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A minimax distribution free procedure for mixed inventory model involving variable lead time with fuzzy demand

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Abstract

In a recent paper, Ouyang and Wu applied the minimax decision approach to solve a continuous review mixed inventory model in which the lead time demand distribution information is unknown but the annual demand is fixed and given. However, in the practical situation, the annual demand probably incurs disturbance due to various uncertainties. In this article, we attempt to modify Ouyang and Wu's model by considering two fuzziness of annual demand (i.e., fuzzy number of annual demand and statistic-fuzzy number of annual demand) and to investigate a computing schema for the continuous review inventory model in the fuzzy sense. We give an algorithm procedure to obtain the optimal ordering strategy for each case.

Scope and purpose

In most of the early literature dealing with inventory problems, either using deterministic or probabilistic models, lead time is viewed as a prescribed constant or a stochastic variable. Recently, some researchers (e.g., Liao and Shyu, Ben-Daya and Raouf, and Ouyang and Wu) incorporated the crashing lead time idea to continuous review inventory models, in which the annual demand is given and fixed. However, in the real situation, the annual demand will probably have a little disturbance due to various uncertainties. The purpose of this article is to modify the Ouyang and Wu's model to accommodate this reality, specifically, we apply the fuzzy set concepts to deal with the uncertain annual demand. We first consider a case where the annual demand is treated as the triangular fuzzy number. Then, we employ the statistical method to construct a confidence interval for the annual demand, and through it to establish the corresponding fuzzy number (namely, the statistic-fuzzy number). For each fuzzy case, we investigate a computing schema for the new model and develop an algorithm to find the optimal ordering strategy.

Keywords: Inventory; Membership function; Extension principle; Fuzzy total cost; Minimax distribution free procedure

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1. Introduction

In most of the early literature dealing with inventory problems, either using deterministic or probabilistic models, lead time is viewed as a prescribed constant or a stochastic variable, which therefore, is not subject to control (see, e.g., Naddor [1] and Silver and Peterson [2]). In 1983, Monden [3] studied the Toyota production system, and pointed out that shortening lead time is a crux of elevating productivity. Also, through the Japanese successful experience of using just-in-time (JIT) production, the advantages and benefits associated with efforts to control the lead time can be clearly perceived.

Recently, several inventory models have been developed to consider lead time as a decision variable. Liao and Shyu [4] first presented a continuous review inventory model in which the order quantity was predetermined and lead time was a unique decision variable. Later, Ben-Daya and Raouf [5] extended Liao and Shyu's [4] model by considering both lead time and order quantity as decision variables where shortages were neglected. In a recent research article, Ouyang and Wu [6] generalized Ben-Daya and Raouf's [5] model and utilized the minimax decision criterion to solve the distribution free model. We note that the annual demand is fixed in above models [4–6]. However, in the real situation, the annual demand will probably have a little disturbance due to various uncertainties. Therefore, if we express the fuzzy annual demand as the neighborhood of the fixed annual demand, then it will match more with the real situation. In fact, the applications of fuzzy set concepts on EOQ inventory models have been proposed by many authors (e.g., Park [7], Chen et al. [8], Yao and Lee [9], Roy and Maiti [10], Chang et al. [11], Lee and Yao [12]). Yao and Lee [9] used the extension principle to solve the inventory model with shortages by fuzzifying the order quantity, in which the shortage quantity was a real variable. Later, Chang et al. [11] fuzzified the shortage quantity in the backorder model, where the order quantity was a real variable. Inventory model without backorder was discussed by Lee and Yao [12], who fuzzified the order quantity to a fuzzy number, and solved the economic order quantity with the extension principle. However, these studies [7–12] are almost concentrated on the simple EOQ forms so that there have few applications in the real inventory systems. The purpose of this paper is to present a more extensive EOQ model, specifically, we modify Ouyang and Wu's [6] model by fuzzifying the annual demand and solve this new inventory model in the fuzzy sense.

In this paper, we introduce two fuzziness of annual demand: fuzzy number of annual demand and statistic-fuzzy number of annual demand which is a new method by utilizing the sample data to obtain the confidence interval of annual demand, and then to get the fuzzy number. Furthermore, we, respectively, investigate a computing schema for the continuous review mixed inventory model in the fuzzy sense. We give an algorithm procedure to obtain the optimal ordering strategy for each case. Two examples are included.

2. Membership function of the fuzzy total cost

Under the assumptions that:

(i) the lead time can be decomposed into n mutually independent components each having a different crashing cost for reducing lead time;

(ii) shortages are allowed and only a fraction, say β $(0 \leqslant \beta \leqslant 1)$, of the demand during the stockout period can be backordered, and the remaining fraction $1-\beta$ is lost,

Ouyang and Wu [6] established the following expected annual total inventory cost function:

$$EAC(Q,L) = \text{ordering cost} + \text{holding cost} + \text{stockout cost} + \text{lead time crashing cost}$$

$$= A\frac{D}{Q} + h\left[\frac{Q}{2} + r - \mu L + (1-\beta)E(X-r)^{+}\right] + \frac{D}{Q}[\pi + \pi_0(1-\beta)]E(X-r)^{+}$$

$$+ \frac{D}{Q}\left[c_i(L_{i-1} - L) + \sum_{j=1}^{i-1} c_j(b_j - a_j)\right], \quad L \in [L_i, L_{i-1}], \tag{1}$$

where D is the average demand per year, A the fixed ordering cost per order, h the inventory holding cost per unit per year, π the fixed penalty cost per unit short, π_0 the marginal profit per unit, Q the order quantity, a decision variable, L the length of lead time, a decision variable, X the lead time demand which has a distribution function F with finite mean μL and standard deviation $\sigma\sqrt{L}(>0)$, r the reorder point, and $r = \mu L + k\sigma\sqrt{L}$ where k is the safety factor and r satisfies $P(X > r) = q$, q represents the allowable stockout probability during L, and q is given, a_i, b_i, c_i the lead time L has n mutually independent components, the ith component has a minimum duration a_i and normal duration b_i, and a crashing cost per unit time c_i, where $c_1 \leqslant c_2 \leqslant \cdots \leqslant c_n$, L_i the length of lead time with components $1, 2, \ldots, i$ crashed to their minimum duration, hence $L_i = \sum_{j=1}^{n} b_j - \sum_{j=1}^{i} (b_j - a_j)$, $i = 1, 2, \ldots, n$. Further, let $L_0 = \sum_{j=1}^{n} b_j$, $E(X-r)^{+}$ the expected demand shortage at the end of cycle, and $EAC(Q,L)$ the expected annual total inventory cost.

