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An EOQ model for perishable items under stock-dependent selling rate and time-dependent partial backlogging

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Abstract

In the present paper, we extend Padmanabhan and Vrat's model by proposing a time-proportional backlogging rate to make the theory more applicable in practice. The existence and uniqueness of the solutions of the relevant systems are examined. Subsequently, a numerical example is presented to illustrate the application of developed model.

Keywords: EOQ; Perishable items; Partial backlogging

1. Introduction

In the classical economic order quantity (EOQ) model, it is often assumed that the shortages are either completely backlogged or completely lost. As a physical phenomenon, some customers may like to prefer backlogging during the shortage period, while the others would not. In 1995, Padmanabhan and Vrat [1] considered an EOQ model for perishable items with stock-dependent demand. Under instantaneous replenishment with zero lead time, they presented three models: without backlogging, with complete backlogging and partial backlogging. Recently, Chung and Dye [2] presented the necessary and sufficient conditions of the existence and uniqueness of the optimal solutions of the profit per unit time functions without backlogging and with complete backlogging. However, the uniqueness of the optimal solution for the partial backlogging case remained further to unexplored.

In Padmanabhan and Vrat's [1] model, the backlogging function was assumed to be dependent on the amount of demand backlogged. Therefore, the more the amount of demand backlogged, the smaller the demand to accept backlogging would be. Their definition of backlogging rate, however, seems to be inappropriate under some circumstances. In real life, for fashionable commodities and high-tech products with short product life cycle, the length of the waiting time for the next replenishment is the main factor for

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deciding whether the backlogging will be accepted or not. The willingness of a customer to wait for backlogging during a shortage period is decline with the length of the waiting time. To reflect this phenomenon, Chang and Dye [3] developed an inventory model in which the proportion of customers who would like to accept backlogging is the reciprocal of a linear function of the waiting time.

Furthermore, the opportunity cost due to lost sales was not taken into account in Padmanabhan and Vrat's [1] analysis. During the shortages period with partial backlogging, we must distinguish between the backorders and lost sales cases. With a lost sale, the customer's demand for the item is lost and presumably filled by a competitor. It can be considered as the loss of profit on the sales. Cost incurred from lost sales, is a fixed cost per unit demanded which cannot be met from inventory. Consequently, the opportunity cost due to lost sales should be considered in the modeling. The opportunity cost due to lost sales is generally defined as opportunity cost = gross profit margin + loss of goodwill [4].

Hence, the main purpose of this note is to amend the paper of Padmanabhan and Vrat [1] with a view to making the model more relevant and applicable in practice. In the next section, the assumptions and notations related to this study are presented. In Section 3, we present the mathematical model and then prove that the optimal replenishment policy not only exists but is also unique. In the last two sections, the model is illustrated with a numerical example adopted from Padmanabhan and Vrat [1] and concluding remarks are provided.

2. Assumptions and notations

Basically, the proposed model is developed under the same assumptions and notations adopted by Padmanabhan and Vrat [1], except the one related to the time-proportional backlogging rate.

1. Replenishment rate is infinite and lead time is zero.
2. The distribution of time to deterioration of the items follows exponential distribution with parameter θ (i.e. constant rate of deterioration).
3. The unit cost C and the inventory carrying cost as fraction i, per unit per unit time, are known and constant.
4. S, the selling price per unit and A, the ordering cost per order, are known and constant.
5. T is the cycle time and t_1 is the time up to which inventory is positive in a cycle.
6. The demand rate function $D(t)$, is deterministic and is a known function of instantaneous stock level $I(t)$; the functional $D(t)$ is given by

$$D(t) = \begin{cases} \alpha + \beta I(t), & 0 \leqslant t < t_1, \\ \alpha, & t_1 \leqslant t < T, \end{cases}$$

   where α, β are positive constants.
7. Shortages are allowed and unsatisfied demand is backlogged at the rate of $1/[1+\delta(T-t)]$. The backlogging parameter δ is a positive constant, and $t_1 \leqslant t < T$. Shortage cost is C_2 per unit per unit time and R is the fixed opportunity cost of lost sales.
8. $P(t_1, T)$ represents the profit function per unit time.

