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Using Randomized Response to Estimate the Proportion and Truthful Reporting Probability in a Dichotomous Finite Population

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ABSTRACT *In this paper, an alternative randomized response procedure is given that allows us to estimate the population proportion in addition to the probability of providing a truthful answer. It overcomes a difficulty associated with traditional randomized response techniques. Properties of the proposed estimators as well as sample size allocations are studied. In addition, an efficiency comparison is carried out to investigate the performance of the proposed technique.*

KEY WORDS: Binomial distribution, estimation of proportion, randomized response

Introduction

The randomized response (RR) technique for asking sensitive questions indirectly to avoid social stigma or fear of reprisals was first introduced by Warner (1965). The objective is to design an effective random device so as to induce each respondent to give truthful answers to sensitive questions without exposing his/her true identity to the interviewer. A number of modifications on Warner's (1965) pioneering technique have been suggested in the literature: see Fox & Tracy (1986), Chaudhuri & Mukherjee (1987, 1988) and Hedayat & Sinha (1991) for the reviews. Some other developments on randomized response sampling in resent years include Kuk (1990), Mangat & Singh (1990), Mangat (1994), Mangat *et al*. (1997), Mahmood *et al*. (1998), Chua & Tsui (2000) and Singh *et al*. (2000).

In practice, incompletely truthful reporting is not an uncommon feature, and it is reasonably assumed that the persons who are members of a sensitive group state honest answers with probability T. An undesirable feature of RR techniques is that under the existing schemes, there is no unbiased estimator of the population proportion π. In addition, one has no way to guess the magnitudes of T and the mean square error of the estimator of π. We attempt to overcome

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these difficulties by suggesting an alternative scheme, which essentially applies Mangat's (1994) technique for two independent sub-samples. The proposed procedure and the principal properties are presented in the next section. Sample size allocations for some practical objectives are given in the section after. In the fourth section, an empirical study is worked out to examine the relative efficiency aspect of the proposed strategy in relation to Mangat's (1994) strategy.

The Proposed Procedure

Consider a dichotomous population in which every individual belongs either to a sensitive group A or to the non-sensitive complement $\bar{A}$. Suppose that we are interested in estimating the population proportion π of individuals who are members of A. Let T be the probability that the respondents belonging to A report the truth. The respondents belonging to $\bar{A}$ have no reason to tell a lie, and thus are completely truthful in their answers.

In the proposed procedure, two independent sub-samples of size n_j, $j=1, 2$, are drawn from the population using simple random sampling with replacement such that $n_1+n_2=n$, the total sample size required. The person in the jth sub-sample is instructed to reply whether he/she is a member of A. If the respondent is not a member of A, he/she is required to use a randomization device R_j consisting of two statements: (a) I am a member of A, and (b) I am a member of $\bar{A}$, represented with probabilities P_j and $(1-P_j)$, $j=1, 2$, respectively. Then the respondent simply gives a 'yes' or 'no' answer depending on the outcome of randomized device R_j without revealing the statement selected. The probability of a 'yes' answer in the jth sub-sample is therefore given by

$$\theta_j = \pi T + (1-\pi)(1-P_j), j=1, 2$$

By the method of moments, the estimator of π is easily shown to be

$$\hat{\pi} = \frac{\bar{Z}_1 - \bar{Z}_2 + P_1 - P_2}{P_1 - P_2}, P_1 \neq P_2$$

and the estimator of T is given by

$$\hat{T} = \frac{(1-P_2)\bar{Z}_1 - (1-P_1)\bar{Z}_2}{\bar{Z}_1 - \bar{Z}_2 + P_1 - P_2}$$

where $\bar{Z}_j = n_j^{-1}\sum_{i=1}^{n_j} Z_{ij}$ is the corresponding observed proportion of 'yes' answers in the jth sub-sample, $j=1, 2$. Here, $\bar{Z}_j$, being the binomial random variable with parameters (n_j, θ_j), is an unbiased estimator of θ_j, $j=1, 2$.

Theorem 1

The estimator $\hat{\pi}$ is unbiased with variance given by

$$Var(\hat{\pi}) = \frac{1}{(P_1-P_2)^2}\left\{\frac{\theta_1(1-\theta_1)}{n_1} + \frac{\theta_2(1-\theta_2)}{n_2}\right\} \quad (1)$$

Proof

Since $\bar{Z}_j$ is distributed as a binomial distribution $B(n_j, \theta_j)$, $j=1, 2$, respectively, the unbiasedness follows immediately. Expression (1) follows from the independence of the sub-samples. This completes the proof.

