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Optimal pricing and ordering policy under permissible delay in payments

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Abstract

Many researchers have assumed that the selling price is the same as the purchase cost, and developed various EOQ models for a retailer when the supplier offers a permissible delay in payments. In this paper, we complement the shortcoming of their models by considering the difference between the selling price and the purchase cost. We then develop an algorithm for a retailer to determine its optimal price and lot size simultaneously when the supplier offers a permissible delay in payments. Our theoretical result contradicts to their conclusion that the economic replenishment interval and order quantity generally increases marginally under the permissible delay in payments.

Keywords: Pricing; Inventory; Permissible delay; Deteriorating items; Finance

1. Introduction

In the traditional economic order quantity (EOQ) model, it assumes that the buyer must pay for the items purchased as soon as the items are received. Typically, a supplier permits the buyer a period of time, say 30 days, to settle the total amount owed to him. Usually, interest is not charged for the outstanding amount if it is paid within the permissible delay period. This credit term in financial management is denoted as "net 30" (e.g., see Brigham, 1995). However, if the payment is not paid within the permissible delay period, then interest is charged on the outstanding amount under the previously agreed terms and conditions. Therefore, a buyer can earn the interest on the accumulated revenue received, and delay the payment up to the last moment of the permissible period allowed by the supplier. The permissible delay in payments reduces the buyer's cost of holding stock because it reduces the amount of capital invested in stock for the duration of the permissible period. Hence, it is a

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marketing strategy for the supplier to attract new customers who consider it to be a type of cost (or price) reduction. However, the strategy of granting credit terms adds not only an additional cost to the supplier but also an additional dimension of default risk to the supplier.

Goyal (1985) developed an EOQ model under conditions of permissible delay in payments. He ignored the difference between the selling price and the purchase cost, and concluded that the economic replenishment interval and order quantity generally increases marginally under the permissible delay in payments. Although Dave (1985) corrected Goyal's model by assuming the fact that the selling price is necessarily higher than its purchase price, his viewpoint did not draw much attention to the recent researchers. Aggarwal and Jaggi (1995) then extended Goyal's model for deteriorating items. Jamal et al. (1997) further generalized the model to allow for shortages and deterioration. Hwang and Shinn (1997) developed the optimal pricing and lot sizing for the retailer under the condition of permissible delay in payments. Liao et al. (2000) developed an inventory model for stock-depend demand rate when a delay in payment is permissible. Recently, Chang and Dye (2001) extended the model by Jamal et al. to allow for not only a varying deterioration rate of time but also the backlogging rate to be inversely proportional to the waiting time. All above models (except Dave, 1985) ignored the difference between unit price and unit cost, and obtained the same conclusion as in Goyal (1985). In contrast, Jamal et al. (2000) and Sharker et al. (2000) amended Goyal's model by considering the difference between unit price and unit cost, and concluded from computational results that the retailer should settle his account relatively sooner as the unit selling price increases relative to the unit cost. Recently, Teng (2002) provided an alternative conclusion from Goyal (1985), and mathematically proved that it makes economic sense for a well-established buyer to order less quantity and take the benefits of the permissible delay more frequently. Chang et al. (2003) then extended Teng's model, and established an EOQ model for deteriorating items in which the supplier provides a permissible delay to the purchaser if the order quantity is greater than or equal to a predetermined quantity.

In this paper, by assuming the selling price is necessarily higher than the purchase cost, we establish an appropriate model for a retailer to determine its optimal price and lot size simultaneously when the supplier offers a permissible delay in payments. As we know, demand is a function of price. Therefore, the retailer first decides upon the unit selling price and determines the expected demand, and then determines the lot size based on the expected demand. However, to solve the problem, we must resort to backward induction. Consequently, we first derive the optimal lot size for a given price, and then find the optimal price that maximizes the total profit. In contrast to the previous results by many other researchers (e.g., Liao et al., 2000; Jamal et al., 1997; Aggarwal and Jaggi, 1995; Goyal, 1985), our theoretical result (i.e., Theorem 2) shows that the economic replenishment interval and order quantity generally decreases under the permissible delay in payments. Finally, our computational results reveal that a higher value of permissible delay period causes a lower unit selling price but a higher profit. In short, when a retailer negotiates a longer permissible delay period from the supplier, the retailer can pass some of the cost savings to the customers by lowering the selling price, while increasing demand and profit.

