*, k^*) is a saddle point.

Case D: On condition C-II if $\lambda f'(T^*) > ih'(T^* + k^*)g(V)$, then

$$\frac{d^2 Z(T,k^*)}{dT^2} = e^{-iT^*}\ [\lambda f'(T^*) - ih'(T^* + k^*)g(V)(1 + k'(T^*)] >$$

$$> -e^{-iT^*}\ ih'(T^* + k^*)g(V)k'(T^*) < 0\,.$$

This condition cannot identify the optimality of the solution to be an optimal point or saddle point.

The above results show that the solutions of equation (4.9) for Case C are saddle points and for Case A, and D are of undetermined status. On condition of Case B, the optimal point is detected.

5. RESULTS AND DISCUSSION

The results and discussion are organized in the following list:

1) Rearranging (4.8), we get $e^{ik^*} = \dfrac{\lambda f(T^* + k^*) - c}{(\lambda f(T^*) - c) - ih(T^* + k^*)g(V) + h'(T^* + k^*)g(V)}$.

As the terms of both e^{ik^*} and $\lambda f(T^* + k^*) - c$ are positive $\lambda f(T^*) - c - ih(T^* + k^*)g(V) +$ $+ h'(T^* + k^*)g(V)$ must be greater than zero. Since $\lambda f(T^*) - c - ih(T^* + k^*)g(V) +$ $+ h'(T^* + k^*)g(V) > 0$, then $\lambda f(T^*) - c + h'(T^* + k^*)g(V) > ih(T^* + k^*)g(V)$, which is interpreted as: the benefit in each time plus the price inflation rate of the invested plant must be more than the interest of the invested plant. When the probability of punishment is too low and the monetary fine is too low compared to the amount of abatement investment, no polluter will devote efforts to improve pollution reduction. As c is a technical factor determined by the technology level and can not be controlled by the government, the policy planner can use three ways: (1) increase the probability of punishment, (2) increase the monetary penalty and (3) decrease the discount rate, to pressure the polluter to do more for pollution reduction.

2) To make sure that the polluter will absolutely start abatement investment in the near future, $Z(T^*,k^*) < Z(0,0)$ and $Z(T^*,k^*) < \lim_{T^* \to \infty} Z(T^*,k^*)$ become necessary conditions which implies $ih(T^* + k^*)g(V) < (\lambda f(T^* + k^*) - c)(1 - e^{-i(T^* + k^*)})$, because of $Z(T^*,k^*) - Z(0,0) = h(T^*,k^*)g(V)e^{-iT^*} + \int_{T^*}^{T^*+k^*} (c - \lambda f(t))dt < 0$, $\int_{T^*}^{T^*+k^*} (c - \lambda f(T^* + k^*))dt < \int_{T^*}^{T^*+k^*} (c - \lambda f(t))dt$, and $\lim_{T^* \to \infty} Z(T^*,k^*) < Z(0,0)$.

3) When T reaches a critical point, say T_c, no polluter will intend to do abatement investment, i.e. $k^*(T_c)=0$. By (4.4), we get

$$h'(T_c)g(V)=(\lambda f(T_c)-c)$$

As optimal point T^* must be greather than zero and $T_c > T^*$, then $T_c > 0$ is a necessary condition to get optimal point $k(T)$. A longer critical point period of abatement investment offers more choice regarding investment timing and technology level for the polluter. Taking the differentiation of (4.15) with respect to λ, c, and V respectively, we get:

$$\frac{dT_c}{d\lambda}=\frac{f(T_c)}{g(V)h''(T_c)-\lambda f'(T_c)}$$

$$\frac{dT_c}{dc}=\frac{f(T_c)}{\lambda f'(T_c)-g(V)h''(T_c)}$$

$$\frac{dT_c}{dV}=\frac{g(V)h'(T_c)+c'(V)}{\lambda f'(T_c)-g(V)h''(T_c)}$$

The above expressions show that the change in direction of the critical point of abatement investment with respect to the change of punishment probability, net operating cost, and emission volume depends on the characteristics of $k(T)$, which is determined by the curve of $h(T+k)$ and $f(T+k)$. As optimal solutions exist only on condition of Case D, which was discussed previously, the constraints of $h''(T+k)g(V)\geq e^{ik}\lambda f'(T+k)$ serve as a criteria of the change in direction of the critical point of abatement investment. Since T_c is defined as the timing of abatement investment when $k(T)=0$. Therefore, $h''(T_c)g(V)\geq e^{ik}\lambda f'(T_c)$, and then we get the following result:

$$\frac{dT_c}{d\lambda}>0,\quad \frac{dT_c}{dc}<0,\quad \frac{dT_c}{dV}<0:$$

These expressions demonstrate that an increase in punishment probability leads to an increase in the critical point of abatement investment, and an increase in net operating cost, and emission volume will result in a decrease in the critical point of abatement investment, and vice versa.