An unbiased estimator of $Var(\hat{\pi})$ can easily be obtained, which is given in the following theorem.

Theorem 2

An unbiased estimator of variance $Var(\hat{\pi})$ is given by

$$var(\hat{\pi}) = \frac{1}{(P_1 - P_2)^2}\left\{\frac{\bar{Z}_1(1-\bar{Z}_1)}{n_1-1} + \frac{\bar{Z}_2(1-\bar{Z}_2)}{n_2-1}\right\}.$$

Next, let us define $d_1 = (1-P_2)\bar{Z}_1-(1-P_1)\bar{Z}_2$ and $d_2=\bar{Z}_1-\bar{Z}_2+P_1-P_2$, then we have $E(d_1)=(P_1-P_2)\pi T$ and $E(d_2)=(P_1-P_2)\pi$. It follows that $\hat{T}=d_1/d_2$ and $T=E(d_1)/E(d_2)$. Further, we define the following quantities:

$$e_1 = \frac{d_1 - E(d_1)}{E(d_1)} \quad \text{and} \quad e_2 = \frac{d_2 - E(d_2)}{E(d_2)}$$

where $|e_2|$ is assumed to be less than unity such that the function $(1+e_2)^{-1}$ can be expressed as power series. Then the estimation error can be written in terms of e_1 and e_2 as

$$\hat{T}-T=T\{e_1-e_2-e_1e_2+e_2^2+O_p(n^{-3/2})\} \tag{2}$$

and we have

$$E(e_1)=E(e_2)=0$$

$$E(e_1e_2)=\frac{1}{(P_1-P_2)^2\pi^2 T}\left\{\frac{(1-P_2)\theta_1(1-\theta_1)}{n_1}+\frac{(1-P_1)\theta_2(1-\theta_2)}{n_2}\right\} \tag{3}$$

$$E(e_1^2)=\frac{1}{(P_1-P_2)^2\pi^2 T^2}\left\{\frac{(1-P_2)^2\theta_1(1-\theta_1)}{n_1}+\frac{(1-P_1)^2\theta_2(1-\theta_2)}{n_2}\right\}$$

$$E(e_2^2)=\frac{1}{(P_1-P_2)^2\pi^2}\left\{\frac{\theta_1(1-\theta_1)}{n_1}+\frac{\theta^2(1-\theta_2)}{n_2}\right\}$$

The expressions for bias and mean square error of the estimator $\hat{T}$ are as follows.

Theorem 3

To the first degree of approximation, the estimator $\hat{T}$ is biased with

$$Bias(\hat{T})=\frac{1}{(P_1-P_2)^2\pi^2}\left\{\frac{(T-1+P_2)\theta_1(1-\theta_1)}{n_1}+\frac{(T-1+P_1)\theta_2(1-\theta_2)}{n_2}\right\} \tag{4}$$

Proof

From equation (2), omitting terms with power in e_i's higher than the second, taking expectation and then substituting the corresponding expected values in equation (3), we get equation (4). Hence we have the following theorem.

Theorem 4

To the first degree of approximation, the mean square error of the estimator $\hat{T}$ is given by

$$MSE(\hat{T}) = \frac{1}{(P_1 - P_2)^2 \pi^2} \left\{ \frac{(T-1+P_2)^2 \theta_1 (1-\theta_1)}{n_1} + \frac{(T-1+P_1)^2 \theta_2 (1-\theta_2)}{n_2} \right\} \tag{5}$$

Proof

Squaring equation (2), neglecting terms with power in ε_i's higher than the second and then taking expectation, we get $MSE(\hat{T}) = T^2 E(e_1^2 - 2e_1 e_2 + e_2^2)$. On using the corresponding expected values from equation (3), and then after some simple algebra the theorem is obtained.

In addition, the estimators of *Bias*($\hat{T}$) and *MSE*($\hat{T}$) can be found straightforwardly. These are outlined in the following theorem.