3. Mathematical formulation

Using above assumptions, the inventory level follows the pattern depicted in Fig. 1. The depletion of the inventory occurs due to the combined effects of the demand and deterioration in the interval $[0, t_1)$ and the

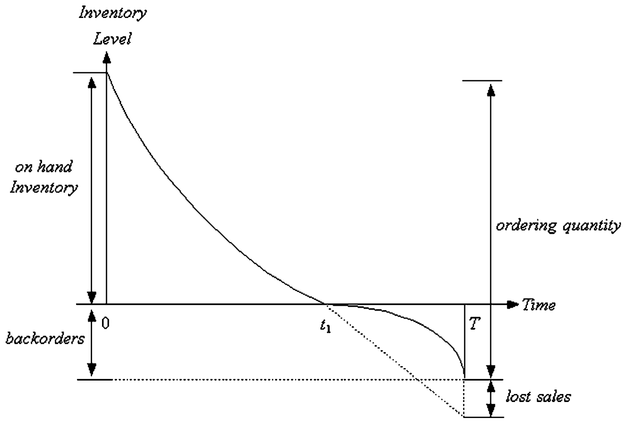

Fig. 1. Graphical representation of inventory system.

demand backlogged in the interval $[t_1, T)$. Hence, the variation of inventory level, $I(t)$, with respect to time can be described by the following differential equation:

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = \begin{cases} -\alpha - \beta I(t) - \theta I(t), & 0 \leqslant t < t_1, \\ -\alpha/[1+\delta(T-t)], & t_1 \leqslant t < T, \end{cases} \tag{1}$$

with boundary condition $I(t_1) = 0$. The solution of (1) is

$$I(t) = \begin{cases} \alpha[\mathrm{e}^{(\beta+\theta)(t_1-t)} - 1]/(\beta+\theta), & 0 \leqslant t < t_1, \\ -\alpha\{\ln[1+\delta(T-t_1)] - \ln[1+\delta(T-t)]\}/\delta, & t_1 \leqslant t < T. \end{cases} \tag{2}$$

Based on (2), the profit function consists of the following elements:

1. Ordering cost per cycle $= A$.
2. Holding cost per cycle $= Ci\int_0^{t_1} \alpha[\mathrm{e}^{(\beta+\theta)(t_1-t)} - 1]/(\beta+\theta)\,\mathrm{d}t$.
3. Shortage cost per cycle $= C_2\int_{t_1}^{T} \alpha\{\ln[1+\delta(T-t_1)] - \ln[1+\delta(T-t)]\}/\delta\,\mathrm{d}t$.
4. Opportunity cost due to lost sales per cycle $= \alpha R\int_{t_1}^{T}\{1 - 1/[1+\delta(T-t)]\}\,\mathrm{d}t$.
5. Purchase cost per cycle $= \alpha C[\mathrm{e}^{(\beta+\theta)t_1} - 1]/(\beta+\theta) + \alpha C\ln[1+\delta(T-t_1)]/\delta$.
6. Sales revenue per cycle $= S\{\int_0^{t_1} \alpha + \beta I(t)\,\mathrm{d}t + \int_{t_1}^{T} \alpha/[1+\delta(T-t)]\,\mathrm{d}t\}$.

Conjunct with the relevant costs mentioned above, the profit per unit time of our model leads to

$$\begin{aligned} P(t_1, T) &= \frac{1}{T}\{\text{sales revenue} - \text{ordering cost} - \text{holding cost} - \text{shortage cost} - \text{opportunity cost} \\ &\qquad - \text{purchase cost}\} \\ &= \frac{\alpha[\beta S - (i+\beta+\theta)C]}{(\beta+\theta)^2 T}[\mathrm{e}^{(\beta+\theta)t_1} - 1 - (\beta+\theta)t_1] + \frac{\alpha(S-C)}{\delta T}\ln[1+\delta(T-t_1)] - \frac{A}{T} \\ &\quad + \frac{\alpha(S-C)t_1}{T} - \frac{\alpha(C_2+R\delta)}{\delta^2 T}\{\delta(T-t_1) - \ln[1+\delta(T-t_1)]\}. \end{aligned} \tag{3}$$

The solutions for optimal t_1^* and T^* are determined from

$$\frac{\partial P(t_1,T)}{\partial t_1} = \frac{\alpha[\beta S-(i+\beta+\theta)C]}{(\beta+\theta)T}[\mathrm{e}^{(\beta+\theta)t_1}-1] + \frac{\alpha[C_2+\delta(R+S-C)](T-t_1)}{[1+\delta(T-t_1)]T} = 0 \tag{4}$$

and

$$\begin{aligned}\frac{\partial P(t_1,T)}{\partial T} &= -\frac{\alpha[\beta S-(i+\beta+\theta)C]}{(\beta+\theta)^2T^2}[\mathrm{e}^{(\beta+\theta)t_1}-1-(\beta+\theta)t_1] - \frac{\alpha(S-C)}{\delta T^2}\ln[1+\delta(T-t_1)] + \frac{\alpha(S-C)}{T[1+\delta(T-t_1)]}\\ &\quad + \frac{A}{T^2} - \frac{\alpha(S-C)t_1}{T^2} + \frac{\alpha(C_2+R\delta)}{\delta^2T^2}\{\delta(T-t_1)-\ln[1+\delta(T-t_1)]\} - \frac{\alpha(C_2+R\delta)(T-t_1)}{T[1+\delta(T-t_1)]} = 0.\end{aligned} \tag{5}$$

