ELSEVIER

Int. J. Production Economics 67 (2000) 269–277

international journal of production economics

www.elsevier.com/locate/dsw

An extended newsboy problem with shortage-level constraints

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Received 18 December 1998; accepted 15 February 2000

Abstract

It is only an order quantity which is decided for a spot selling time in the classical newsboy problem. Both the purchase timing and the time-variant variance of the demand are neglected. When the vendor gives a price discount for early purchase, the buyer purchases the quantities of price discount at the cost of forecast bias. The buyer has to forecast the demand early before purchase ahead of schedule, but it may increase the variance of the forecasted demand, which is a forecast bias. This time-variant variance of the demand is embedded into the model. When the average shortage-level is specified to an upper limit, both the purchase timing and the order quantity are simultaneously considered as the decision variables of the extended newsboy problem in this paper. The resultant outcomes could apply to some cases in the futures contracts. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Inventory; Newsboy problem; Forecast bias

1. Introduction

We may consider the classical newsboy problem as a scenario on the basis of demand forecasted at time t ahead of time T, and therefore it only decides the order quantity on the tradeoff between over- and under-stocking to minimize cost (or maximize profit).

Since the classical newsboy problem has only one decision variable in it, it is oversimplified. We improved it to be an extended one with two decision variables: (1) when to order (to decide the purchase timing); (2) how much to order (to decide the order quantity). The purchase timing was necessarily considered because it affects not only the purchase cost but also the accuracy of the forecasted demand. The later the purchase timing is, i.e., near the delivering time, the more accurate it is, i.e., the less the variance of the forecasted demand is. The larger variance of the forecasted demand results from forecasting the demand earlier. This characteristic exists mostly in the future contracts. It is different about the assumptions and applications between this paper and the classical newsboy problem, which are summarized as follows:

(1) The classical newsboy problem assumed that the purchase cost per unit is fixed. We assumed that the vendor gives more price discount to stimulate the buyer to purchase earlier for decreasing the inventory level. Based on the different purchase discount at different purchase timing, the model is completely different from

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0925-5273/00/$ - see front matter
PII: S0925-5273(00)00025-6

the other models in the literature [1–4] which is formulated contrarily on the different sales discount at different sales quantity. Although Eeckhout et al. [5] proposed that buyer can reorder the inventory by an expensive purchase price, it was different from this paper which is only one order.

The assumption in price discount is different from the applications in the literature of the classical newsboy problem. For example, Khouja [6] formulated a model on the effects of sale price discount on the demand. Contrarily, we formulated our model on the effects of purchase price discount at different purchase timing on purchase cost.

(2) There is a difference in conducting under-stocking between the classical newsboy problem and this paper. Since the shortage cost includes penalty cost specified in the contract, discount asked by customers for shortage of goods; customers asking some discount for inconvenience; imputation loss; and loss from losing potential customers, it is really difficult to measure shortage cost by experiences. Therefore, some researchers like Aardal et al. [7], Moon and Choi [8] formulated their inventory models with a specified service-level constraint to deal with under-stocking. To widen the application of this model, we formulated the inventory model with a specified average shortage-level constraint.

(3) It was assumed that the variance of the demand was fixed in the literature of the newsboy problem [9–12] namely, the variance is invariant with time. When the buyer forecasts the demand, the nearer the time between delivery and purchase the less the variance of the forecasted demand is; i.e., the less the forecast bias of the buyer is. The effects of the forecast bias on the expected inventory level and the expected shortage of quantity were simultaneously considered in the model of this paper. This tract has not been adopted by other literature relating to the newsboy problem.

The model in this paper could be used in making decisions for some kinds of the futures contracts. It is also suitable for the wholesaler to decide how many and when to buy the seasonal agriculture products. When to order roses and how many roses to order on the eve of Valentine's Day, is a typical example in use.

