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ORIGINAL ARTICLE

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A neural network application for reliability modelling and condition-based predictive maintenance

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Abstract Traditionally, decisions on the use of machinery are based on previous experience, historical data and common sense. However, carrying out an effective predictive maintenance plan, information about current machine conditions must be made known to the decision-maker. In this paper, a new method of obtaining maintenance information has been proposed. By integrating traditional reliability modelling techniques with a real-time, online performance estimation model, machine reliability information such as hazard rate and mean time between failures can be calculated. Essentially, this paper presents an innovative method to synthesise low level information (such as vibration signals) with high level information (like reliability statistics) to form a rigorous theoretical base for better machine maintenance.

Keywords Cerebellar model articulation controller · Neural network · Predictive maintenance · Weibull proportional hazards model

1 Introduction

It is well known that 99% of machine failures are preceded by some indicators [1]. Therefore, condition-based predictive maintenance is probably the most economical way to maintain machinery. Its idea is to allow the determination of machinery health in a real-time, online fashion. As such, faults can be predicted before they take place. Maintenance can then be scheduled as needed. Reported benefits of predictive maintenance include reduced downtime, lower maintenance costs, and reduction of unexpected catastrophic failures.

The objective of this paper is to combine traditional reliability modelling methods with vibration-based monitoring techniques and artificial neural network technologies in an integrated system to determine the health status of machinery, namely, cerebellar model articulation controller neural network-based machine performance estimation model (CMAC-PEM). In order to verify our methodology, the Weibull proportional hazards model (WPHM) has also been implemented. Additionally, a bearing deterioration experiment was conducted. Its data were used to test both the CMAC-PEM and the WPHM methodologies. Their comparison results and analyses are given in this paper.

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2 Literature review

There are three main types of maintenance, namely improvement maintenance, preventive maintenance, and corrective maintenance. The efforts of improvement maintenance are to reduce or eliminate the need for maintenance entirely. By contrast, corrective maintenance is the repair actions executed after failure occurrence and preventive maintenance denotes all actions intended to keep equipment in good operating condition and to avoid failures [2].

Condition-based maintenance (CBM) is dynamic preventive maintenance in practice. Markov and semi-Markov models have been the preferred approach in simulation and evaluation of CBM [3–5]. Other approaches, like Monte Carlo modelling [6, 7] or an artificial intelligence approach [8], have also been proposed by several researchers.

As a method of modelling the process of age-dependent failures, the concept of hazard modelling is well known in maintenance literature. The proportional hazards model (PHM), developed by Cox [9], extends the traditional hazard model to include correlative information supplied by diagnostic variables. It has been widely used in medical diagnostics, and was first applied to engineering reliability problems by Jardine and Anderson [10]. Kobbacy et al. [11, 12] proposed a heuristic approach for implementing a proportional hazards model to schedule the next maintenance interval on the basis of the equipment's full condition history.

Albus first proposed CMAC [13, 14]. It is capable of very fast learning, and contains certain features of interpolation and approximation [15]. It can learn nonlinear relationships from a very broad category of functions, and converges in a small number of training iterations. Many researchers have applied CMAC to find the solutions of various problems. Lee and Kramer [16] developed a pattern discrimination model based on a CMAC network to analyse robot system degradation. Basically, a CMAC network is defined by a series of mappings, which then forms a special memory association table. It is a supervised learning network, so, in the training stage, the desired output of each input vector must be given. Training consists of adjusting the values in the weight table based on the error between their present output and the desired output.

Vibration analysis has been widely accepted as a most reliable technique for predicting machinery problems [17–19]. Furthermore, vibration trending is a good way to translate a signal into a measure for comparison of machinery health status. Several trending techniques have been developed and studied in the literature [20–22]. In this paper, vibration signals are used for machine condition monitoring.

3 Condition-based reliability modelling system

For predictive maintenance, the determination of timing for maintenance is crucial. It obviously relies on instruments with specific sensors to monitor operations, and to analyse these signals by comparing them with baseline data. Lack of quantitative methods to describe the overall machine condition is the bottleneck of the problem. Even though sensory readings are taken, it is hard to known how "good" or "bad" the machine is, and how long it can run. All these problems lead to the development of a condition-based reliability modelling system, CMAC-PEM. With sensor readings, the CMAC fuses the processed information and estimates machine reliability measures. These measures are then used to form a reliability model for the current machine condition, and then important maintenance information can be derived easily.