After rearranging the terms in (4) and (5), we thus get

$$\frac{\beta S-(i+\beta+\theta)C}{\beta+\theta}[\mathrm{e}^{(\beta+\theta)t_1}-1] = -\frac{[C_2+\delta(R+S-C)](T-t_1)}{1+\delta(T-t_1)} \tag{6}$$

and

$$\begin{aligned}&-\frac{\alpha[\beta S-(i+\beta+\theta)C]}{(\beta+\theta)^2}[\mathrm{e}^{(\beta+\theta)t_1}-1-(\beta+\theta)t_1] - \frac{\alpha(S-C)}{\delta}\ln[1+\delta(T-t_1)] + \frac{\alpha(S-C)T}{1+\delta(T-t_1)}\\ &\quad + A - \alpha(S-C)t_1 + \frac{\alpha(C_2+\delta R)}{\delta^2}\{\delta(T-t_1)-\ln[1+\delta(T-t_1)]\} - \frac{\alpha(C_2+\delta R)T(T-t_1)}{1+\delta(T-t_1)} = 0.\end{aligned} \tag{7}$$

From the analysis carried out so far, we have the following propositions:

Proposition 1. *If $\beta S-(i+\beta+\theta)C \geqslant 0$, then the optimal solution of maximum profit, $P(t_1,T)$, does not exist.*

Proof. If $\beta S-(i+\beta+\theta)C > 0$, we can get (6) holds if and only if $(T-t_1) < 0$. It is a contradiction, because by assumption, we have $t_1 \leqslant T$. Similarly, if $\beta S-(i+\beta+\theta)C = 0$, then, from (6), we obtain $T = t_1$. Substituting this into (7), we get $A = 0$, which is contradictory, because the ordering cost per cycle, A, is positive. Therefore, if $\beta S-(i+\beta+\theta)C \geqslant 0$, the optimal solution of $P(t_1,T)$ does not exist. □

A simple managerial implication of the result follows: $\beta S-(i+\beta+\theta)C = \beta(S-C)-(i+\theta)C$, where $\beta(S-C)$ is the benefit received from a unit of inventory and $(i+\theta)C$ is the cost due to a unit of inventory. Consequently, if $\beta(S-C)-(i+\theta)C \geqslant 0$, then building inventory is profitable. Thus, we should display inventory to maximum as possible as we can, and, in this case, the profit per unit time $P(t_1,T)$ will reach to infinite. It is impossible. Therefore, when $\beta S-(i+\beta+\theta)C \geqslant 0$, we can not find a finite value of (t_1,T), such that $P(t_1,T)$ is maximum and finite.

Proposition 2. *If $\beta S-(i+\beta+\theta)C < 0$, then the point (t_1^*,T^*) which solves (6) and (7) simultaneously not only exists but is also unique.*

Proof. To prove the uniqueness of the solution, by using (6), we set $x = T-t_1$ and let

$$F(x) = \frac{\beta S-(i+\beta+\theta)C}{\beta+\theta}[\mathrm{e}^{(\beta+\theta)t_1}-1] + \frac{[C_2+\delta(R+S-C)]x}{1+\delta x}, \quad x \geqslant 0. \tag{8}$$

Taking the first derivative of $F(x)$ with respect to x, we have

$$\frac{\mathrm{d}F(x)}{\mathrm{d}x} = \frac{C_2 + (R + S - C)\delta}{(1 + \delta x)^2} > 0.$$

Hence, $F(x)$ is a strictly increasing function in $x \in [0, \infty)$.