When the distributional form of the lead time demand X is unknown, they utilized the minimax decision criterion to solve this problem and obtained the optimal order quantity, Q^*, and the optimal lead time, L^*.

Note that the annual demand, D, in model (1) is viewed as a fixed constant. However, in the real situation, the annual demand probably will have some changes due to various uncertainties. Hence, to match more realistic situation, now we attempt to modify Ouyang and Wu's [6] model (1) by considering the fuzzy annual demand. We here use the same notations and assumptions as in their model to avoid any possible confusion.

For any $Q > 0$ and $L \in [L_i, L_{i-1}]$, we may express the expected annual total cost function (1) as

$$F_{(Q,L)}(D) \equiv EAC(Q,L)$$

$$= \frac{D}{Q}[W + c_i(L_{i-1} - L)] + h\left[\frac{Q}{2} + r - \mu L + (1-\beta)E(X-r)^{+}\right], \tag{2}$$

where $W = A + [\pi + \pi_0(1-\beta)]E(X-r)^{+} + \sum_{j=1}^{i-1} c_j(b_j - a_j)$.

We now replace the annual demand, D, by the fuzzy number $\tilde{D}$, and consider the fuzzy number $\tilde{D}$ as the triangular fuzzy number, $\tilde{D} = (D - \Delta_1, D, D + \Delta_2)$, where $0 < \Delta_1 < D$ and $0 < \Delta_2$, Δ_1

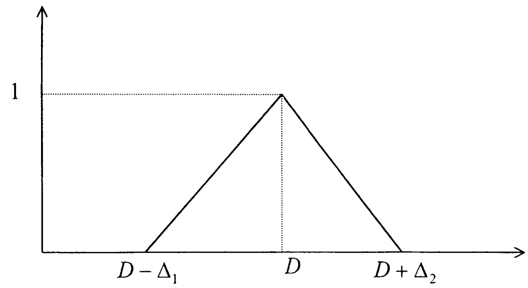

Fig. 1. Triangular fuzzy number $\tilde{D}$.

and Δ_2 are determined by the decision makers. Also, here we describe the membership function of $\tilde{D}$ as follows:

$$\mu_{\tilde{D}}(x) = \begin{cases} \dfrac{x - D + \Delta_1}{\Delta_1} & \text{if } D - \Delta_1 \leqslant x \leqslant D, \\ \dfrac{D + \Delta_2 - x}{\Delta_2} & \text{if } D \leqslant x \leqslant D + \Delta_2, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

See Fig. 1. Then the centroid of $\mu_{\tilde{D}}(x)$ is

$$D^* = D + \tfrac{1}{3}(\Delta_2 - \Delta_1). \tag{4}$$

We regard this value as the estimate of annual demand in the fuzzy sense.

For any $Q > 0$ and $L \in [L_i, L_{i-1}]$, we let $F_{(Q,L)}(x) = y(>0)$. By extension principle [13,14], the membership function of the fuzzy cost $F_{(Q,L)}(\tilde{D})$ is given by

$$\mu_{F_{(Q,L)}(\tilde{D})}(y) = \begin{cases} \sup\limits_{x \in F^{-1}_{(Q,L)}(y)} \mu_{\tilde{D}}(x) & \text{if } F^{-1}_{(Q,L)}(y) \neq \phi, \\ 0 & \text{if } F^{-1}_{(Q,L)}(y) = \phi. \end{cases} \tag{5}$$

From $F_{(Q,L)}(x) = y$ and Eq. (2), we get

$$\frac{x}{Q}[W + c_i(L_{i-1} - L)] + h\left[\frac{Q}{2} + r - \mu L + (1-\beta)E(X-r)^+\right] = y. \tag{6}$$

Consequently,

$$x = \frac{Q}{W + c_i(L_{i-1} - L)}\left\{y - h\left[\frac{Q}{2} + r - \mu L + (1-\beta)E(X-r)^+\right]\right\}. \tag{7}$$

Therefore, from (3) and (7), the membership function of $F_{(Q,L)}(\tilde{D})$ can be written as

$$\mu_{F_{(Q,L)}(\tilde{D})}(y) = \begin{cases} \dfrac{Qy - Qh[Q/2 + r - \mu L + (1-\beta)E(X-r)^+]}{[W + c_i(L_{i-1} - L)]\Delta_1} - \dfrac{D - \Delta_1}{\Delta_1} & \text{if } y_1 \leqslant y \leqslant y_2, \\ \dfrac{D + \Delta_2}{\Delta_2} + \dfrac{Qh[Q/2 + r - \mu L + (1-\beta)E(X-r)^+] - Qy}{[W + c_i(L_{i-1} - L)]\Delta_2} & \text{if } y_2 \leqslant y \leqslant y_3, \\ 0 & \text{otherwise,} \end{cases} \tag{8}$$

where

$$y_1 = \frac{(D - \Delta_1)[W + c_i(L_{i-1} - L)]}{Q} + h\left[\frac{Q}{2} + r - \mu L + (1-\beta)E(X-r)^+\right],$$

$$y_2 = \frac{D[W + c_i(L_{i-1} - L)]}{Q} + h\left[\frac{Q}{2} + r - \mu L + (1-\beta)E(X-r)^+\right]$$

and

$$y_3 = \frac{(D + \Delta_2)[W + c_i(L_{i-1} - L)]}{Q} + h\left[\frac{Q}{2} + r - \mu L + (1-\beta)E(X-r)^+\right].$$

The pictorial of the membership function of $F_{(Q,L)}(\tilde{D})$ is shown in Fig. 2.