2. Assumptions and notation

To develop the mathematical model, the following assumptions are being made:

(1) The demand for the item is a downward sloping function of the price. There are three major popular demand functions: constant elasticity, negative exponential, and linear. For simplicity, we assume that demand is a constant elasticity function of the price.
(2) Shortages are not allowed.
(3) In reality, the retailer has numerous ways to spend the profit from sales, such as expansion, R/D, new product development, hardware and software upgrade, etc. For simplicity, we

assume here that the retailer spends the profit in other activities than paying off the loan. During the time the account is not settled, generated sales revenue is deposited in an interest bearing account. At the end of this period, the retailer pays off all units sold, keeps the rest for the use of the other activities, and starts paying for the interest charges on the items in stocks. In the meantime, the retailer starts accumulating profit for the use of the other activities. As a matter of fact, the reader can easily develop a similar model in which the retailer pays off the loan whenever he/she has money, such as in Jamal et al. (2000) and Sharker et al. (2000).

(4) Time horizon is infinite.

In addition, the following notation is used throughout this paper:

h	the unit holding cost per year excluding interest charges
p	the selling price per unit
c	the unit purchasing cost, with $c<p$
I_c	the interest charged per \$ in stocks per year by the supplier
I_d	the interest earned per \$ per year
s	the ordering cost per order
m	the period of permissible delay in settling account; that is, the trade credit period
Q	the order quantity
θ	the constant deterioration rate, where $0\leqslant\theta<<1$
$I(t)$	the level of inventory at time t, $0\leqslant t\leqslant T$
T	the replenishment time interval
D	the annual demand, as a decreasing function of price; we set $D(p)=\alpha p^{-\beta}$, where $\alpha>0$ and $\beta>1$
$Z(T,p)$	the total annual profit

The total annual profit consists of: (a) the sales revenue, (b) cost of placing orders, (c) cost of purchasing, (d) cost of carrying inventory (excluding interest charges), (e) cost of interest payable for items unsold after the permissible delay m (note that this cost occurs only if $T>m$), and (f) interest earned from sales revenue during the permissible period.

3. Mathematical formulation

The level of inventory $I(t)$ gradually decreases mainly to meet demands and partly due to deterioration. Hence, the variation of inventory with respect to time can be described by the following differential equations:

$$\frac{dI(t)}{dt}+\theta I(t)=-D,\quad 0\leqslant t\leqslant T, \tag{1}$$

with the boundary condition $I(T)=0$. Consequently, the solution of (1) is given by

$$I(t)=\frac{D}{\theta}[e^{\theta(T-t)}-1],\quad 0\leqslant t\leqslant T, \tag{2}$$

and the order quantity is

$$Q=I(0)=\frac{D}{\theta}(e^{\theta T}-1). \tag{3}$$

The total annual profit consists of the following:

(a) Sales revenue $=Dp$, (4)

(b) Cost of placing orders $=s/T$, (5)

(c) Cost of purchasing $=cQ/T=\dfrac{cD}{\theta T}(e^{\theta T}-1)$, (6)

(d) Cost of carrying inventory $=h\int_0^T I(t)\,dt/T$

$$=\frac{hD}{\theta^2 T}(e^{\theta T}-1)-\frac{hD}{\theta}. \tag{7}$$

Regarding interests payable and earned (i.e., costs of (e) and (f)), we have the following two possible cases based on the values of T and m. These two cases are depicted graphically in Fig. 1.