Because, $\beta S - (i + \beta + \theta) < 0$, thus, from (8), we get $F(0) < 0$. Besides, it can be shown that if

$$t_1 < \hat{t}_1 \equiv \frac{1}{\beta + \theta} \ln \left[1 - \frac{(\beta + \theta)[C_2 + \delta(R + S - C)]}{\delta[\beta S - (i + \beta + \theta)C]}\right],$$

then

$$\lim_{x \to \infty} F(x) = \frac{\beta S - (i + \beta + \theta)C}{\beta + \theta}[\mathrm{e}^{(\beta+\theta)t_1} - 1] + \frac{C_2 + \delta(R + S - C)}{\delta} > 0.$$

Therefore, there exists a unique $x^* \in (0, \infty)$ such that $F(x^*) = 0$. From this, we can obtain the following result: Once we get the value t_1, then, from the value $x^* \in (0, \infty)$, $T\ (> t_1)$ can be uniquely determined as a function of t_1.

Next, in order to prove the existence of the solution, by taking implicit differentiation on (6) with respect to t_1, we have

$$[\beta S - (i + \beta + \theta)C]\mathrm{e}^{(\beta+\theta)t_1} = -\frac{C_2 + \delta(R + S - C)}{[1 + \delta(T - t_1)]^2}\left(\frac{\mathrm{d}T}{\mathrm{d}t_1} - 1\right). \tag{9}$$

Since $C_2 + \delta(R + S - C) > 0$ and $\beta S - (i + \beta + \theta)C < 0$, the previous equation holds if and only if $\mathrm{d}T/\mathrm{d}t_1 - 1 > 0$. Furthermore, from (7), we let

$$\begin{aligned} G(t_1) = &-\frac{\alpha[\beta S - (i + \beta + \theta)C]}{(\beta + \theta)^2}[\mathrm{e}^{(\beta+\theta)t_1} - 1 - (\beta + \theta)t_1] - \frac{\alpha(S - C)}{\delta}\ln[1 + \delta(T - t_1)] \\ &+ \frac{\alpha(S - C)T}{1 + \delta(T - t_1)} + A - \alpha(S - C)t_1 + \frac{\alpha(C_2 + \delta R)}{\delta^2}\{\delta(T - t_1) - \ln[1 + \delta(T - t_1)]\} \\ &- \frac{\alpha(C_2 + \delta R)T(T - t_1)}{1 + \delta(T - t_1)}. \end{aligned} \tag{10}$$

Due to the relations shown in (6) and $\mathrm{d}T/\mathrm{d}t_1 - 1 > 0$, we get

$$\begin{aligned} \frac{\mathrm{d}G(t_1)}{\mathrm{d}t_1} = &-\frac{\alpha[\beta S - (i + \beta + \theta)C]}{\beta + \theta}[\mathrm{e}^{(\beta+\theta)t_1} - 1] - \frac{\alpha[C_2 + \delta(R + S - C)](T - t_1)}{1 + \delta(T - t_1)} \\ &- \frac{\alpha[C_2 + \delta(R + S - C)]T}{[1 + \delta(T - t_1)]^2}\left(\frac{\mathrm{d}T}{\mathrm{d}t_1} - 1\right) = -\frac{\alpha[C_2 + \delta(R + S - C)T}{[1 + \delta(T - t_1)]^2}\left(\frac{\mathrm{d}T}{\mathrm{d}t_1} - 1\right) < 0. \end{aligned}$$

As a result, $G(t_1)$ is a strictly decreasing function of t_1.

Because

$$\begin{aligned} \lim_{t_1 \to \widehat{t_1}^-} G(t_1) = \lim_{t_1 \to \widehat{t_1}^-} \Bigg\{ &-\frac{\alpha[\beta S - (i + \beta + \theta)C]}{(\beta + \theta)^2}[\mathrm{e}^{(\beta+\theta)t_1} - 1 - (\beta + \theta)t_1] - \frac{\alpha(S - C)}{\delta}\ln[1 + \delta(T - t_1)] \\ &+ \frac{\alpha(S - C)T}{1 + \delta(T - t_1)} + A - \alpha(S - C)t_1 + \frac{\alpha(C_2 + \delta R)}{\delta^2}\{\delta(T - t_1) - \ln[1 + \delta(T - t_1)]\} \\ &- \frac{\alpha(C_2 + \delta R)T(T - t_1)}{1 + \delta(T - t_1)} \Bigg\} \end{aligned}$$