3.1 Machine reliability modelling

The random variable used to quantify machine reliability is the "time to failure" denoted by T. The reliability function $R(t)$ of a system is defined in Eq. 1:

$$\begin{aligned} R(t) &= \Pr[T > t] \\ &= 1 - \Pr[T \leq t] \\ &= 1 - F_T(t). \end{aligned} \tag{1}$$

Many parametric failure models have been proposed to describe the behaviour of machine degradation, such as the Poisson, exponential, Weibull and log-normal distributions. The Weibull model is the most popular one because it can accommodate several types of behaviour, such as infant mortality and the various aging defects found in a "bath-tub curve" [23]. The probability density function of a Weibull distribution is defined in Eq. 2:

$$f_T(t) = \frac{\beta}{\theta}\left(\frac{t-c}{\theta}\right)^{\beta-1} \exp\left[-\left(\frac{t-c}{\theta}\right)^{\beta}\right] \tag{2}$$

where β is the shape or slope parameter, θ is the scale parameter and c is the location constant. Once the underlying model of a system is defined, we can easily derive the other reliability functions. As shown in Eq. 3, where the expected value $E[T]$ is also called "mean time between failures" (MTBF), and $\Gamma(x)$ is the well-known gamma function. From Eq. 3, $R(t)$ can be rearranged in the form shown in Eq. 4:

$$\begin{aligned} F_T(t) &= 1 - \exp\left[-\left(\frac{t-c}{\theta}\right)^{\beta}\right] \\ R(t) &= \exp\left[-\left(\frac{t-c}{\theta}\right)^{\beta}\right] \\ h(t) &= \frac{\beta}{\theta}\left(\frac{t-c}{\theta}\right)^{\beta-1} \\ E[T] &= MTBF = c + \theta\Gamma\left(1+\frac{1}{\beta}\right) \\ \mathrm{Var}[T] &= \theta^2\left[\Gamma\left(1+\frac{2}{\beta}\right) - \left(\Gamma\left(1+\frac{1}{\beta}\right)\right)^2\right] \\ \Gamma(x) &= \int_0^{\infty} t^{x-1}e^{-t}dt\ , \mathrm{x} > 1 \end{aligned} \tag{3}$$

$$\ln\left(\ln\left(\frac{1}{R(t)}\right)\right) = \beta\ln(t-c) - \beta\ln(\theta)\,. \tag{4}$$

If the estimates of $R(t)$ can be obtained, validation of the model may be accomplished by computing the linear regression line of $\ln(\ln(1/R(t)))$ with respect to $\ln(t-c)$ as shown in Eq. 4. The slope of the linear regression line gives β and the intercept on the Y-axis gives θ. The value of c is usually zero, but could take different values if $\ln(\ln(1/R(t)))$ values have some kind of curvature rather than a straight line. In such cases, a suitable value for c must be chosen in order to fit a straight line. The chi-square test may be used to validate the model fit for a given confidence level.

After β, θ and c are obtained, $h(t)$, MTBF and $\mathrm{Var}[T]$ can be calculated through Eq. 3. These values will provide useful guidelines for further maintenance scheduling.

3.2 CMAC performance estimation model

The problem at hand is how to calculate the reliability measure, $R(t)$. Since CMAC can provide a very good estimate of machine degradation levels [16], it is assumed that the machine degradation levels estimated from CMAC are equivalent to the machine's reliability measures. A unique CMAC feature, the CMAC confidence level, explained in a later section justifies this assumption.

Therefore, given the reliability measures and the well-defined reliability equations described previously, important maintenance information such as $h(t)$, MTBF, and Var$[T]$ can be computed. In this paper, vibration signal trending techniques such as root mean square (RMS), matched filter root mean square (MFRMS) and Kurtosis were chosen to use as inputs to the CMAC-PEM system.