2. The notation and assumptions

2.1. Notation

2.1.1. Parameters

- T delivering timing
- δ purchase discount per unit ahead of one unit time
- c_t purchase cost per unit at time t; in which c_t is a linear function of $(T-t)$ satisfying $c_T = c$, i.e., $c_t = c - \delta(T-t)$ $0 \leqq t \leqq T$
- v salvage value per unit unsold at time T
- h unit holding cost per time

2.1.2. Function

$\phi(z) = 1/\sqrt{2\pi}\mathrm{e}^{-z^2/2}$ is a standard normal probability density function

$\Phi(z) = \int_{-\infty}^{z} \phi(x)\,\mathrm{d}x$ is a standard normal distribution function

X_t the demand within period $[0, T]$ which is forecasted at the purchase timing t, satisfying $E(X_t) = \mu$, $\mathrm{Var}(X_t) = \mathrm{Var}$ (demand within period $[0, T]$ forecasted at time t) $= (((T-t)/T)\sigma)^2$.

2.1.3. Decision variable

- t purchase timing, $0 \leqslant t \leqslant T$
- q order quantity that has been decided by the buyer at time t

2.1.4. Operator

$[x]^+ = \max\{0, x\}$.

2.2. Assumptions

(1) When a price discount for early purchase is offered by the vendor within period $[0, T]$, the buyer has to decide the purchase timing t at time 0 and then decide the order quantity q.
(2) The buyer allows his customer to preorder products. The customer's preordered products have to be sent to the customer by the buyer at time T in order to avoid losing the opportunity for the sale.
(3) The consumer's preorder timing is evenly distributed within period $[0, T]$. Hence, once the purchase timing for the buyer has been decided at time t, the mean of the demand within period $[0, T]$ forecasted at time t is μ and the variance is $[\sigma((T-t)/T)]^2$. As to decision time point 0, the demand within period $[0, t]$ is a random variable (with mean $\mu(t/T)$ and variance $[\sigma(t/T)]^2$); Yet, as to the decision time point t, the demand within period $[0, t]$ is a deterministic value (an observable historical data).

3. The proposed model

To maintain the sales stable, the vendor always sells products on a price discount to stimulate the buyer to buy ahead of schedule. Since the purchase and holding costs for (t, q) minus the ones for $(t - \Delta t, q)$ equal to

$$[c_t q + h(T-t)q] - [c_{t-\Delta t} q + h(T - t + \Delta t)q] = (\delta - h)q\Delta t, \quad \forall q,$$

therefore, we could assume $\delta > h$; otherwise, it makes this model same as the classical newsboy problem and the optimal purchase timing t^* must be T.

Let the order quantity q be decided by the buyer at time t. If order quantity is greater than the demand, the expected salvage value of unsold products would be $v[q - X_t]^+$. If order quantity is less than the demand, the expected shortage of quantity would be $E[X_t - q]^+$. If the average shortage-level, the ratio of expected shortage of quantity to the expected demand quantity, is specified to an upper limit $\beta (0 \leqq \beta \leqq 1)$ by the buyer, then

$$\frac{E[X_t - q]^+}{E[X_t]} = \frac{\int_{T(q-\mu)/\sigma(T-t)}^{\infty} (\mu + x\sigma(1 - t/T) - q)\phi(x)\,\mathrm{d}x}{\mu} \leqslant \beta, \tag{1}$$

would be the constraint of the model. Under the constraint of average shortage-level, if the buyer decides the inventory policy (t, q) to minimize the expected total cost $L(t, q)$, the mathematical model could be formulated as:

Model (I):

$$\operatorname*{Min}_{(t,q)} \; L(t, q) = (c - \delta(T-t))q + h(T-t)q - v\int_{-\infty}^{T(q-\mu)/\sigma(T-t)} \left(q - x\sigma\left(1 - \frac{t}{T}\right) - \mu\right)\phi(x)\,\mathrm{d}x, \tag{2}$$

$$\text{s. t.} \quad \int_{T(q-\mu)/\sigma(T-t)}^{\infty} (\mu + x\sigma(1 - t/T) - q)\phi(x)\,\mathrm{d}x \leqslant \beta\mu, \qquad 0 \leqslant t \leqslant T, \quad q \geqslant 0, \tag{3}$$

in which the first item of $L(t, q)$ is purchase cost, the second is holding cost, and the third is expected salvage value of unsold products. The objective function of model (I) does not include setup cost in it because the setup cost included in the expected total cost does not affect the optimal solution of the model.