Theorem 5

The estimators of *Bias*($\hat{T}$) and *MSE*($\hat{T}$) are respectively given by

$$bias(\hat{T}) = \frac{1}{(P_1 - P_2)^2 \hat{\pi}^2} \left\{ \frac{(\hat{T}-1+P_2)\bar{Z}_1(1-\bar{Z}_1)}{n_1} + \frac{(\hat{T}-1+P_1)\bar{Z}_2(1-\bar{Z}_2)}{n_2} \right\}$$

$$MSE(\hat{T}) = \frac{1}{(P_1 - P_2)^2 \hat{\pi}^2} \left\{ \frac{(\hat{T}-1+P_2)^2\bar{Z}_1(1-\bar{Z}_1)}{n_1} + \frac{(\hat{T}-1+P_1)^2\bar{Z}_2(1-\bar{Z}_2)}{n_2} \right\}$$

Sample Size Allocations

In sample surveys, the total sample size n is fixed from a consideration of available resources. In what follows, we study the appropriate selection of sub-sample sizes with a constraint $n_1 + n_2 = n$ for some practical objectives.

Case 1

Consider the situation where the researcher is interested in minimizing the variance of $\hat{\pi}$. Through a simple application of the Cauchy–Schwarz inequality, the optimum choices of n_1 and n_2 are given by

$$n_1 = \frac{\sqrt{\theta_1(1-\theta_1)}}{\sqrt{\theta_1(1-\theta_1)}+\sqrt{\theta_2(1-\theta_2)}}.n$$

and

$$n_2 = \frac{\sqrt{\theta_2(1-\theta_2)}}{\sqrt{\theta_1(1-\theta_1)}+\sqrt{\theta_2(1-\theta_2)}}.n$$

and the resulting minimum variance is

$$Var_{\min}(\hat{\pi}) = \frac{\{\sqrt{\theta_1(1-\theta_1)} + \sqrt{\theta_2(1-\theta_2)}\}^2}{(P_1 - P_2)^2 n}$$

Case 2

It can easily be verified that the mean square error of the estimator $\hat{T}$ is minimized if the ratio of n_1 and n_2 is chosen to be

$$\frac{n_1}{n_2} = \frac{|T-1+P_2|\sqrt{\theta_1(1-\theta_1)}}{|T-1+P_1|\sqrt{\theta_1(1-\theta_2)}}$$

and the expression for the minimum mean square error is given by

$$MSE_{\min}(\hat{T}) = \frac{\{|T-1+P_2|\sqrt{\theta_1(1-\theta_1)}+|T-1+P_1|\sqrt{\theta_2(1-\theta_2)}\}^2}{(P_1 - P_2)^2 \pi^2 n}$$

Case 3

One may be concerned with the selection of n_1 and n_2 so as to estimate π and T simultaneously and precisely. That is, an attempt is made to minimize VM (say), the product of $Var(\hat{\pi})$ and $MSE(\hat{T})$. From expressions (1) and (5), using the Cauchy–Schwarz inequality, it follows that

$$VM \geqslant \frac{1}{(P_1 - P_2)^4 \pi^2} \left\{ \frac{|T-1+P_2|\theta_1(1-\theta_1)}{n_1} + \frac{|T-1+P_1|\theta_2(1-\theta_2)}{n_2} \right\}^2$$

with equality if and only if

$$|T-1+P_2| = |T-1+P_1|$$

Thus, the sample size allocation for which VM attains its minimum is given by

$$\frac{n_1}{n_2} = \frac{\sqrt{\theta_1(1-\theta_1)}}{\sqrt{\theta_2(1-\theta_2)}}$$

In addition, the minimum value of VM corresponding to this ratio of n_1 and n_2 is

$$VM_{\min} = \frac{1}{(P_1 - P_2)^4 \pi^2 n^2} \{\sqrt{|T-1+P_2|\theta_1(1-\theta_1)} + \sqrt{|T-1+P_1|\theta_2(1-\theta_2)}\}^4$$

Efficiency Comparison

To have an idea about the magnitude of the relative efficiency of the proposed strategy in relation to Mangat's (1994) strategy, we resort to an empirical investigation. Without loss of generality, it is supposed that $P_1 > P_2$. The relative efficiency of the proposed estimator $\hat{\pi}$ with respect to Mangat's estimator $\hat{\pi}_M$ is defined as

$$RE = \frac{MSE(\hat{\pi}_M)}{Var_{\min}(\hat{\pi})} = \frac{\{\theta_M(1-\theta_M) + n\pi^2(1-T)^2\}(P_1 - P_2)^2}{P_M^2\{\sqrt{\theta_1(1-\theta_1)} + \sqrt{\theta_2(1-\theta_2)}\}^2}$$

where

$$\theta_M = \pi T + (1-\pi)(1-P_M)$$

It is well known that the value P_M should be chosen close to unity. In addition, it can be verified that the optimum choice of P_1 remains the same as P_M while P_2 should be chosen as close to zero as practicable. For that reason, we simply set $P_M = P_1$ and $P_2 = 1 - P_1$. The relative efficiency figures are shown in the following tables for the practicable choices of P_1, π and T. Tables 1 and 2 are respectively appropriate for the case where the sample sizes considered are 1000 and 2000.