We now derive the centroid of $\mu_{F_{(Q,L)}(\tilde{D})}(y)$ as follows:

$$\begin{aligned} M(Q,L) &= \frac{\int_{-\infty}^{\infty} y\mu_{F_{(Q,L)}(\tilde{D})}(y)\,\mathrm{d}y}{\int_{-\infty}^{\infty} \mu_{F_{(Q,L)}(\tilde{D})}(y)\,\mathrm{d}y} \\ &= \frac{1}{3}\{y_1 + y_2 + y_3\} \\ &= EAC(Q,L) + \frac{W + c_i(L_{i-1} - L)}{3Q}(\Delta_2 - \Delta_1). \end{aligned}$$

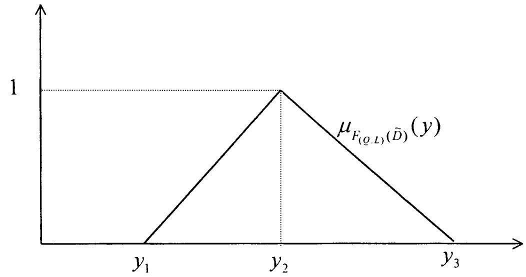

Fig. 2. Triangular fuzzy number $F_{(Q,L)}(\tilde{D})$.

Thus, we obtain the following property.

Property 1. *For any $Q > 0$ and $L \in [L_i, L_{i-1}]$, the estimate of the expected annual total inventory cost in the fuzzy sense is*

$$M(Q,L) = EAC(Q,L) + \frac{W + c_i(L_{i-1} - L)}{3Q}(\Delta_2 - \Delta_1). \tag{9}$$

Remark 1. In the traditional inventory models, the annual demand, D, is fixed and given. However, in the real situation, the annual demand may slightly change due to various uncertainties. Hence, if we regard the crisp value $EAC(Q,L)$ as a true expected annual total inventory cost, then it may lose some realities. In order to match more realistic situation, we here replace the fixed annual demand, D, by the triangular fuzzy number, $\tilde{D} = (D - \Delta_1, D, D + \Delta_2)$, where $0 < \Delta_1 < D$ and $0 < \Delta_2$, and use the value $M(Q,L)$ as the estimate of the expected annual total inventory cost. Furthermore, if we let $S = (W + c_i(L_{i-1} - L))/(3Q \cdot EAC(Q,L))(\Delta_2 - \Delta_1)$, then from (9) we obtain $(M(Q,L) - EAC(Q,L))/EAC(Q,L) \times 100\% = S \times 100\%$, which implies

$$[M(Q,L) - EAC(Q,L)] \times 100\% = S \times EAC(Q,L) \times 100\%. \tag{10}$$

Remark 2. If $\Delta_1 = \Delta_2$, then Fig. 1 is the isosceles triangle, and hence, Eq. (9) reduces to $M(Q,L) = EAC(Q,L)$, this implies that the fuzzy case becomes the crisp case; i.e., Ouyang and Wu's [6] inventory model is a special case of our new fuzzy annual demand inventory model. If $\Delta_1 < \Delta_2$, then the triangle in Fig. 1 is skewed to the right, and in this case $M(Q,L) > EAC(Q,L)$, where the increment of $M(Q,L)$ is $S\%$ of $EAC(Q,L)$ (from (10)). If $\Delta_1 > \Delta_2$, the triangle is skewed to the left, and $M(Q,L) < EAC(Q,L)$, where the decrement of $M(Q,L)$ is $|S|\%$ of $EAC(Q,L)$ (from (10)).

3. Optimal solution

When the probability distribution of the lead time demand X is unknown, it can not get the exact value of $E(X - r)^+$, and hence, the optimal value of (Q,L) which minimizes the fuzzy expected annual total inventory cost $M(Q,L)$ cannot be found. Now, we attempt to use the minimax distribution free procedure to solve this problem. For convenience, we let $\mathscr{F}$ denote the class of the distribution function F of X which has finite mean μL and standard deviation $\sigma\sqrt{L}$. Then the minimax distribution free approach for this problem is to find the most unfavorable distribution function F in $\mathscr{F}$ for each (Q,L), and then to minimize the fuzzy expected annual total cost function over (Q,L). That is, our problem is to solve

$$\min_{Q>0,L>0} \max_{F\in\mathscr{F}} M(Q,L). \tag{11}$$

By using $r = \mu L + k\sigma\sqrt{L}$ and Lemma 1 in Gallego and Moon [15], we have

$$E(X - r)^+ \leqslant \tfrac{1}{2}\sigma\sqrt{L}(\sqrt{1 + k^2} - k) \quad \text{for any } F \in \mathscr{F}. \tag{12}$$

Then problem (11) is reduced to minimize

$$M^u(Q,L)=\frac{1}{Q}\left\{A+[\pi+\pi_0(1-\beta)]\frac{\sigma\sqrt{L}(\sqrt{1+k^2}-k)}{2}+c_i(L_{i-1}-L)+\sum_{j=1}^{i-1}c_j(b_j-a_j)\right\}$$
$$\left(D-\frac{\Delta_1-\Delta_2}{3}\right)+h\left[\frac{Q}{2}+k\sigma\sqrt{L}+\frac{1}{2}\sigma\sqrt{L}(1-\beta)(\sqrt{1+k^2}-k)\right],$$
$$L\in[L_i,L_{i-1}], \tag{13}$$

where $M^u(Q,L)$ is the least upper bound of $M(Q,L)$.