Let the optimal solution of the model (I) exist, and let

(t^*, q^*) be the optimal solution of the model (I). (4)

Hereafter, we could assume

$$\frac{c - T(\delta - h)}{v} \geqslant 1. \tag{5}$$

First, if $[c - T(\delta - h)]/v < 1$ holds, it implies that the salvage value v per unit unsold at time T is more than its purchase and holding cost $c - T(\delta - h)$, which was purchased at time 0. Hence, the more quantity the purchaser orders at time 0, the more profit he earns. It makes the optimal solution of model (I) nonexistent. By (2) and (5), the following inequality holds:

$$\frac{\partial L(t, q)}{\partial q} = \frac{c - (T - t)(\delta - h)}{v} - \Phi\left(\frac{T(q - \mu)}{(T - t)\sigma}\right) > \frac{c - T(\delta - h)}{v} - 1 \geqslant 0, \quad \forall t \in [0, T]. \tag{6}$$

Additionally, it is obvious that the left item, $E[X_t - q]^+$, of (3) is a decreasing function of q for a fixed t. Since $L(t, q)$ can be proved to be an increasing function of q by (5) and (6), model (I) is equivalent to model (II).

Model (*II*):

$$\operatorname*{Min}_{(t,q)} \quad L(t, q) = (c - (\delta - h)(T - t))q - v\int_{-\infty}^{T(q-\mu)/\sigma(T-t)} \left(q - x\sigma\left(1 - \frac{t}{T}\right) - \mu\right)\phi(x)\,\mathrm{d}x \tag{7}$$

$$\text{s. t.} \quad \int_{T(q-\mu)/\sigma(T-t)}^{\infty} \left(\mu + x\sigma\left(1 - \frac{t}{T}\right) - q\right)\phi(x)\,\mathrm{d}x = \beta\mu, \qquad 0 \leqslant t \leqslant T,\ q \geqslant 0. \tag{8}$$

Let

$$z = \frac{(q - \mu)}{\sigma(1 - t/T)} = \frac{T(q - \mu)}{\sigma(T - t)}. \tag{9}$$

Model (II) can be formulated as follows:

Model (*III*):

$$\operatorname*{Min}_{(t,z)} \quad L(t, z) = (c - (\delta - h)(T - t))\left(z\,\sigma\left(1 - \frac{t}{T}\right) + \mu\right) - v\,\sigma\left(1 - \frac{t}{T}\right)\int_{-\infty}^{z} (z - x)\phi(x)\,\mathrm{d}x \tag{10}$$

$$\text{s. t.} \quad -z\sigma\left(1 - \frac{t}{T}\right)[1 - \Phi(z)] + \sigma\left(1 - \frac{t}{T}\right)\phi(z) = \beta\mu, \qquad 0 \leqslant t \leqslant T. \tag{11}$$

4. The optimal solution

Since $\phi(z) = (1/\sqrt{2\pi})\mathrm{e}^{-z^2/2}$,

$$\phi'(z) = \frac{\mathrm{d}}{\mathrm{d}z}\left[\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-z^2/2}\right] = -z\phi(z). \tag{12}$$

Differentiating (11) with respect to t and using (12), we can get

$$\left(-z'(t)\sigma\left(1 - \frac{t}{T}\right) + z(t)\sigma\frac{1}{T}\right)(1 - \Phi(z(t))) - \sigma\frac{1}{T}\phi(z(t)) = 0. \tag{13}$$

Let

$$H(z) = \frac{\phi(z)}{1-\Phi(z)}, \tag{14}$$

we get

$$H'(z) = H(z)(H(z)-z) > 0, \ \forall z. \tag{15}$$

Arranging (13) and utilizing (15)

$$(T-t)z'(t) = z(t) - H(z) = -\frac{H'(z)}{H(z)} < 0 \tag{16}$$

holds.