From these tables, the proposed strategy is more efficient than Mangat's (1994) strategy under certain conditions. Even though some RE values are less than unity, one may regard as a trade-off for being able to obtain an estimator of T and a variance estimator of $\hat{\pi}$ in using our proposed technique instead of using the usual Mangat's (1994) technique. Moreover, it can be observed that the relative efficiency increases with increasing P_1. And the growth of total sample size results in more gain in efficiency. Thus, as far as the efficiency of $\hat{\pi}$ is concerned, larger efficiency is expected for larger P_1 or n.

To sum up, the proposed procedure makes it possible to get admissible estimators for π and T simultaneously and to derive the variance estimation for the estimator of π. One may then check which survey technique is superior in practical application. In addition, the proposed strategy results in better performance than the strategy given by Mangat (1994) for most of the situations. Consequently, the proposed technique is recommended for use in practical sample surveys.

Practical Example

In a survey to estimate the proportion of habitual gamblers among industrial works. Two samples, each of size 50, were taken and each interviewee is required to reply to the direct question, whether he/she belongs to sensitive group A. If

Table 1. Relative efficiency of $\hat{\pi}$ with respect to $\hat{\pi}_M$ for $n = 1000$

		π								
P_1	T	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	0.1	3.14	12.25	27.79	50.87	83.56	129.44	194.89	292.18	448.02
	0.2	2.49	9.56	21.29	38.02	60.41	89.54	127.11	175.76	239.79
	0.3	1.92	7.26	15.97	28.05	43.64	63.01	86.61	115.07	149.29
	0.4	1.43	5.33	11.61	20.18	31.03	44.18	59.72	77.81	98.65
0.7	0.5	1.02	3.72	8.05	13.94	21.34	30.24	40.67	52.66	66.31
	0.6	0.68	2.42	5.21	9.02	13.84	19.68	26.59	34.63	43.89
	0.7	0.42	1.41	3.01	5.23	8.09	11.65	15.98	21.21	27.49
	0.8	0.24	0.68	1.42	2.47	3.87	5.69	8.03	11.07	15.07
	0.9	0.13	0.24	0.44	0.73	1.13	1.70	2.50	3.70	5.62
	0.1	6.64	24.88	54.60	97.26	156.07	236.84	349.99	515.69	778.61
	0.2	5.25	19.30	41.45	71.80	111.22	161.40	225.14	307.00	414.74
	0.3	4.04	14.61	30.90	52.57	79.70	112.75	152.50	200.24	257.86
	0.4	3.00	10.69	22.38	37.67	56.44	78.76	104.88	135.19	170.31
0.8	0.5	2.13	7.45	15.51	25.99	38.77	53.86	71.35	91.46	114.46
	0.6	1.42	4.84	10.04	16.84	25.18	35.10	46.70	60.17	75.78
	0.7	0.87	2.82	5.81	9.79	14.78	20.85	28.14	36.91	47.48
	0.8	0.47	1.36	2.75	4.65	7.12	10.25	14.23	19.34	26.07
	0.9	0.24	0.48	0.85	1.39	2.12	3.11	4.51	6.55	9.78
	0.1	13.82	47.05	96.34	162.72	250.11	365.98	523.99	750.89	1108.58
	0.2	10.83	35.91	71.47	116.91	173.25	242.77	329.43	439.89	585.88
	0.3	8.27	26.89	52.54	84.33	122.38	167.50	221.08	285.32	363.55
	0.4	6.10	19.54	37.76	59.96	86.05	116.34	151.42	192.19	239.95
0.9	0.5	4.30	13.58	26.10	41.26	58.99	79.42	102.90	129.94	161.23
	0.6	2.84	8.82	16.92	26.79	38.38	51.84	67.43	85.55	106.76
	0.7	1.71	5.14	9.86	15.70	22.69	30.97	40.81	52.59	66.95
	0.8	0.90	2.49	4.72	7.57	11.09	15.43	20.83	27.72	36.83
	0.9	0.42	0.86	1.48	2.31	3.40	4.82	6.76	9.55	13.92