$$
\begin{aligned}
&= -\frac{\alpha[\beta S-(i+\beta+\theta)C]}{(\beta+\theta)^2}[e^{(\beta+\theta)\hat{t}_1}-1-(\beta+\theta)\hat{t}_1]+A\\
&\quad+\lim_{t_1\to\hat{t}_1^-}\left\{-\frac{\alpha[C_2+\delta(R+S-C)]}{\delta^2}\ln[1+\delta(T-t_1)]+\frac{\alpha(S-C)(T-t_1)}{1+\delta(T-t_1)}\right.\\
&\qquad-\frac{\alpha\delta(S-C)(T-t_1)t_1}{1+\delta(T-t_1)}+\frac{\alpha(C_2+\delta R)(T-t_1)}{\delta}-\frac{\alpha(C_2+\delta R)(T-t_1)^2}{1+\delta(T-t_1)}\\
&\qquad\left.-\frac{\alpha(C_2+\delta R)(T-t_1)t_1}{1+\delta(T-t_1)}\right\}\\
&= -\frac{\alpha[\beta S-(i+\beta+\theta)C]}{(\beta+\theta)^2}[e^{(\beta+\theta)\hat{t}_1}-1-(\beta+\theta)\hat{t}_1]+A-\infty<0\\
&\quad\left(\text{since we have }\lim_{t_1\to\hat{t}_1^-}(T-t_1)=\infty\text{ from (6)}\right).
\end{aligned}
$$

And, from (6), $T=t_1=0$ as $t_1=0$, it gets $G(0)=A>0$. Therefore, there exists a unique $t_1^*\in(0,\hat{t}_1)$ such that $G(t_1^*)=0$. This completes the proof. □

Proposition 3. *If $\beta S-(i+\beta+\theta)C<0$, then the (t_1^*,T^*) which solves Eqs.* (6) *and* (7) *simultaneously is the global maximum of the profit per unit time.*

Proof. Now, we examine the corresponding second-order sufficient conditions for the optimal solutions. Since

$$
\left.\frac{\partial^2P(t_1,T)}{\partial t_1^2}\right|_{(t_1,T)=(t_1^*,T^*)}=\frac{\alpha[\beta S-(i+\beta+\theta)C]}{T^*}e^{(\beta+\theta)t_1^*}-\frac{\alpha[C_2+\delta(R+S-C)]}{T^*[1+\delta(T^*-t_1^*)]^2}<0,
$$

$$
\left.\frac{\partial^2P(t_1,T)}{\partial T^2}\right|_{(t_1,T)=(t_1^*,T^*)}=-\frac{\alpha[C_2+\delta(R+S-C)]}{T^*[1+\delta(T^*-t_1^*)]^2}<0,
$$

and

$$
\left.\frac{\partial^2P(t_1,T)}{\partial t_1\partial T}\right|_{(t_1,T)=(t_1^*,T^*)}=\frac{\alpha[C_2+\delta(R+S-C)]}{T^*[1+\delta(T^*-t_1^*)]^2},
$$

it can be easily verified that

$$
\left|\left.\frac{\partial^2P(t_1,T)}{\partial t_1^2}\right|_{(t_1,T)=(t_1^*,T^*)}\right|>\left|\left.\frac{\partial^2P(t_1,T)}{\partial t_1\partial T}\right|_{(t_1,T)=(t_1^*,T^*)}\right|,
$$

$$
\left|\left.\frac{\partial^2P(t_1,T)}{\partial T^2}\right|_{(t_1,T)=(t_1^*,T^*)}\right|=\left|\left.\frac{\partial^2P(t_1,T)}{\partial t_1\partial T}\right|_{(t_1,T)=(t_1^*,T^*)}\right|,
$$

and

$$
|\mathbf{H}|=\left.\frac{\partial^2P(t_1,T)}{\partial t_1^2}\right|_{(t_1,T)=(t_1^*,T^*)}\times\left.\frac{\partial^2P(t_1,T)}{\partial T^2}\right|_{(t_1,T)=(t_1^*,T^*)}-\left[\left.\frac{\partial^2P(t_1,T)}{\partial t_1\partial T}\right|_{(t_1,T)=(t_1^*,T^*)}\right]^2>0.
$$

Hence, the Hessian matrix $\mathbf{H}$ at point (t_1^*, T^*) is negative definite. Consequently, we can conclude that the stationary point for our optimization problem is a global maximum. This completes the proof. □

4. Numerical example

Because t_1^* and T^* cannot be determined in closed forms from (6) and (7), they have to be solved numerically using some computer algorithm. It is trivial from (6) that

$$T = t_1 - \frac{[\beta S - (i+\beta+\theta)C][\mathrm{e}^{(\beta+\theta)t_1} - 1]}{[C_2 + \delta(R+S-C)](\beta+\theta) + \delta[\beta S - (i+\beta+\theta)C][\mathrm{e}^{(\beta+\theta)t_1} - 1]}. \tag{11}$$

Substituting the result into (10), we can obtain the value of t_1^* from $G(t_1) = 0$ by using Newton–Raphson Method (or any bisection method). We then get T^* by using (11). Therefore, we have the corresponding maximum profit per unit time $P(t_1^*, T^*)$ which can be obtained by (3).