To justify the equivalence between CMAC outputs and reliability measures, a special feature of the CMAC algorithm must be investigated and implemented. According to research by Lee and Kramer [16], one can calculate the percentage of shared-weight cells in a trained CMAC when a test vector is presented to the CMAC. This percentage value, called the "confidence level", represents the closeness between the test vector and the training vector. The confidence value is regarded as the level of behavioural change of the system being monitored. It is computed as follows:

$$\text{Confidence Level} = (\text{Repeatable\_Weight\_Location\_Count})/K \tag{5}$$

where K is the number of quantising functions, which is actually the total number of cells used for storing the distributed weights of any input vector in a CMAC network. The Repeatable_Weight_Location_Count is the number of shared cells between the trained CMAC and a test vector.

In the context of machine condition monitoring, the input vectors to CMAC are readings from sensors, which are formed by vibration trending indices. If we train a CMAC network, using the readings from the normal condition of a machine, then in the CMAC test stage, the confidence value may be seen as the machine's current health index compared to its normal condition. Hence, using this percentage deviation to represent a machine's reliability measure is a logical consequence. In other words, a low confidence level means the machine has deviated from the normal condition, and requires close monitoring. These confidence values are therefore treated as machine reliability measures in this paper.

3.3 Weibull proportional hazards model: a validation tool

A diagnostic variable in PHM is defined as a measurement reflecting conditions critical to its system performance, such as the level of machine vibration in our case. In this research, PHM was implemented as a validation tool to verify the results from CMAC-PEM.

The assumption of PHM is that the hazard rate of a system is the product of a baseline hazard rate function $h_0(t)$, depending on system age, and a positive function Φ depending only on the values of a set of diagnostic variables. Thus, if z is the vector of diagnostic variables, the hazard rate function is

$$h\,(t, z) = h_0\,(t)\,\Phi\,(z)\,. \tag{6}$$

The most common form for $\Phi(z)$ is

$$\Phi\,(z) = \exp\,(\gamma \cdot z)\,. \tag{7}$$

Note that $\gamma \cdot z$ is the dot product of the vectors γ and z. In Eq. 7 γ represents the coefficients vector of the separate diagnostic variables used in the model. Cox suggests estimating the parameters of $\Phi(z)$ and γ by maximising an expression he calls conditional likelihood or partial likelihood [9]. In the approach taken by Cox, the baseline hazard rate is estimated directly from empirical data. In the work of Jardine and Anderson [10], the underlying baseline hazard rate is assumed to be Weibull distribution, and the empirical data is used to estimate the shape and scale parameters of the Weibull distribution alone with the coefficients of the diagnostic variables [24]. In our paper, since the Weibull assumption was applied, a Weibull-based hazard rate was utilised as a baseline hazard rate.

The hazard rate of the Weibull distribution is given by Eq. 8:

$$\lambda_0(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} \tag{8}$$

where β is the shape parameter and θ is the scale parameter. The Weibull proportional hazards model is

$$\lambda(t, z) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} \exp(\gamma \cdot z) \tag{9}$$

Thus, the partial likelihood function for WPHM is specified as Eq. 10 [24]:

$$L(\beta, \theta, \gamma) = \left[\prod_i \left(\frac{\beta}{\theta}\left(\frac{t_i}{\theta}\right)^{\beta-1} \exp(\gamma \cdot z_i)\right)\right] \cdot \left[\prod_j \exp\left(-\left(\frac{t_j}{\theta}\right)^{\beta-1} \exp(\gamma \cdot z_j)\right)\right]. \tag{10}$$

It is numerically more tractable to utilise the log of Eq. 10 for determining the optimal parameters. Therefore the log-likelihood, as shown in Eq. 11, is the quantity actually maximised.

$$l(\beta, \theta, \gamma) = N \ln(\frac{\beta}{\theta}) + \sum_i \ln\left(\frac{t_i}{\theta}\right)^{\beta-1} + \sum_i \gamma \cdot z_i - \sum_j \left(\left(\frac{t_j}{\theta}\right)^{\beta-1} \exp(\gamma \cdot z_j)\right) \tag{11}$$

Here, N is the total number of running times available. The maximum likelihood estimate is the set of parameter values, which maximises the log-likelihood function.