Let (t^*, z^*), $z^* = z(t^*)$, be the optimal solution of model (III). Differentiating (10) with respect to t and using (16), the following equation holds as $t = t^*$:

$$\begin{aligned}
0 &= \frac{\mathrm{d}L(t, z(t))}{\mathrm{d}t} \\
&= \left[(\delta-h)\left(z\sigma\left(1-\frac{t}{T}\right)+\mu\right) - (c-(\delta-h)(T-t))\frac{z\sigma}{T} + \frac{v\sigma}{T}\int_{-\infty}^{z}(z-x)\phi(x)\,\mathrm{d}x\right] \\
&\quad + \left[(c-(\delta-h)(T-t))\sigma\left(1-\frac{t}{T}\right) - v\sigma\left(1-\frac{t}{T}\right)\int_{-\infty}^{z}\phi(x)\,\mathrm{d}x\right]\frac{z-H}{T-t} \\
&= \frac{1}{T}\left\{(\delta-h)(z\sigma(T-t)+T\mu) - (c-(\delta-h)(T-t))\sigma H + v\sigma\int_{-\infty}^{z}(H-x)\phi(x)\,\mathrm{d}x\right\}
\end{aligned} \tag{17}$$

and the following inequality holds as $t = t^*$:

$$\begin{aligned}
0 &\leqslant \frac{\mathrm{d}^2L(t, z(t))}{\mathrm{d}t^2} \\
&= \frac{\sigma}{T}\begin{bmatrix} -(\delta-h)(z'(T-t)-z) - H'z'(c-(\delta-h)(T-t)) \\ -H(\delta-h) - v(z-H)\phi(z)z' + vH'z'\Phi(z)\end{bmatrix} \quad \text{by (15) and (16)} \\
&= \frac{\sigma}{T}\left\{\begin{matrix} -(\delta-h)H - \dfrac{H(H-z)(z-H)}{T-t}(c-(\delta-h)(T-t)) - (\delta-h)H \\ -\dfrac{v(z-H)^2\phi(z)}{T-t} + \dfrac{vH(H-z)(z-H)\Phi(z)}{T-t}\end{matrix}\right\} \\
&= \frac{\sigma H}{T(T-t)}\left\{\begin{matrix} -2(\delta-h)(T-t) + (H-z)^2(c-(\delta-h)(T-t)) - \\ (H-z)^2\left(\dfrac{\phi(z)}{H}+\Phi(z)\right)v\end{matrix}\right\} \quad \text{by (14)} \\
&= \frac{\sigma H}{T(T-t)}[(H-z)^2(c-(\delta-h)(T-t)-v) - 2(\delta-h)(T-t)].
\end{aligned} \tag{18}$$

Using (11) and (17) together, we get t^* and z^*. By (9), the optimal solution q^* can be presented as

$$q^* = \sigma\left(1-\frac{t^*}{T}\right)z^* + \mu. \tag{19}$$

5. Example and sensitivity analysis

Example 1. Supposed $\mu = 10\,000$ units, $\sigma = 2000$ units,$T = 60$ days, $h = 1.2$ dollars/per day, $\delta = 1.5$ dollars/per day, $c = 100$ dollars/per unit, $v = 20$ dollars/per unit, $s = 120$ dollars/per unit, $\beta = 0.05$. Using (11), (17) and Maple V software, the optimal purchase timing t^* is 5.15 and then using (9) the optimal order quantity is 11 828 units.

5.1. The effect of changing the standard deviation σ of the demand

The z, $z = z(t, \sigma)$, in (11) is variant to (t, σ). Differentiating (11) with respect to σ and using (12) and (15),

$$\frac{\partial z}{\partial \sigma} = \frac{H(z) - z}{\sigma} > 0 \quad \forall (t, \sigma) \tag{20}$$

holds.