Case 1: $T\leqslant m$

In this case, the customer sells DT units in total by the end of the replenishment cycle time T, and has cDT to pay the supplier in full by the end of the credit period m. Consequently, there is no interest payable. However, the interest earned per year is

$$pI_d\left[\int_0^T Dt\,dt+DT(m-T)\right]/T=pI_dD(m-T/2). \tag{8}$$

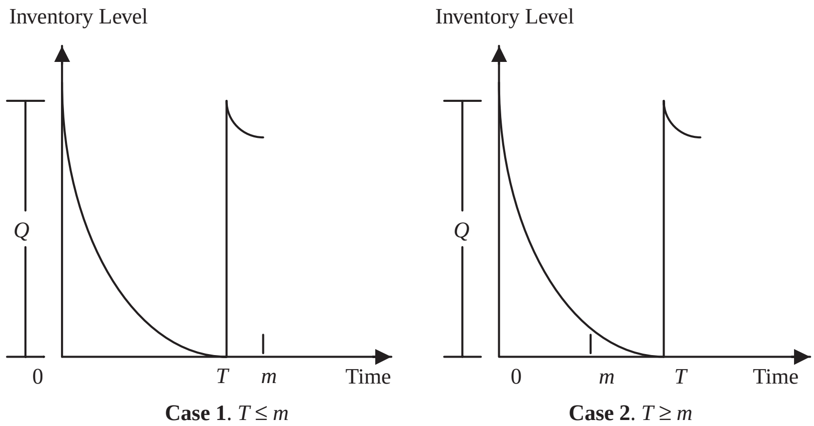

Fig. 1. Graphical representation of two inventory systems.

As a result, the total annual profit $Z_1(T,p)$ is

$$Z_1(T,p) = pD - \frac{s}{T} - \frac{D(h+c\theta)}{\theta^2 T}(e^{\theta T}-1) + \frac{hD}{\theta} + pI_d D(m - T/2). \quad (9)$$

Case 2: $T \geqslant m$

As stated in Assumption 3 above, the buyer sells Dm units in total by the end of the permissible delay m, and has cDm to pay the supplier. The items in stock are charged at interest rate I_c by the supplier starting at time m. Thereafter, the buyer gradually reduces the amount of financed loan from the supplier due to constant sales and revenue received. As a result, the interest payable per year is

$$cI_c \int_m^T I(t)\,dt/T = \frac{cI_c D}{\theta^2 T}[e^{\theta(T-m)} - 1] - \frac{cI_c D}{\theta T}(T-m). \quad (10)$$

Next, during the permissible delay period, the buyer sells products and deposits the revenue into an account that earns I_d per dollar per year. Therefore, the interest earned per year is

$$pI_d \int_0^m Dt\,dt/T = \frac{pI_d D}{2T}m^2. \quad (11)$$

Hence, the total annual profit $Z_2(T,p)$ is

$$Z_2(T,p) = pD - \frac{s}{T} - \frac{D(h+c\theta)}{\theta^2 T}(e^{\theta T}-1) + \frac{hD}{\theta} - \frac{cI_c D}{\theta^2 T}[e^{\theta(T-m)} - 1] + \frac{cI_c D}{\theta T}(T-m) + \frac{pI_d D}{2T}m^2. \quad (12)$$

Note that there are many different ways to calculate the interest payable as well as interest earned, such as Goyal (1985), Aggarwal and Jaggi (1995), and Teng (2002). For simplicity, we use Goyal's approach throughout this paper.

Hence, the total annual profit $Z(T,p)$ is written as

$$Z(T,p) = \begin{cases} Z_1(T,p) & \text{for} \quad T \leqslant m, \\ Z_2(T,p) & \text{for} \quad T \geqslant m. \end{cases}$$

Although $Z_1(m,p) = Z_2(m,p)$, $Z(T,p)$ is a continuous function of T either in (0, m) or in (m, ∞), but not in both.