As discussed in that of Ouyang and Wu [6], we can show that $M^u(Q,L)$ is concave in $L\in[L_i,L_{i-1}]$ for fixed Q. Hence, for fixed Q, the minimum upper bound of the expected annual total cost in the fuzzy sense will occur at the end points of the interval $[L_i,L_{i-1}]$. In addition, it can be shown that $M^u(Q,L)$ is convex in Q for fixed $L\in[L_i,L_{i-1}]$. Upon setting $\partial M^u(Q,L)/\partial Q=0$, we get

$$Q=\left[\frac{2}{h}\left(D-\frac{\Delta_1-\Delta_2}{3}\right)\left\{\begin{array}{c}A+\frac{1}{2}\sigma\sqrt{L}[\pi+\pi_0(1-\beta)](\sqrt{1+k^2}-k)\\+c_i(L_{i-1}-L)+\sum_{j=1}^{i-1}c_j(b_j-a_j)\end{array}\right\}\right]^{1/2}. \tag{14}$$

Theoretically, for given L_i $(i=0,1,2,\dots,n)$, $\Delta_1,\Delta_2,D,A,\pi,\pi_0,h,\sigma,\beta$, and k (which is depended on the allowable stockout probability q and the distribution function F of the lead time demand), from (14), we can get the value of (Q_i,L_i), Furthermore, using Eq. (13), we can obtain the corresponding expected annual total cost $M^u(Q_i,L_i)$ in the fuzzy sense for $i=0,1,2,\dots,n$. And thus, the minimum expected annual total cost in the fuzzy sense is $\min_{i=0,1,2,\dots,n}\{M^u(Q_i,L_i)\}$. However, in practice, since the form of distribution function F is unknown, and even if the value of the allowable stockout probability q is given, we cannot get the exact value of k. Fortunately, by using Proposition 2 in Ouyang and Wu [6] and $r=\mu L+k\sigma\sqrt{L}$, we have

$$P(X>r)\leqslant\frac{1}{1+k^2}. \tag{15}$$

Thus, from $q=P(X>r)$ and (15), we get $0\leqslant k\leqslant\sqrt{1/q-1}$. Furthermore, it is easy to verify that $M^u(Q,L)$ has a smooth curve for $k\in[0,\sqrt{1/q-1}]$. Hence, we can establish the following algorithm to obtain the suitable k and thus the optimal values of Q and L.

Algorithm 1. For given q, we let $k_0=0$, $k_N=\sqrt{1/q-1}$ and $k_j=k_{j-1}+(k_N-k_0)/N$, $j=1,2,\dots,N-1$, where N is a dividing number and is large enough. We execute the following procedure:

Step 1: For give L_i, $i=0,1,2,\dots,n$, perform (i)–(iii).

(i) For each $k_j\in\{k_0,k_1,\dots,k_N\}$, $j=0,1,2,\dots,N$, and L_i, we can use Eq. (14) to compute Q_{k_j}.

(ii) For each pair (Q_{k_j},L_i), compute the corresponding fuzzy expected annual total cost $M^u(Q_{k_j},L_i)$, $j=0,1,2,\dots,N$.

(iii) Find $\min_{j=0,1,\dots,N}M^u(Q_{k_j},L_i)$. If $M^u(Q_{k(i)},L_i)=\min_{j=0,1,\dots,N}M^u(Q_{k_j},L_i)$, then $(Q_{k(i)},L_i)$ is the local optimal solution in the fuzzy sense for fixed L_i.

Step 2: Find $\min_{i=0,1,\dots,n} M^u(Q_{k(i)}, L_i)$. If $M^u(Q_{\tilde{D}}, L_{\tilde{D}}) = \min_{i=0,1,\dots,n} M^u(Q_{k(i)}, L_i)$, then $(Q_{\tilde{D}}, L_{\tilde{D}})$ is the optimal solution in the fuzzy sense.

If we let Q^*, L^* and $EAC^u(Q^*, L^*)$ denote the optimal order quantity, optimal lead time and minimum expected annual total cost for the crisp case, respectively, then the relative error in the fuzzy sense for order quantity, lead time and minimum expected annual total cost are, respectively, given by

$$Rel\,Q = \frac{Q_{\tilde{D}} - Q^*}{Q^*} \times 100\%, \quad Rel\,L = \frac{L_{\tilde{D}} - L^*}{L^*} \times 100\%,$$

$$Rel\,C = \frac{M^u(Q_{\tilde{D}}, L_{\tilde{D}}) - EAC^u(Q^*, L^*)}{EAC^u(Q^*, L^*)} \times 100\%, \tag{16}$$

where $(Q_{\tilde{D}}, L_{\tilde{D}})$ is the optimal solution of $M^u(Q, L)$.

Example 1. In order to illustrate the above solution procedure, let us consider an inventory system with the data used in Ouyang and Wu [6]: $D = 600$ units/yr, $A = \$200$ per order, $h = \$20$ per unit per year, $\pi = \$50$ per unit short, $\pi_0 = \$150$ per unit lost, $\sigma = 7$ units/week, and the lead time has three components with data as shown in Table 1.

Here, we consider the cases when $\beta = 0, 0.5, 0.8, 1$. We solve each case for some values of (Δ_1, Δ_2) and $q = 0.2$ (in this situation, we have $k_0 = 0$, $k_N = 2$) and $k_j = k_{j-1} + (k_N - k_0)/N$, $j = 1, 2, \dots, N-1$, where $N = 200$. Applying the proposed procedure (Algorithm 1) yields the optimal results as shown in Table 2. Moreover, in order to find the values of relative error in the fuzzy sense (as defined in (16)), we first list the result of crisp case [6] in Table 3.

Then, using the values in Tables 2 and 3, and formulas in (16), we obtain the results as shown in Table 4.

Roughly speaking, Table 4 displays when the value of D^* approaches the crisp annual demand D, the relative error in the fuzzy sense for order quantity, lead time and minimum expected annual total cost approaches zero, respectively; that is, when $D^* \to 600$, we have $Rel\,Q \to 0$, $Rel\,L \to 0$ and $Rel\,C \to 0$.