4) Taking the differentiation of equation (4.9) with respect to λ, c, i and V respectivaly, we get:

$$\frac{d(T^*+k^*)}{d\lambda}=\frac{f(T^*)(1+k^*)}{ig(V)h'(T^*+k^*)(1+k^*)-\lambda f'(T^*)}$$

$$\frac{d(T^*)}{d\lambda}=\frac{f(T^*)}{ig(V)h'(T^*+k^*)(1+k^*)-\lambda f'(T^*)}$$

$$\frac{d(k^*)}{d\lambda}=\frac{f(T^*)k^*}{ig(V)h'(T^*+k^*)(1+k^*)-\lambda f'(T^*)}$$

$$\frac{d(T^*+k^*)}{dc}=\frac{(1+k^*)}{\lambda f'(T^*)-ig(V)h'(T^*+k^*)(1+k^*)}$$

$$\frac{d(T^*)}{dc}=\frac{1}{\lambda f'(T^*)-ig(V)h'(T^*+k^*)(1+k^*)}$$

$$\frac{d(k^*)}{dc}=\frac{k^*}{\lambda f'(T^*)-ig(V)h'(T^*+k^*)(1+k^*)}$$

$$\frac{d(T^*+k^*)}{dV}=\frac{ih(T^*+k^*)g'(V)(1+k^*)+c'(V)}{\lambda f'(T^*)-ig(V)h'(T^*+k^*)(1+k^*)}$$

$$\frac{d(T^*)}{dV}=\frac{ih(T^*+k^*)g'(V)(1+k^*)+c'(V)}{[\lambda f'(T^*)-ig(V)h'(T^*+k^*)(1+k^*)](1+k^*)}$$

$$\frac{d(k^*)}{dV}=\frac{[ih(T^*+k^*)g'(V)(1+k^*)+c'(V)]k^*}{[\lambda f'(T^*)-ig(V)h'(T^*+k^*)(1+k^*)](1+k^*)}$$

$$\frac{d(T^*+k^*)}{di}=\frac{g(V)h(T^*+k^*)(1+k^*)}{\lambda f'(T^*)-ig(V)h'(T^*+k^*)(1+k^*)}$$

$$\frac{d(T^*)}{di}=\frac{g(V)h(T^*+k^*)}{\lambda f'(T^*)-ig(V)h'(T^*+k^*)(1+k^*)}$$

$$\frac{d(k^*)}{di}=\frac{g(V)h(T^*+k^*)k^*}{\lambda f'(T^*)-ig(V)h'(T^*+k^*)(1+k^*)}$$

Similar to the discussion in 2), the constraint of Case A, B, C and D will be put on the above equations as a criteria to judge direction change with respect to parameter change.

In Case A, B, and D, when the optimal point is obtained, then

(1) $\frac{d(T^* + k^*)}{d\lambda} < 0$, $\frac{d(T^*)}{d\lambda} < 0$, $\frac{dk^*}{d\lambda} > 0$: an increase in punishment probability will decrease the overall technology level, the timing of abatement investment and increase the technology level of the invested plant, and vice versa.

(2) $\frac{d(T^* + k^*)}{dc} > 0$, $\frac{d(T^*)}{dc} > 0$, $\frac{dk^*}{dc} < 0$: an increase in the net operating cost will increase the overall technology level and the timing of abatement investment, and decrease the technology level of the invested plant, and vice versa.

(3) $\frac{d(T^* + k^*)}{dV} > 0$, $\frac{d(T^*)}{dV} > 0$, $\frac{dk^*}{dV} < 0$: an increase in emission volume will increase the overall technology level and the timing of abatement investment, and decrease the technology level of the invested plant, and vice versa.

(4) $\frac{d(T^* + k^*)}{di} > 0$, $\frac{d(T^*)}{di} > 0$, $\frac{dk^*}{di} < 0$: an increase in interest rate will increase the overall technology level and the timing of abatement investment, and decrease the technology level of the invested plant, and vice versa.

The above results are expressed in Table 5.1.

Table 5.1: The change direction of the overall technology level, timing, technology level with respect to parameters.

	overall technology level	abatement investment timing	technology level
λ : punishment probability	−	−	+
c : net operating cost	+	+	−
V : emission volume	+	+	−
i : interest rate	+	+	−

In Case A, C, and D, when a saddle point is obtained, then the minimum value will take place at $T = 0$. The solution for these cases will be obtained in Section 6.

6. THE OPTIMAL SOLUTION THEN PENALTY IS CONSTANT

When $f'(t) = 0$, equation (4.5) becomes $k'(T) = \frac{-h''(T+k)g(V)}{h''(T+k)g(V) + ih'(T+k)g(V)}$.