Differentiate (17) with respect to σ and it becomes

$$\begin{aligned} &T\frac{\partial}{\partial \sigma}\left(\frac{\partial L(t, z(t, \sigma))}{\partial t}\right) \\ &= \frac{\partial z}{\partial \sigma}\{\sigma(\delta - h)(T - t) - \sigma H'[c - (\delta - h)(T - t)] + v\sigma(H - z)\phi(z) + v\sigma H'\Phi(z)\} \\ &\quad + z(\delta - h)(T - t) - H(c - (\delta - h)(T - t)) + v\left[H\Phi(z) - \int_{-\infty}^{z} x\phi(x)\mathrm{d}x\right] \quad \text{by (12), (15) and (20)} \\ &= (H - z)\{(\delta - h)(T - t) - H(H - z)(c - (\delta - h)(T - t)) + v(H - z)[\phi(z) + H\Phi(z)]\} \\ &\quad + z(\delta - h)\,(T - t) - H[c - (\delta - h)(T - t)] + v[H\Phi(z) + \phi(z)] \quad \text{by (14)} \\ &= \; - H\{(H - z)^2[c - (\delta - h)(T - t) - v] - 2(\delta - h)(T - t) + (c - v)\}. \end{aligned} \tag{21}$$

Let $\dot{L}(t, \sigma) = (\partial/\partial t)L(t, z(t, \sigma))$. By (17),

$$0 = \frac{\mathrm{d}}{\mathrm{d}\sigma}\dot{L}(t^*(\sigma), \sigma) = \frac{\partial \dot{L}(t^*, \sigma)}{\partial t}\frac{dt^*}{d\sigma} + \frac{\partial \dot{L}(t^*, \sigma)}{\partial \sigma},$$

hence,

$$\begin{aligned} \frac{\mathrm{d}t^*(\sigma)}{\mathrm{d}\sigma} &= -\frac{(\partial/\partial\sigma)\dot{L}(t^*, \sigma)}{(\partial/\partial t)\dot{L}(t^*, \sigma)} \quad \text{let } H^* = H(z^*) \text{ and by (18) and (21)} \\ &= \frac{T - t^*}{\sigma} \cdot \frac{\{(H^* - z^*)^2(c - (T - t^*)(\delta - h) - v) - 2(\delta - h)(T - t^*) + (c - v)\}}{\{(H^* - z^*)^2(c - (T - t^*)(\delta - h) - v) - 2(\delta - h)(T - t^*)\}} \\ &= \frac{T - t^*}{\sigma}\left\{1 + \frac{1}{(H^* - z^*)^2} \cdot \frac{c - v}{(c - v) - (1 + 2(H^* - z^*)^{-2})(T - t^*)(\delta - h)}\right\} \\ &\geqslant \frac{T - t^*}{\sigma}\left[1 + \frac{1}{(H^* - z^*)^2}\right] \geqslant 0, \end{aligned} \tag{22}$$

i.e., if the standard deviation of the demand increases, the buyer must delay the purchase timing.

Differentiate (19) with respect to σ and it becomes

$$\frac{dq^*}{d\sigma} = \left(1 - \frac{t^*}{T}\right)z^* - \frac{\sigma z}{T}\frac{dt}{d\sigma} + \sigma\left(1 - \frac{t^*}{T}\right)\left[\frac{\partial z^*}{\partial t}\frac{dt^*}{d\sigma} + \frac{\partial z^*}{\partial \sigma}\right] \quad \text{by (16) and (20)}$$

$$= -\frac{\sigma H^*}{T}\cdot\frac{dt^*}{d\sigma} + \left(1 - \frac{t^*}{T}\right)H^* \quad \text{by (22)}$$

$$\leqslant -H^*\left(1 - \frac{t^*}{T}\right)\frac{1}{(H^* - z^*)^2} \leqslant 0, \tag{23}$$

i.e., if the standard deviation of the demand increases, the buyer must decrease the order quantity.

5.2. The effect of changing the expected demand quantity μ

Let z, $z = z(t, \mu)$, in (11) be variant to (t, μ). Differentiating (11) with respect to μ and using (12), it becomes

$$\frac{\partial z}{\partial \mu} = \frac{-\beta T}{\sigma(1 - \Phi(z))(T - t)} < 0 \quad \forall (t, \mu). \tag{24}$$