Table 1
Lead time data

Lead time component i	Normal duration b_i (days)	Minimum duration a_i (days)	Unit crashing cost c_i (\$/day)
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

Table 2
Summary of the optimal procedure solution[a]

β	Δ_1	Δ_2	$\Delta_2 - \Delta_1$	D^*	k	$Q_{\tilde{D}}$	$L_{\tilde{D}}$	$M^u(Q_{\tilde{D}}, L_{\tilde{D}})$
	20	80	60	620	2.00	184	3	4185.34
	30	80	50	617	2.00	183	3	4175.46
	40	80	40	613	2.00	183	3	4165.55
	50	80	30	610	2.00	182	3	4155.61
	60	80	20	607	2.00	182	3	4145.64
	70	80	10	603	2.00	181	3	4135.65
0.0	80	80	0	600	2.00	181	3	4125.64
	80	70	− 10	597	2.00	180	3	4115.59
	80	60	− 20	593	2.00	180	3	4105.51
	80	50	− 30	590	2.00	179	3	4095.41
	80	40	− 40	587	2.00	179	3	4085.27
	80	30	− 50	583	2.00	178	3	4075.11
	80	20	− 60	580	2.00	178	3	4064.92
	20	80	60	620	2.00	164	3	3788.64
	30	80	50	617	2.00	164	3	3779.79
	40	80	40	613	2.00	164	3	3770.91
	50	80	30	610	2.00	163	3	3762.01
	60	80	20	607	2.00	163	3	3753.08
	70	80	10	603	2.00	162	3	3744.13
0.5	80	80	0	600	2.00	162	3	3735.16
	80	70	− 10	597	2.00	161	3	3726.15
	80	60	− 20	593	2.00	161	3	3717.13
	80	50	− 30	590	2.00	160	3	3708.08
	80	40	− 40	587	2.00	160	3	3699.00
	80	30	− 50	583	2.00	160	3	3689.90
	80	20	− 60	580	2.00	159	3	3680.77
	20	80	60	620	1.83	154	3	3524.90
	30	80	50	617	1.83	153	3	3516.62
	40	80	40	613	1.82	153	3	3508.32
	50	80	30	610	1.82	153	3	3499.99
	60	80	20	607	1.82	152	3	3491.64
	70	80	10	603	1.81	152	3	3483.26
0.8	80	80	0	600	1.81	152	3	3474.87
	80	70	− 10	597	1.81	151	3	3466.43
	80	60	− 20	593	1.80	151	3	3457.97
	80	50	− 30	590	1.80	150	3	3449.49
	80	40	− 40	587	1.80	150	3	3440.99
	80	30	− 50	583	1.79	150	3	3432.45
	80	20	− 60	580	1.79	149	3	3423.89

(*continued on next page*)

Table 2 (*continued*)

β	Δ_1	Δ_2	$\Delta_2 - \Delta_1$	D^*	k	$Q_{\tilde{D}}$	$L_{\tilde{D}}$	$M^u(Q_{\tilde{D}}, L_{\tilde{D}})$
	20	80	60	620	1.40	144	4	3272.48
	30	80	50	617	1.40	144	4	3264.72
	40	80	40	613	1.40	143	4	3256.95
	50	80	30	610	1.39	143	4	3249.15
	60	80	20	607	1.39	143	4	3241.33
	70	80	10	603	1.39	142	4	3233.48
1.0	80	80	0	600	1.38	142	4	3225.61
	80	70	− 10	597	1.38	142	4	3217.71
	80	60	− 20	593	1.38	141	4	3209.80
	80	50	− 30	590	1.38	141	4	3201.85
	80	40	− 40	587	1.37	140	4	3193.88
	80	30	− 50	583	1.37	140	4	3185.89
	80	20	− 60	580	1.37	140	4	3177.87

[a] $D^* = D + \frac{1}{3}(\Delta_2 - \Delta_1)$ is the estimation of annual demand in the fuzzy sense.

Table 3
The optimal solution for the crisp case (Ouyang and Wu [6])

β	k	(Q^*, L^*)	$EAC''(Q^*, L^*)$
0.0	2.00	(181, 3)	4125.64
0.5	2.00	(162, 3)	3735.16
0.8	1.81	(151, 3)	3474.87
1.0	1.38	(142, 4)	3225.61

Table 4
The relative error in the fuzzy sense with the crisp case (%)

β	D^*	*Rel Q*	*Rel L*	*Rel C*
	620	1.66	0	1.45
	617	1.10	0	1.21
	613	1.10	0	0.97
	610	0.55	0	0.73
	607	0.55	0	0.48
	603	0.00	0	0.24
0.0	600	0.00	0	0.00
	597	− 0.55	0	− 0.24
	593	− 0.55	0	− 0.49
	590	− 1.10	0	− 0.73
	587	− 1.10	0	− 0.98
	583	− 1.66	0	− 1.22
	580	− 1.66	0	− 1.47

Table 4 (*continued*)

β	D^*	$Rel\,Q$	$Rel\,L$	$Rel\,C$
	620	1.23	0	1.43
	617	1.23	0	1.19
	613	1.23	0	0.96
	610	0.62	0	0.72
	607	0.62	0	0.48
	603	0.00	0	0.24
0.5	600	0.00	0	0.00
	597	− 0.62	0	− 0.24
	593	− 0.62	0	− 0.48
	590	− 1.23	0	− 0.73
	587	− 1.23	0	− 0.97
	583	− 1.23	0	− 1.21
	580	− 1.85	0	− 1.46
	620	1.32	0	1.44
	617	0.66	0	1.20
	613	0.66	0	0.96
	610	0.66	0	0.72
	607	0.00	0	0.48
	603	0.00	0	0.24
0.8	600	0.00	0	0.00
	597	− 0.66	0	− 0.24
	593	− 0.66	0	− 0.49
	590	− 1.32	0	− 0.73
	587	− 1.32	0	− 0.98
	583	− 1.32	0	− 1.22
	580	− 1.97	0	− 1.47
	620	1.41	0	1.45
	617	1.41	0	1.21
	613	0.70	0	0.97
	610	0.70	0	0.73
	607	0.70	0	0.49
	603	0.00	0	0.24
1.0	600	0.00	0	0.00
	597	0.00	0	− 0.24
	593	− 0.70	0	− 0.49
	590	− 0.70	0	− 0.74
	587	− 1.41	0	− 0.98
	583	− 1.41	0	− 1.23
	580	− 1.41	0	− 1.48

4. Using the sample data to fuzzify the annual demand

If the actual annual demand D is unknown, we can not utilize (14) to determine the optimal ordering strategy. In this section, we attempt to use the random sample data of annual demand during past time to find the interval estimation of the real annual demand, and then employ it to get the statistic-fuzzy annual demand $\tilde{D}^*$. The procedure of estimating is as follows:

Assume that the actual annual demand D (which can be regarded as the population mean of the annual demand) is unknown, and suppose that we have collected m random sample data of annual demand during past time, say $d_1, d_2, \ldots, d_m$, then the sample mean is $\bar{d} = 1/m\sum_{i=1}^{m} d_i$, and the sample variance is $s^2 = 1/(m-1)\sum_{i=1}^{m}(d_i - \bar{d})^2$. It can be shown that $\bar{d}$ is a good point estimator of the annual demand D. However, in practical situations, when the inventory planning is completed, the real annual demand may be not equal to $\bar{d}$ but just close to it. This scenario can be expressed in fuzzy language as "$\tilde{D}^*$ = real annual demand D is around $\bar{d}$". In order to deal with such an inventory problem, we need to combine the statistics and fuzzy technologies. First, we use the statistical method of interval estimation to get a $(1-\alpha)\times 100\%$ confidence interval for D as follows:

$$\left[\bar{d} - t_{m-1}(\alpha_1)\frac{s}{\sqrt{m}},\ \bar{d} + t_{m-1}(\alpha_2)\frac{s}{\sqrt{m}}\right], \tag{17}$$

where $\alpha_1,\ \alpha_2 > 0,\ \alpha_1 + \alpha_2 = \alpha$, and $t_{m-1}(\alpha_i)$, $i = 1,2$, is the tabulated upper α_i point of the t-distribution with $m-1$ degrees of freedom; that is, if T be a random variable distributed as t-distribution with $m-1$ degrees of freedom, then $t_{m-1}(\alpha_i)$ is the value that satisfies the following condition:

$$P[T > t_{m-1}(\alpha_i)] = \alpha_i, \quad i = 1,\ 2. \tag{18}$$

Then, we take any point (denoted by $\bar{d}_0$) from the inside of above confidence interval (17). If $\bar{d}_0 = \bar{d}$, then the error of estimation $|\bar{d}_0 - \bar{d}| = 0$; in this case, we take the *confidence level* as 1. In contrast, the further the point $\bar{d}_0$ is from $\bar{d}$, the larger the error of estimation $|\bar{d}_0 - \bar{d}|$ to be, that is, the smaller the *confidence level* will be given. If $\bar{d}_0$ is one of the end points of the confidence interval, then the error of estimation $|\bar{d}_0 - \bar{d}|$ is in the largest; in this case, the *confidence level* is 0. On the other hand, we can employ (17) to express the statistical-fuzzy annual demand $\tilde{D}^*$ as the following triangular fuzzy number

$$\tilde{D}^* = \left(\bar{d} - t_{m-1}(\alpha_1)\frac{s}{\sqrt{m}},\ \bar{d},\ \bar{d} + t_{m-1}(\alpha_2)\frac{s}{\sqrt{m}}\right), \tag{19}$$

where $\alpha_1 + \alpha_2 = \alpha$. Note that the decision makers can determine α_1 and α_2 so as to satisfy $\bar{d} - t_{m-1}(\alpha_1)s/\sqrt{m} > 0$. Thus we can use the fuzzy number (19) together with the membership grade to represent the confidence level.

The membership function of statistic-fuzzy annual demand $\tilde{D}^*$ is given by

$$\mu_{\tilde{D}^*}(x) = \begin{cases} \dfrac{x - \bar{d} + t_{m-1}(\alpha_1)s/\sqrt{m}}{t_{m-1}(\alpha_1)s/\sqrt{m}} & \text{if } \bar{d} - t_{m-1}(\alpha_1)\dfrac{s}{\sqrt{m}} \leqslant x \leqslant \bar{d}, \\ \dfrac{\bar{d} + t_{m-1}(\alpha_2)s/\sqrt{m} - x}{t_{m-1}(\alpha_2)s/\sqrt{m}} & \text{if } \bar{d} \leqslant x \leqslant \bar{d} + t_{m-1}(\alpha_2)\dfrac{s}{\sqrt{m}}, \\ 0 & \text{otherwise.} \end{cases} \tag{20}$$

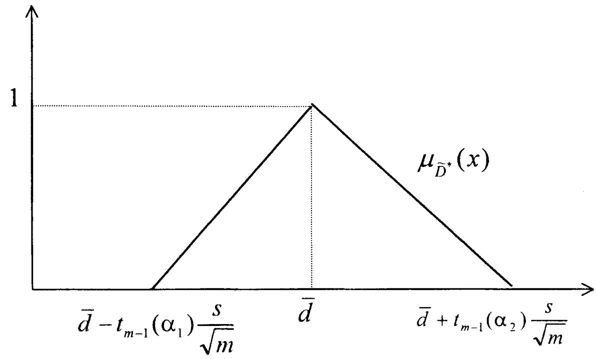

Fig. 3. Triangular fuzzy number $\tilde{D}^*$.

See Fig. 3. Then the centroid of $\mu_{\tilde{D}^*}(x)$ is

$$D^{**} = \bar{d} + \frac{1}{3}[t_{m-1}(\alpha_2) - t_{m-1}(\alpha_1)]\frac{s}{\sqrt{m}}. \tag{21}$$

We regard this value as the estimate of annual demand in the fuzzy sense. Obviously, $D^{**} > 0$ and D^{**} belongs to the interval (17). For the special case $\alpha_1 = \alpha_2 = \alpha/2$, it gets $D^{**} = \bar{d}$.