4. Determination of the optimal replenishment time for any given price

For low deterioration rates, we can assume

$$e^{\theta T} \approx 1 + \theta T + (\theta T)^2/2. \quad (13)$$

Hence, the total annual profit will be given by

$$Z_1(T,p) \approx AZ_1(T,p) = Dp[1 + I_d(m - T/2)] - \frac{s}{T} - Dc - \frac{DT}{2}(h + c\theta) \tag{14}$$

and

$$Z_2(T,p) \approx AZ_2(T,p) = D[p - c(1 - I_c m)] - \frac{1}{T}\left[s + \frac{Dm^2(cI_c - pI_d)}{2}\right] - \frac{DT}{2}(h + c\theta + cI_c). \tag{15}$$

That is, the approximation of total annual profit $AZ(T,p)$ is written as

$$AZ(T,p) = \begin{cases} AZ_1(T,p) & \text{for} \quad T \leqslant m, \\ AZ_2(T,p) & \text{for} \quad T \geqslant m. \end{cases}$$

Although $AZ_1(m,p) = AZ_2(m,p)$, $AZ(T,p)$ is a continuous function of T either in (0, m) or in (m, ∞). However, we know from Theorem 1 below that $AZ(T,p)$ is not continuous in (0, ∞) because only one case of $AZ_i\,(T,p)$ can occur. For example, $h = \$0.65$/unit/year, $I_c = 0.09$/\$/year, $I_d = 0.06$/\$/year, $c = \$5.0$ per unit, $\theta = 0.05$, $p = \$10$ per unit, $m = 0.1$ year, $s = 50$, $\alpha = 1000000$ and $\beta = 2.0$, we obtain from Theorem 1 below that $2s \leqslant (h + c\theta + pI_d)Dm^2 = 150$, $AZ(T,p) = AZ_1(T,p)$, and the optimal $T^* = 0.081650 < m$, as shown in Fig. 2. For an example of Case 2 (i.e., $AZ(T,p) = AZ_2(T,p)$), we assume that $h = \$0.65$/unit/year, $I_c = 0.09$/\$/year, $I_d = 0.06$/\$/year, $c = \$5.0$ per unit, $\theta = 0.05$, $p = \$10$ per unit, $m = 0.1$ year, $s = 100$, $\alpha = 1000000$ and $\beta = 2.0$. Then we obtain from Theorem 1 that $2s \geqslant (h + c\theta + pI_d)Dm^2 = 150$, $AZ(T,p) = AZ_2(T,p)$, and the optimal $T^* = 0.117063 > m$, as shown in Fig. 3.

Note that the purpose of this approximation is to obtain the unique closed-form solution for the optimal T. By taking the first- and second-order derivatives of $AZ_i\,(T,p)$, for $i = 1$ and 2, with respect to T, we obtain

$$\frac{\partial AZ_1(T,p)}{\partial T} = \frac{1}{T^2}s - \frac{D}{2}(h + c\theta + pI_d), \tag{16}$$

$$\frac{\partial AZ_2(T,p)}{\partial T} = \frac{1}{T^2}\left[s + \frac{Dm^2(cI_c - pI_d)}{2}\right] - \frac{D}{2}(h + c\theta + cI_c), \tag{17}$$

$$\frac{\partial^2 AZ_1(T,p)}{\partial T^2} = -2\frac{1}{T^3}s < 0 \tag{18}$$

and

$$\frac{\partial^2 AZ_2(T,p)}{\partial T^2} = -2\,\frac{1}{T^3}\left[s + \frac{Dm^2(cI_c - pI_d)}{2}\right]. \tag{19}$$

Consequently, for a fixed p, $AZ_1(T,p)$ is a strictly concave function of T. Thus, there exists a unique value of T_1 which maximizes $AZ_1(T,p)$ as

$$T_1 = \sqrt{2s/D(h + c\theta + pI_d)} = \sqrt{2s/Dg_1}, \tag{20}$$

where

$$g_1 = h + c\theta + pI_d. \tag{21}$$

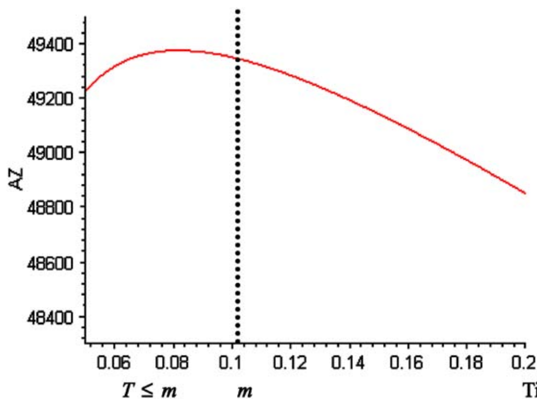

Fig. 2. Graph of $AZ(T,p) = AZ_1(T,p)$ with $T \leqslant m$.