Once calculated, by substituting the values of parameters β, θ, and γ into Eq. 9, the proportional hazard rate can be found. The cumulative hazard rate $H(t)$ may then be calculated as follows:

$$H\,(t) = \int_0^t h\,(t)\,\mathrm{d}t. \tag{12}$$

Then, the reliability measures, $R(t)$, may be calculated as

$$R\,(t) = \exp\,(-H\,(t))\,. \tag{13}$$

WPHM provides another way to calculate the Weibull parameters, which may allow for comparison of its estimates to the results from CMAC-PEM.

4 Analysis of the bearing deterioration process

As shown in Fig. 1, a bearing deterioration experiment was conducted to test the CMAC-PEM methodology. During the experiment, an extra constant load was added to a test bearing using a turnbuckle, and thereby driving the bearing housing downward. This created a constant stress on the top of the test bearing to accelerate the bearing's deterioration process.

A bearing with a single point inner-race defect was created and used as the test bearing. The motor was turned on and ran continuously while the experiment was proceeding. Vibration signals, collected in the time waveform from the test bearing, were saved into a personal computer for later analysis. Data sets were collected four to five times per day. The entire running time of the experiment was about one week.

After the data collection stage, a band pass filter was used to extract the most critical information from the raw data about the inner race deterioration. The band width of the filter is specified to be between 1300 and 2000 Hz, which is based on the bearing geometry and the test stand fundamental shaft speed. By doing so, the vibration trending indices of the band-passed-filtered signals are able to better indicate the growth of the inner race defect. Figure 2 displays the plot of trending indices from the band pass filtered signals. As can be seen from Fig. 2, the trending indices from the extracted signals show an increasing pattern. In addition, when the increasing pattern of each index in Fig. 2 is compared, the trend of RMS and MFRMS indices appeared to be better than that of Kurtosis. In other words, RMS and MFRMS are more consistent with the physical change inside the bearing, and are able to perform as more dominant factors in testing and training a CMAC network.

Based on the procedure of the CMAC-PEM, the data shown in Fig. 2 were used to train and test a CMAC network. First, we defined the data sets collected at the beginning of the experiment as the normal condition of the test stand. They were used to train a CMAC network. The remaining data sets were used as test vectors for the trained CMAC. The test results are shown in CMAC-$R(t)$ curve of Fig. 3, where the Y-axis represents the estimated test stand confidence level and X-axis represents the running time. As expected, Fig. 3 depicted a decreasing trend since the test stand confidence level was declining due to the growing inner race defect. Using CMAC-$R(t)$ curve as the CMAC estimated reliability measures, the underlying failure model, the Weibull model, and its model parameters β, θ, and c, could be obtained. Table 2 lists the computed results.

Once we have these parameters, the Weibull model reliability curve can be drawn as shown in the Estimated-$R(t)$ curve of Fig. 3. The chi-square test for the estimated parameters was performed, which validated a significant model fit for a 95% confidence level. In Fig. 3, the Estimated-$R(t)$ curve indicates that the predicted probability of the machine surviving over 150 h is very close to zero. In fact, during the experiment, the test stand stopped running due to a broken shaft and the experiment was terminated at about 140 running hours.

To verify the results of CMAC-PEM, the same bearing deterioration data set was used again. The WPHM method was formed to fit the data set. The coefficients of the model (β, θ, and γ) may be found via maximum likelihood estimation.