We let $F_{(Q,L)}(x) = z$. By extension principle [13,14], the membership function of the fuzzy cost $F_{(Q,L)}(\tilde{D}^*)$ is

$$\mu_{F_{(Q,L)}(\tilde{D}^*)}(z) = \begin{cases} \sup\limits_{x \in F^{-1}_{(Q,L)}(z)} \mu_{\tilde{D}}(x) & \text{if } F^{-1}_{(Q,L)}(z) \neq \phi, \\ 0 & \text{if } F^{-1}_{(Q,L)}(z) = \phi. \end{cases} \tag{22}$$

From $F_{(Q,L)}(x) = z$ and Eqs. (2), (20), and (22), we obtain the membership function of $F_{(Q,L)}(\tilde{D}^*)$ as follows:

$$\mu_{F_{(Q,L)}(\tilde{D}^*)}(z) = \begin{cases} \dfrac{\{Qz - Qh[Q/2 + r - \mu L + (1-\beta)E(X-r)^+]\}}{[W + c_i(L_{i-1} - L)]t_{m-1}(\alpha_1)s/\sqrt{m}} - \dfrac{[\bar{d} - t_{m-1}(\alpha_1)s/\sqrt{m}]}{t_{m-1}(\alpha_1)s/\sqrt{m}} & \text{if } z_1 \leqslant z \leqslant z_2, \\ \dfrac{[\bar{d} + t_{m-1}(\alpha_2)s/\sqrt{m}]}{t_{m-1}(\alpha_2)s/\sqrt{m}} + \dfrac{\{Qh[Q/2 + r - \mu L + (1-\beta)E(X-r)^+] - Qz\}}{[W + c_i(L_{i-1} - L)]t_{m-1}(\alpha_2)s/\sqrt{m}} & \text{if } z_2 \leqslant z \leqslant z_3, \\ 0 & \text{otherwise,} \end{cases} \tag{23}$$

where

$$z_1 = \frac{[\bar{d} - t_{m-1}(\alpha_1)s/\sqrt{m}][W + c_i(L_{i-1} - L)]}{Q} + h\left[\frac{Q}{2} + r - \mu L + (1-\beta)E(X-r)^+\right],$$

$$z_2 = \frac{\bar{d}[W + c_i(L_{i-1} - L)]}{Q} + h\left[\frac{Q}{2} + r - \mu L + (1-\beta)E(X-r)^+\right]$$

and

$$z_3 = \frac{[\bar{d} + t_{m-1}(\alpha_2)s/\sqrt{m}][W + c_i(L_{i-1} - L)]}{Q} + h\left[\frac{Q}{2} + r - \mu L + (1-\beta)E(X-r)^+\right].$$

Therefore, the centroid of $\mu_{F_{(Q,L)}(\tilde{D}^*)}(z)$ can be obtained and is given by

$$\begin{aligned} R(Q,L) &= \frac{\int_{-\infty}^{\infty} z\mu_{F_{(Q,L)}(\tilde{D}^*)(z)}\,\mathrm{d}z}{\int_{-\infty}^{\infty} \mu_{F_{(Q,L)}(\tilde{D}^*)}(z)\,\mathrm{d}z} \\ &= \frac{1}{3}(z_1 + z_2 + z_3) \\ &= EAC^*(Q,L) + \frac{W + c_i(L_{i-1} - L)}{3Q}[t_{m-1}(\alpha_2) - t_{m-1}(\alpha_1)]\frac{s}{\sqrt{m}}, \end{aligned}$$

where

$$EAC^*(Q,L) = \frac{\bar{d}}{Q}[W + c_i(L_{i-1} - L)] + h\left[\frac{Q}{2} + r - \mu L + (1-\beta)E(X-r)^+\right].$$

Thus, we get the following property.

Property 2. *For any $Q > 0$, $L \in [L_i, L_{i-1}]$, and given $\alpha_1 > 0$, $\alpha_2 > 0$ and $\alpha_1 + \alpha_2 = \alpha$, the estimate of the expected annual total inventory cost in the fuzzy sense is*

$$R(Q,L) = EAC^*(Q,L) + \frac{W + c_i(L_{i-1} - L)}{3Q}[t_{m-1}(\alpha_2) - t_{m-1}(\alpha_1)]\frac{s}{\sqrt{m}}. \tag{24}$$

Remark 3. It notes that the difference between $EAC^*(Q,L)$ and $EAC(Q,L)$ (defined in (2)) is that D in (2) is replaced by $\bar{d}$. If we let

$$S^* = \frac{W + c_i(L_{i-1} - L)}{3Q \cdot EAC^*(Q,L)}[t_{m-1}(\alpha_2) - t_{m-1}(\alpha_1)]\frac{s}{\sqrt{m}}, \tag{25}$$

then, from (24), the relative error of the expected annual total cost in the fuzzy sense is

$$\frac{R(Q,L) - EAC^*(Q,L)}{EAC^*(Q,L)} \times 100\% = S^* \times 100\%, \tag{26}$$

which implies

$$[R(Q,L) - EAC^*(Q,L)] \times 100\% = S^* \times EAC^*(Q,L) \times 100\%. \tag{27}$$

Thus, from (25) and (27), we have the following results.

Case 1: If $0 < \alpha_2 < \alpha_1 < 1$, then $t_{m-1}(\alpha_1) < t_{m-1}(\alpha_2)$, which implies $R(Q,L) > EAC^*(Q,L)$, and the increment of $R(Q,L)$ is $S^*\%$ of $EAC^*(Q,L)$.

Case 2: If $0 < \alpha_1 < \alpha_2 < 1$, then $t_{m-1}(\alpha_1) > t_{m-1}(\alpha_2)$, which implies $R(Q,L) < EAC^*(Q,L)$, and the decrement of $R(Q,L)$ is $|S^*|\%$ of $EAC^*(Q,L)$.

Case 3: If $\alpha_1 = \alpha_2 = \alpha/2$, then $t_{m-1}(\alpha_1) = t_{m-1}(\alpha_2)$, which implies $R(Q,L) = EAC^*(Q,L)$. That is, the total cost $EAC^*(Q,L)$ obtained by point estimate $\bar{d}$ is consistent with the total cost $R(Q,L)$ obtained by fuzzy number $\tilde{D}^*$ defined in (19).