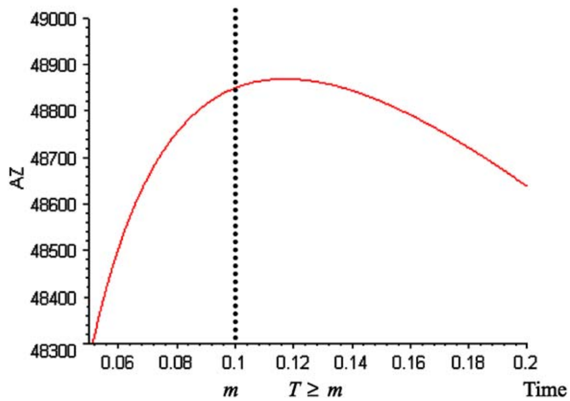

Fig. 3. Graph of $AZ(T,p) = AZ_2(T,p)$ with $T \geqslant m$.

To ensure $T_1 \leqslant m$, we substitute (20) into inequality $T_1 \leqslant m$, and obtain that

$$\text{if and only if } 2s \leqslant (h + c\theta + pI_d)Dm^2, \text{ then } T_1 \leqslant m. \tag{22}$$

Hence, substituting (20) into (3), the optimal EOQ for Case 1 (i.e., $T_1 \leqslant m$) is

$$\begin{aligned} Q^*(T_1) &= \frac{D}{\theta}(e^{\theta T_1} - 1) \approx D(T_1 + \theta T_1^2/2) \\ &= \sqrt{2sD/(h + c\theta + pI_d)} + \theta s/(h + c\theta + pI_d). \end{aligned} \tag{23}$$

Substituting (20) into (14), we obtain

$$\begin{aligned} AZ_1(p) &= AZ_1(T_1(p), p) \\ &= D[p(1 + I_d m) - c] - \sqrt{2g_1 Ds}. \end{aligned} \tag{24}$$

From (22), we know that $T_2 \geqslant m$ implies $2s \geqslant (h + c\theta + pI_d)Dm^2$. Consequently, we obtain: $2s + Dm^2(cI_c - pI_d) \geqslant (h + c\theta + cI_c)Dm^2 > 0$, and

$$\frac{\partial^2 AZ_2(T,p)}{\partial T^2} = -[2s + Dm^2(cI_c - pI_d)]/T^3 < 0. \tag{25}$$

Therefore, for a fixed p, $AZ_2(T,p)$ is also a strictly concave function of T. Likewise, we obtain the optimal solution to $AZ_2(T,p)$ as

$$\begin{aligned} T_2 &= \sqrt{[2s + Dm^2(cI_c - pI_d)]/[D(h + c\theta + cI_c)]} \\ &= \sqrt{2s_1/Dg_2}, \end{aligned} \tag{26}$$

where

$$s_1 = s + \frac{Dm^2(cI_c - pI_d)}{2} \text{ and } g_2 = h + c\theta + cI_c. \tag{27}$$

To ensure T_2 is existent and $T_2 \geqslant m$, we substitute (26) into inequality $T_2 \geqslant m$, and obtain that

$$\text{if and only if } 2s \geqslant (h + c\theta + pI_d)Dm^2, \text{ then } T_2 \geqslant m. \tag{28}$$