For the special case, $\delta \to \infty$ (i.e. complete lost sales), from (11), we get $T^* = t_1^*$. In this situation, the optimal solution does not allow shortage.

To solve the nonlinear equation, $G(t_1) = 0$, the subroutine "FindRoot" in the Mathematica version 4.0 is used to find the solution. To illustrate the proposed model, we consider the data as the example in Padmanabhan and Vrat [1]: $\alpha = 600$, $\beta = \{0.2, 0.3, 0.4\}$, $A = 250$, $\theta = 0.05$, $C = 5$, $i = 0.35$, $C_2 = 3$ and $S = 7$. Besides, $R = 5$ and $\delta = \{1, 5, 10, 25, 50\}$. The computed results are shown in Table 1. Also, the results of special cases, $\delta = 0$ (i.e. complete backlogging) and $\delta \to \infty$ (i.e. complete lost sales) are listed in the same table.The following inferences can be made from the results in Table 1.

1. For fixed β, increasing the value of δ will result in a decrease in the optimal profit, yet the optimal service rate (t_1^*/T^*) is increasing and approaches to 1.
2. For fixed β, the maximum profit occurred at $\delta = 0$ (i.e. complete backlogging), and the minimum profit occurred at $\delta \to \infty$ (i.e. complete lost sales).
3. The optimal profit with partial backlogging is more sensitive to δ when its value is small.
4. For fixed δ, if the value of β is increasing, then the optimal profit and the optimal service rate (t_1^*/T^*) are increasing simultaneously.

Table 1
Effects of β and δ on profit

β		Complete backlogging ($\delta = 0$)	δ					Complete lost sales ($\delta \to \infty$)
			1	5	10	25	50	
0.2	t_1^*	0.5519	0.6296	0.6661	0.6733	0.6781	0.6798	0.6816
	T^*	0.8674	0.7521	0.7021	0.6925	0.6861	0.6839	0.6816
	t_1^*/T^*	0.6362	0.8372	0.9487	0.9723	0.9884	0.9941	1.0000
	$P(t_1^*, T^*)$	631.9	545.4	504.2	496.0	490.6	488.6	486.6
0.3	t_1^*	0.5877	0.6617	0.6955	0.7021	0.7065	0.7080	0.7096
	T^*	0.8921	0.7781	0.7295	0.7201	0.7140	0.7118	0.7096
	t_1^*/T^*	0.6588	0.8504	0.9535	0.9750	0.9895	0.9947	1.0000
	$P(t_1^*, T^*)$	651.9	574.5	538.5	531.4	526.7	525.1	523.4
0.4	t_1^*	0.6308	0.7006	0.7315	0.7374	0.7414	0.7428	0.7442
	T^*	0.9226	0.8102	0.7632	0.7542	0.7483	0.7463	0.7442
	t_1^*/T^*	0.6837	0.8646	0.9585	0.9777	0.9907	0.9953	1.0000
	$P(t_1^*, T^*)$	674.9	607.0	576.3	570.3	566.4	565.0	563.6

5. Concluding remarks

In this paper, an inventory model is developed for deteriorating items with stock-dependent demand, permitting shortages and time-proportional backlogging rate. In particular, the backlogging rate is considered to be a decreasing function of the waiting time for the next replenishment. This assumption is more realistic. In practice, we can observe periodically the proportion of demand which would accept backlogging and the corresponding waiting time for the next replenishment. Then the statistical techniques, such as the nonlinear regression method, can be used to estimate the backlogging rate. The analytical formulations of the problem on the general framework described have been given. The condition which guarantees the unique solution is obtained and the complete proof of corresponding second-order sufficient conditions for optimum is also provided.

Furthermore, the results of above sensitivity analysis are consistent with the intuitive reasoning. For fixed β, the larger the value of δ is, the smaller the proportion of customers who would accept backlogging at time t. Hence, in order to maximize the profit per unit time, the retailer should add the fraction of each cycle in which there is no shortage. We also find that the optimal profit per unit time with partial backlogging is more sensitive to δ when its value is small. As the value of δ increases, the optimal profit per unit time becomes close to the optimal profit per unit time without shortage.

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