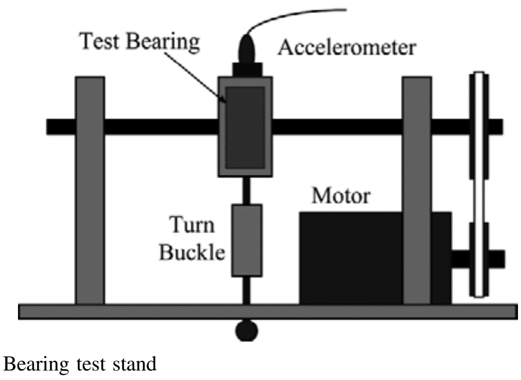

Fig. 1. Bearing test stand

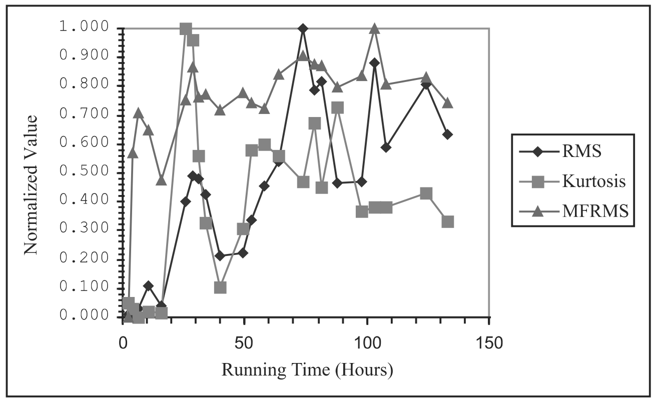

Fig. 2. Vibration trending indices plot of the test bearing band pass filtered signals

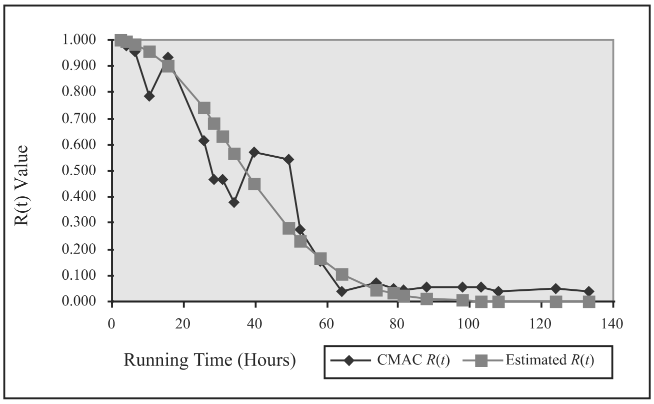

Fig. 3. CMAC reliability curve of the inner race defect bearing and calculated Weibull model reliability curve

Table 1. Summary of Weibull proportional hazards modelling for the bearing data

	Concomitant parameters			Weibull parameters		
	RMS (γ_1)	Kurtosis (γ_2)	MFRMS (γ_3)	β	θ	LL*
Model 1	0.9090	−0.0985	1.3501	2.0961	50.1005	−19.201
Model 2	0.8860	—	1.2951	2.0820	50.0987	−19.265
Model 3	1.3260	—	—	1.8874	51.7405	−21.695

*Log likelihood

The Weibull proportional hazards model is fitted to the data and the model parameters are calculated through the LIFEREG routine in SAS [25].

The model may be refitted in order to improve the parameter estimates by dropping the non-statistical-significant diagnostic variables.

Choose the estimated parameters from the best-fitted model. Calculate the necessary reliability functions and compare results with estimates from the CMAC-PEM method.

Since we used three diagnostic variables (RMS, Kurtosis and MFRMS), the first model derived was

$$h(t,z) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}\exp(\gamma\cdot z)$$
$$= \exp\left[\gamma_1(\text{RMS}) + \gamma_2(\text{Kurtosis}) + \gamma_3(\text{MFRMS})\right] \times \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} \qquad (14)$$

where γ_l, γ_2 and γ_3 are coefficients of the three trending indices and t is the running time variable. The analysis results of Model 1 shows that the chi-square statistic for parameter γ_2 is not significantly larger than its approximate standard error. Therefore, the corresponding concomitant variable of γ_2 was removed from the model.

The second model that removed the Kurtosis variable was then refitted to the data. The second model is

$$h(t,z) = \exp\left[\gamma_1\,(\text{RMS}) + \gamma_3\,(\text{MFRMS})\right]\frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}. \qquad (15)$$

Table 1 summarises the WPHM results for the test bearing data. From the table, one can see that although the Kurtosis variable has been removed from the first model to form the second model, the estimated parameters are not significantly different between Model 1 and Model 2. Also, the values of log likelihood (LL) are shown to be slightly reduced, from −19.201 to −19.265. These indicate that Kurtosis is a redundant variable in the model. In other words, RMS and MFRMS are the dominant variables. This conclusion is consistent with the findings shown in Fig. 2, where the RMS and MFRMS plots show better increasing trends that reflect the physical changes inside the bearing than that of Kurtosis.