Let $\dot{L}(t, \mu) = (\partial/\partial t)L(t, z(t, \mu))$. Differentiating (17) with respect to μ and using (15), we get

$$T\frac{\partial \dot{L}}{\partial \mu} = (\delta - h)\left(\frac{\partial z}{\partial \mu}\sigma(T - t) + T\right) - \sigma H'\frac{\partial z}{\partial \mu}(c - (\delta - h)(T - t))$$

$$+ v\sigma((H - z)\phi(z) + \int_{-\infty}^{z} H'\phi(x)\,dx)\frac{\partial z}{\partial \mu} \quad \text{by (15)}$$

$$= \sigma\frac{\partial z}{\partial \mu}\left\{\begin{aligned}&(\delta - h)(T - t) - (c - (\delta - h)(T - t))H(H - z)\\ &+ v(H - z)\phi(z) + vH(H - z)\Phi(x)\end{aligned}\right\} + T(\delta - h) \quad \text{by (14)}$$

$$= \sigma\frac{\partial z}{\partial \mu}\{(\delta - h)(T - t) - (c - (\delta - h)(T - t) - v)H(H - z)\} + T(\delta - h) \quad \text{by (24)}$$

$$= \frac{\beta T}{(1 - \Phi(z))(T - t)}\{H(H - z)[c - (\delta - h)(T - t) - v] - (\delta - h)(T - t)\} + T(\delta - h). \tag{25}$$

From (17), since

$$0 = \frac{d}{d\mu}\dot{L}(t^*(\mu), \mu) = \frac{\partial \dot{L}(t^*, \mu)}{\partial t}\frac{dt^*}{d\mu} + \frac{\partial \dot{L}(t^*, \mu)}{\partial \mu},$$

we have

$$\frac{dt^*(\mu)}{d\mu} = -\frac{(\partial/\partial\mu)\dot{L}(t^*, \mu)}{(\partial/\partial t)\dot{L}(t^*, \mu)} \quad \text{by (18) and (25)}$$

$$= -\frac{\beta T}{(1 - \Phi(z^*))\sigma H^*}$$

$$\times\left[1 + \frac{[c - (\delta - h)(T - t^*) - v]H^*(H^* - z^*) + (\delta - h)(T - t^*)[1 + (1 - \Phi(z^*))\beta^{-1}]}{(H^* - z^*)^2(c - (\delta - h)(T - t^*) - v) - 2(\delta - h)(T - t^*)}\right]$$

$$< -\frac{\beta T}{(1 - \Phi(z^*))\sigma H^*} < 0, \tag{26}$$

i.e., if the expected demand quantity increases, the buyer must advance the purchase timing.

Differentiating (19) with respect to μ, it becomes

$$\begin{aligned}\frac{\mathrm{d}q^*}{\mathrm{d}\mu} &= -\frac{\sigma}{T}\frac{\mathrm{d}t^*}{\mathrm{d}\mu}z^* + \sigma\left(1-\frac{t^*}{T}\right)\left[\frac{\partial z^*}{\partial t}\frac{\mathrm{d}t^*}{\mathrm{d}\mu} + \frac{\partial z^*}{\partial \mu}\right] + 1 \quad \text{by (16) and (24)}\\ &= \left(\frac{\sigma H^*}{T}\right)\left(-\frac{dt^*}{d\mu}\right) - \frac{\beta}{1-\Phi(z^*)} + 1 \quad \text{by (26)}\\ &> 1 \end{aligned} \tag{27}$$

i.e., if the expected demand quantity increases, the buyer must increase the order quantity.

5.3. *The effect of changing shortage-level upper limit* β

Let z, $z = z(t, \beta)$, in (11) be variant to (t, β). Differentiating (11) with respect to β and using (12),

$$\frac{\partial z}{\partial \beta} = -\frac{T\mu}{\sigma(1-\Phi(z(t)))(T-t)} < 0 \quad \forall (t, \beta) \tag{28}$$

holds.