Because the distribution function of the lead time demand X is unknown, it can not get the exact value of $E(X-r)^+$. By using the similar arguments as discussed in Section 3, we obtain the upper bound of expected annual total inventory cost $R(Q,L)$ in the fuzzy sense as follows:

$$R^u(Q,L) = \frac{1}{Q}\left\{A + [\pi + \pi_0(1-\beta)]\frac{\sigma\sqrt{L}(\sqrt{1+k^2}-k)}{2} + c_i(L_{i-1}-L) + \sum_{j=1}^{i-1} c_j(b_j - a_j)\right\}$$
$$\times\left(\bar{d} - \frac{t_{m-1}(\alpha_1) - t_{m-1}(\alpha_2)}{3}\frac{s}{\sqrt{m}}\right) + h\left[\frac{Q}{2} + k\sigma\sqrt{L} + \frac{\sigma\sqrt{L}(1-\beta)}{2}(\sqrt{1+k^2}-k)\right],$$
$$L \in [L_i, L_{i-1}]. \tag{28}$$

Now, we seek to find the optimal value of (Q,L) such that $R^u(Q,L)$ has minimum value. As discussed in the previous section, we can show that $R^u(Q,L)$ is concave in $L \in [L_i, L_{i-1}]$ for fixed Q. Hence, for fixed Q, the minimum upper bound of the fuzzy expected annual total cost will occur at the end points of the interval $[L_i, L_{i-1}]$. Moreover, it can be shown that $R^u(Q,L)$ is convex in Q for fixed $L \in [L_i, L_{i-1}]$. Upon setting $\partial R^u(Q,L)/\partial Q = 0$, we get

$$Q = \left[\frac{2}{h}\left[\bar{d} - \frac{t_{m-1}(\alpha_1) - t_{m-1}(\alpha_2)}{3}\frac{s}{\sqrt{m}}\right]\left\{\begin{array}{l} A + \frac{1}{2}\sigma\sqrt{L}[\pi + \pi_0(1-\beta)](\sqrt{1+k^2}-k) \\ + c_i(L_{i-1}-L) + \sum_{j=1}^{i-1} c_j(b_j - a_j) \end{array}\right\}\right]^{1/2}. \tag{29}$$

Therefore, for given $\alpha_1 > 0$, $\alpha_2 > 0$, and $\alpha_1 + \alpha_2 = \alpha$, we can establish the following algorithm to obtain the optimal ordering strategy in the fuzzy sense.

Algorithm 2. For given q, we let $k_0 = 0$, $k_N = \sqrt{1/q - 1}$ and $k_j = k_{j-1} + (k_N - k_0)/N$, $j = 1, 2, \ldots, N-1$, where N is a dividing number and is large enough. We execute the following procedure:

Step 1: Collect m sample data of annual demand, say $d_1, d_2, \ldots, d_m$, and then evaluate sample mean $\bar{d} = 1/m\sum_{i=1}^m d_i$ and sample standard deviation

$$s = \left[\frac{1}{m-1}\sum_{i=1}^m (d_i - \bar{d})^2\right]^{1/2}.$$

Table 5
Summary of the optimal procedure solution

β	k	$(Q_{\tilde{D}^*}, L_{\tilde{D}^*})$	$R^u(Q_{\tilde{D}^*}, L_{\tilde{D}^*})$
0.0	2.00	(181, 3)	\$4130.28
0.5	2.00	(162, 3)	3739.32
0.8	1.81	(152, 3)	3478.76
1.0	1.39	(142, 4)	3229.26

In addition, for given α_1 and α_2 ($\alpha_1 + \alpha_2 = \alpha$), consulting the t-distribution table to find the values of $t_{m-1}(\alpha_1)$ and $t_{m-1}(\alpha_2)$, where $t_{m-1}(\alpha_i)$ is the upper α_i point of the t-distribution with $m-1$ degrees of freedom, $i = 1, 2$.

Step 2: For given L_i, $i = 0, 1, 2, \ldots, n$, perform (i)–(iii).

(i) For each $k_j \in \{k_0, k_1, \ldots, k_N\}$, $j = 0, 1, 2, \ldots, N$, and L_i, using (29) to compute Q_{k_j}.

(ii) For each pair (Q_{k_j}, L_i), compute the corresponding fuzzy expected annual total cost $R^u(Q_{k_j}, L_i)$, $j = 0, 1, 2, \ldots, N$.

(iii) Find $\min_{j=0,\ 1\ldots,N} R^u(Q_{k_j}, L_i)$. If $R^u(Q_{k(i)}, L_i) = \min_{j=0,\ 1,\ldots,N} R^u(Q_{k_j}, L_i)$, then $(Q_{k(i)}, L_i)$ is the local optimal solution in the fuzzy sense for fixed L_i.

Step 3: Find $\min_{i=0,\ 1,\ldots,n} R^u(Q_{k(i)}, L_i)$. If $R^u(Q_{\tilde{D}^*}, L_{\tilde{D}^*}) = \min_{i=0,\ 1,\ldots,n} R^u(Q_{k(i)}, L_i)$, then $(Q_{\tilde{D}^*}, L_{\tilde{D}^*})$ is the optimal solution in the fuzzy sense.

Example 2. We use the same data as in Example 1, but the random sample of size 9 yields the sample mean of annual demand $\bar{d} = 600$ units/yr and sample standard deviation $s = 30$ units/yr. We seek to find the optimal ordering strategy in the fuzzy sense, where $\alpha_1 = 0.1$ and $\alpha_2 = 0.05$. We solve the cases when $\beta = 0, 0.5, 0.8, 1$, $q = 0.2$ and $N = 200$ (in this situation, we have $k_0 = 0$, $k_{200} = 2$). Consulting the t-distribution table, we find $t_8(0.1) = 1.397$ and $t_8(0.05) = 1.860$. The results of the solution procedure are summarized in Table 5.

5. Concluding remarks

This paper considers the inventory problems accommodating the practical situation. In Section 2, we discuss how to apply the fuzzy concepts to deal with the problem in which no statistical data can be used. On the other hand, when the statistical data are available, we discuss how to combine the statistics and fuzzy technologies to deal with such a problem in Section 4. We note that the optimal solution derived from the total cost function in [6] may not match the real situation, while using the optimal solution derived from the total cost through properties 1 and 2 in this article does.

In future research on this problem, it would be of interest to deal with the problem of fuzzifying the crisp random variable of lead time demand X to a fuzzy random variable and viewing the annual demand D as the crisp variable. Another possible extension of this work may be conducted by fuzzifying both of X and D, and to investigate the optimal ordering strategy for such a case.

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