When both Kurtosis and MFRMS indices are not in the model, the log likelihood value drops 2.43 points from −19.265 in Model 2 to −21.695 in Model 3, indicating that the parameter estimation does not improve by dropping MFRMS. Also, from Model 2 to Model 3, the parameters change significantly (see γ_1 in Table 1), showing the importance of MFRMS index in the model. Therefore, the best model to fit the test stand data would be Model 2.

There are two purposes of implementing WPHM. The first is to investigate the individual influence of each diagnostic variable in the model, which has been performed by the model fitting. The second is to verify the proposed CMAC-PEM methodology for machine reliability estimation. Table 2 lists the Weibull parameters estimated from Model 2 and CMAC-PEM. For the slope parameter, β, the two estimates are only slightly different. For the scale parameter, θ, the estimate from CMAC-PEM is 44.1968, which is less than the 50.0987 from WPHM. Note that in WPHM RMS and MFRMS these are variables – not constants. In order to plot the curves, extreme diagnostic variable values were specified. Fig. 4 displays the original CMAC estimated reliability measures and two boundary reliability curves. The U-WPHM-$R(t)$ curve was obtained by assuming the RMS and MFRMS values were from the initial state of the test stand.

Table 2. Weibull parameter estimates for CMAC-PEM and WPHM

Parameter	CMAC-PEM estimate	WPHM estimate*
β	2.1765	2.0820
θ	44.1968	50.0987
c	0	0

*From model 2

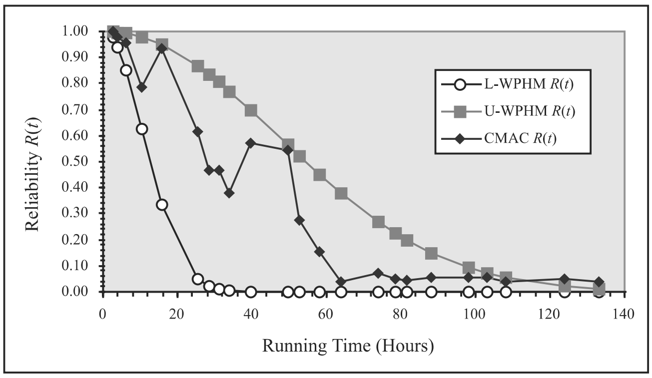

Fig. 4. CMAC reliability curve and two boundary WPHM reliability curves

In contrast, the L-WPHM-$R(t)$ curve was calculated by using the last-state RMS and MFRMS readings, which were the values taken before the shaft broke. These two curves represent the worst and best case reliability curves for the test stand used in the WPHM method. Note that most CMAC estimated reliability values are located inside the area covered by the two boundary WPHM reliability curves. By examining Fig. 4, one can see that the CMAC estimated reliability measures are inside the reliability area calculated from the WPHM. These results verify the robustness of the CMAC-PEM methodology in estimating machine reliability.

5 Conclusions

The major content presented in this paper is the proposed CMAC-PEM methodology for machine reliability modelling. Our objective is to combine the traditional reliability modelling method and vibration-based machine condition monitoring techniques to estimate machine reliability. In the paper, a CMAC neural network-based machine performance estimation model has been introduced. CMAC-PEM was used to fuse sensory data and to predict machine reliabilities. Then, the underlying Weibull reliability model of the machine can be established. The model provides several reliability statistics, which may be used as basic information for further predictive maintenance. A bearing deterioration experiment has been conducted to generate a real-world data set to test the robustness of CMAC-PEM. A validation tool, the Weibull proportional hazards model, has been used to verify CMAC-PEM results. In-depth analyses of WPHM on the bearing data have been provided in the paper.

From this paper, it can be concluded that the CMAC-PEM methodology is a reliable and robust system for online machine reliability analysis. Most importantly, CMAC-PEM can realise the concept of condition-based predictive maintenance.

For future research, besides the implemented vibration trend indices, other information, such as temperature, pressure, and oil analysis, can also be added into CMAC input vectors to enrich the information base. By doing so, an advanced sensor fusion implementation would be a possible extension to the current work.

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