Let $\dot{L}(t, \beta) = (\partial/\partial t)L(t, z(t, \beta))$. Differentiating (17) with respect to β and using (15) and (16), it holds

$$\begin{aligned} T\frac{\partial \dot{L}}{\partial \beta} &= \sigma\frac{\partial z}{\partial \beta}\left\{\begin{matrix}(T-t)(\delta-h) - [c-(T-t)(\delta-h)]H(H-z)\\ +v(H-z)\phi(z) + vH(H-z)\Phi(z)\end{matrix}\right\} \quad \text{by (14)}\\ &= \sigma\frac{\partial z}{\partial \beta}\{(T-t)(\delta-h) - [c-(T-t)(\delta-h)-v]H(H-z)\} \quad \text{by (28)}\\ &= \frac{-T\mu}{(1-\Phi(z))(T-t)}\{(T-t)(\delta-h) - [c-(T-t)(\delta-h)-v]H(H-z)\} \quad \forall (t, \beta). \end{aligned} \tag{29}$$

From (17),

$$0 = \frac{\mathrm{d}}{\mathrm{d}\beta}\dot{L}(t^*(\beta), \beta) = \frac{\partial \dot{L}(t^*, \beta)}{\partial t}\frac{\mathrm{d}t^*}{\mathrm{d}\beta} + \frac{\partial \dot{L}(t^*, \beta)}{\partial \beta}$$

holds. Hence

$$\begin{aligned}\frac{\mathrm{d}t^*}{\mathrm{d}\beta} &= -\frac{(\partial/\partial\beta)\dot{L}(t^*,\beta)}{(\partial/\partial t)\dot{L}(t^*, \beta)} \quad \text{by (18) and (29)}\\ &= \frac{T\mu\{(\delta-h)(T-t^*) - [c-(\delta-h)(T-t^*)-v]H^*(H^*-z^*)\}}{(1-\Phi(z^*))\sigma H^*\{(H^*-z^*)^2[c-(\delta-h)(T-t^*)-v] - 2(T-t^*)(\delta-h)\}} \quad \text{by (18)}\\ &< 0, \end{aligned} \tag{30}$$

i.e., if the buyer sets up a higher shortage-level upper limit, the buyer must advance the purchase timing.

Differentiating (19) with respect to β, we obtain

$$\begin{aligned}\frac{\mathrm{d}q^*}{\mathrm{d}\beta} &= \sigma\left\{\left(1-\frac{t^*}{T}\right)\frac{\partial z(t^*, \mu)}{\partial t}\frac{\mathrm{d}t^*}{\mathrm{d}\beta} + \left(-\frac{1}{T}\right)z^*\frac{\mathrm{d}t^*}{\mathrm{d}\beta}\right\} \quad \text{by (16)}\\ &= \frac{\partial t^*}{\partial \beta}\sigma\left(-\frac{H^*}{T}\right) \quad \text{by (30)}\\ &> 0, \end{aligned} \tag{31}$$

i.e., if the buyer sets up a higher shortage-level upper limit, the buyer must increase the order quantity.

6. Conclusion

This paper is mainly about an extended newsboy problem with a specified average shortage-level constraint. The buyer must simultaneously decide the optimal purchase timing and the optimal order quantity like the presented scenario. We set up a mathematical model, which could be concretely discussed, and present the method and procedure to find the optimal solution in this paper. Since the buyer has to forecast the demand for early purchase, it will make the variance of the forecasted demand (forecast bias) as large as possible. This characteristic was embedded in the model.

The resulting outcomes in this paper are: (i) The more the expected demand quantity μ is (i.e., ceteris paribus, μ increases), the earlier the optimal purchase timing t^* is (26) and the more the optimal order quantity q^* is (27). (ii) The less accurate in forecasting demand (i.e., ceteris paribus, σ increases), the more later the optimal purchase timing t^* is (22) and the less optimal order quantity q^* is (23). (iii) The more the upper shortage-level limit (i.e., ceteris paribus, β increases), the earlier the optimal purchase timing t^* is (30) and the more the optimal order quantity q^* is (31).

The effect of deteriorating rate on the inventory could be considered in the future research. It may be extended to be a newsboy problem with distribution free in another future research.

Acknowledgements

This published paper, a partial fulfillment of the requirements for Ph.D. Degree in management sciences at Tamkang University, is taken from the dissertation of Mr. Chuang, a candidate for Ph.D. Degree. We thank the referees for offering brilliant comments and we ourselves are responsible for the mistakes if there is any.

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