As $h''(T+k) > 0$, $h'(T+k) > 0$, so $k'(T) < 0$, and $|k'(T)| < 1$ which leads to $\frac{d^2 Z(T, k(T))}{dT^2} = e^{-iT}[-ih'(T+k)g(V)(1 + k'(T)] < 0$. In this case, we conclude that the solution of equation (4.9) is a saddle point when $f'(t)=0$. The solution for the saddle point for Cases A, C, D of increasing penalty scheme or constant penalty scheme, will exist at $T^*=0$. Therefore, problem (3.1) is identical to

$$\min_k Z = h(k)g(V) + \int_0^k e^{-it} c\,dt + \int_k^{\infty} e^{-it} \lambda f(t)\,dt \tag{6.1}$$

The solution of (6.1) is obtained:

$$e^{ik^*} = \frac{\lambda f - c}{h'(k^*)g(V)}$$

Secondary condition: $\dfrac{d^2 Z(0,k)}{dT^2} = h''(k)g(V) + i(\lambda f - c)e^{-ik} > 0$. This proves the existence of a minimum value.

Taking the differentiation of equation (6.2) with respect to λ, c, i and V respectively, we get

$$\frac{dk^*}{d\lambda} = \frac{f}{e^{ik^*}(ih'(k^*) + h''(k^*))g(V)}$$

$$\frac{dk^*}{dc} = \frac{-1}{e^{-ik^*}(ih'(k^*) + h''(k^*))g(V)}$$

$$\frac{dk^*}{dV} = \frac{-(e^{ik^*}(ih'(k^*)g(V) + c'(V))}{e^{-ik^*}(ih'(k^*) + h''(k^*))g(V)}$$

$$\frac{dk^*}{di} = \frac{-h'(k^*)k^*}{ih'(k^*) + h''(k^*)}$$

The above equations demonstrate that an increase in punishment probability will increase the technology level of the invested plant, and an increase in net operating cost, emission volume, and interest rate will decrease the technology level of the invested plant. The above results are expressed in Table 6.1.

Table 6.1: Change in direction of the technology level with respect to parameters in case of constant penalty.

	technology level
λ : punishment probability	+
c : net operating cost	–
V : emission volume	–
I : interest rate	–

Compared to the optimal solution for the scheme of increasing penalty, these result show the change in direction of technology with respect to parameters is identical. In the meantime, to make sure that the firm will absolutely start the abatement investment in the future, i.e. $k^* > 0$, $Z(0,k^*) < Z(0,0)$ becomes a necessary condition which implies $ih(k^*)g(V) << (\lambda f(k^*) - c)(1 - e^{-ik^*})$.

7. CONCLUSION

The foregoing discussions lead to following conclusions:

(1) In both cases of constant monetary penalty and increase monetary penalty, two factors serve as the necessary conditions for the determination of the entrance timing and technology level of the abatement investment: $\lambda f(T + k^*) - c > 0$, and $ih(T^* + k^*)g(V) < (\lambda f(T^* + k^*) - c)(1 - e^{-i(T^* + k^*)})$. For the scheme of increasing monetary penalty which assures the existence of an optimal point, the condition $\lambda f(T^*) - c + h'(T^* + k^*)g(V) > ih(T^* + k^*)g(V)$ is added as a constraint to solve the solution. The three factors serve as good criteria for the policy planner to set up the penalty scheme.

(2) From Table 5.1, an increase λ will shorten the timing of abatement investment. Policy planners can take advantage of this feature to put pressure upon firms to start abatement investment earlier by reinforcing the observation frequency to increase punishment probability.

(3) In order to push the firms to start abatement investment earlier, a policy planner can design a penalty scheme by reducing the increasing rate of penalty.

On condition of Case B, the optimal solution exists. When $f'(t)$ decreases, Case B will become Case A in which either an optimal point or a saddle point exist. A further reduction in the penalty increasing rate will lead to a saddle point solution which implies the immediate start-up of abatement investment will be beneficial to polluters.

On condition of Case D which may be a saddle point or an optimal point, a continual decrease in $f'(t)$ will change Case D into Case C which assures existence of a saddle point.

(4) An the increase in emission volume results in a delay in abatement investment. Since high emission volume will result in high accumulated pollutants in the environment and damage environmental quality, a penalty based on emission level (concentration) seems to be not efficient enough to improve the environmental situation. A further study is needed to extend the analysis to a comparison between the penalty based on emission level, and on both emission level and emission volume.

In this paper, to simplify our analysis, we rejected the possibility of the modification of installed plant to meet up-revised regulatory standards. Actually, many industries pay constant attention to new technology progress and may modify their existing plants to meet the current standard or future trends. Future study can be focused on the effect of repeated abatement investment (modification of an existing installed anti-pollution equipment) on pollution reduction.

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