the answer of interviewee is 'no', he/she is required to use a randomization device R_j which consists of a jar containing balls of two different colours, say, red and white. In the first (second) jar it contains eight (two) red balls and two (eight) white balls, that is, $P_1 = 0.8$ ($P_2 = 0.2$). The interviewee is requested to report 'yes' or 'no' according to the outcome of this randomization device and the actual status that he/she has with respect to attribute A if this ball is red. On the contrary, the interviewee is requested to report 'yes' or 'no' according to the outcome of this randomization device and the actual status that he/she with respect to attribute $\bar{A}$ if the ball is white. The whole procedure is completed by the respondent unobserved by the interviewer and the response is just one of 'yes' or 'no'. The observations thus obtained are shown in Table 3.

In such a case, we have and $\bar{Z}_1 = (7+8)/50 = 0.3$ and $\bar{Z}_2 = (6+28)/50 = 0.68$. Using the proposed estimators and after simple computation, it follows that $\hat{\pi} = 0.36667$ and $\hat{T} = 0.47273$. In addition, by Theorem 2 and Theorem 5 one can obtain $var(\hat{\pi}) = 0.02424$, $bias(\hat{T}) = -0.00387$ and $mse(\hat{T}) = 0.01598$.

Table 2. Relative efficiency of $\hat{\pi}$ with respect to $\hat{\pi}_M$ for $n = 2000$

		π								
P_1	T	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	0.1	6.21	24.42	55.51	101.68	167.06	258.81	389.72	584.30	895.96
	0.2	4.90	19.04	42.51	75.97	120.75	179.01	254.15	351.46	479.51
	0.3	3.76	14.45	31.87	56.02	87.20	125.95	173.15	230.07	298.50
	0.4	2.78	10.58	23.14	40.28	61.97	88.28	119.37	155.55	197.23
0.7	0.5	1.96	7.36	16.03	27.80	42.60	60.40	81.25	105.24	132.54
	0.6	1.28	4.75	10.33	17.95	27.59	39.28	53.10	69.18	87.70
	0.7	0.76	2.73	5.93	10.36	16.09	23.21	31.88	42.33	54.90
	0.8	0.39	1.27	2.74	4.84	7.64	11.28	15.97	22.05	30.05
	0.9	0.16	0.39	0.78	1.35	2.16	3.29	4.91	7.31	11.15
	0.1	13.15	49.64	109.09	194.42	312.05	473.58	699.88	1031.27	1557.11
	0.2	10.37	38.48	82.78	143.48	222.33	322.70	450.17	613.88	829.35
	0.3	7.94	29.09	61.67	105.02	159.29	225.37	304.88	400.34	515.58
	0.4	5.86	21.24	44.63	75.21	112.76	157.40	209.62	270.24	340.48
0.8	0.5	4.11	14.76	30.87	51.84	77.41	107.58	142.57	182.77	228.78
	0.6	2.69	9.53	19.92	33.52	50.21	70.04	93.25	120.19	151.41
	0.7	1.58	5.48	11.46	19.43	29.40	41.53	56.13	73.66	94.82
	0.8	0.79	2.56	5.34	9.13	14.06	20.33	28.30	38.52	51.98
	0.9	0.32	0.78	1.53	2.59	4.04	6.04	8.83	12.91	19.39
	0.1	27.50	93.98	192.56	325.33	500.11	731.85	1047.85	1501.65	2215.00
	0.2	21.50	71.68	142.79	233.69	346.36	485.39	658.71	879.62	1171.58
	0.3	16.36	53.61	104.92	168.49	244.60	334.84	442.00	570.46	726.92
	0.4	12.02	38.90	75.35	119.75	171.93	232.50	302.66	384.20	479.70
0.9	0.5	8.40	26.96	52.00	82.33	117.78	158.64	205.61	259.68	322.26
	0.6	5.47	17.42	33.63	53.36	76.55	103.47	134.65	170.88	213.32
	0.7	3.20	10.05	19.48	31.16	45.14	61.71	81.38	104.97	133.69
	0.8	1.57	4.73	9.18	14.87	21.91	30.59	41.41	55.21	73.44
	0.9	0.59	1.44	2.67	4.32	6.48	9.34	13.23	18.82	27.59

Table 3. Practical example

Technique used	Sample 1		Sample 2	
Direct response	YES 7	NO 43	YES 6	NO 44
Randomized response	YES 8	NO 35	YES 28	NO 16
Sample Size	50		50	

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