Hence, the optimal EOQ for Case 2 (i.e., $T_2 \geqslant m$) is

$$\begin{aligned} Q^*(T_2) &\approx D(T_2 + \theta T_2^2/2) \\ &= \sqrt{[2sD + D^2m^2(cI_c - pI_d)]/[h + c\theta + cI_c]} \\ &\quad + \theta[2s + Dm^2(cI_c - pI_d)]/\{2[h + c\theta + cI_c]\}. \end{aligned} \tag{29}$$

Substituting (26) into (15), we obtain

$$\begin{aligned} AZ_2(p) &= AZ_2(T_2(p), p) \\ &= D[p - c(1 - I_c m)] - \sqrt{2g_2 Ds_1}. \end{aligned} \tag{30}$$

In the classical EOQ model, the supplier must be paid for the items as soon as the customer receives them. Therefore, it is a special case of Case 2 with $m = 0$. As a result, the classical optimal EOQ is

$$\begin{aligned} Q^* = \frac{D}{\theta}(e^{\theta T*} - 1) &\approx \sqrt{2sD/(h + c\theta + cI_c)} \\ &\quad + [\theta s/(h + c\theta + cI_c)]. \end{aligned} \tag{31}$$

By comparing (22) and (28), we have the following theorem.

Theorem 1. *For low deterioration rates, we can obtain the following results.*

(1) *If* $2s < (h + c\theta + pI_d)Dm^2$, *then* $T^* = T_1$.

(2) *If* $2s > (h + c\theta + pI_d)Dm^2$, *then* $T^* = T_2$.

(3) *If* $2s = (h + c\theta + pI_d)Dm^2$, *then* $T^* = m$.

Proof. It immediately follows from (22) and (28).

Similarly, from (23), (29) and (31), we have the following theorem.

Theorem 2. *For low deterioration rates, we get the following results*:

(a) *If* $cI_c < pI_d$, *then* $Q^*(T_1)$ *and* $Q^*(T_2) < Q^*$.

(b) *If* $cI_c > pI_d$, *then* $Q^*(T_1)$ *and* $Q^*(T_2) > Q^*$.

(c) *If* $cI_c = pI_d$, *then* $Q^*(T_1)$ *and* $Q^*(T_2) = Q^*$.

Proof. It is obvious from (23), (29) and (31).

Note that Theorems 1 and 2 here are a generalization of the corresponding Theorems 1 and 2 of Teng (2002), in which the deterioration rate is zero. By assuming that $p = c$ and $I_c > I_d$ (i.e., Part (b) of Theorem 2), many recent researchers (e.g., Liao et al., 2000; Jamal et al., 1997; Aggarwal and Jaggi, 1995) concluded that $Q^*(T_1)$ and $Q^*(T_2) > Q^*$. However, in reality, pI_d is in general greater than cI_c. Consequently, we know from Part (a) of Theorem 2 that $Q^*(T_1)$ and $Q^*(T_2) < Q^*$.

5. Determination of the optimal price

Taking the first derivative of $(h+c\theta+pI_{\rm d})D(p)m^2$ with respect to p, we obtain

$$I_{\rm d}D(p)m^2+(h+c\theta+pI_{\rm d})D'(p)m^2 = m^2[(h+c\theta)D'(p)-I_{\rm d}(\beta-1)D(p)]<0. \quad (32)$$

Hence, $(h+c\theta+pI_{\rm d})D(p)m^2$ is a strictly decreasing function of p. Using the facts in (22) and (28), we set P_0 such that

$$2s=(h+c\theta+p_0I_{\rm d})D(p_0)m^2. \quad (33)$$

Consequently, we know from (22) and (28) that

$$AZ(p)=\begin{cases} AZ_1(p)=AZ_1(T_1(p),p) & \text{for } p\leqslant p_0,\\ AZ_2(p)=AZ_2(T_2(p),p) & \text{for } p\geqslant p_0. \end{cases} \quad (34)$$

To obtain the optimal price, taking the first derivative of (24) with respect to p and setting the result to be zero, we have

$$\frac{{\rm d}AZ_1(p)}{{\rm d}p}=D[(1+I_{\rm d}m)(1-\beta)+p^{-1}\beta c + (2g_1Ds)^{-1/2}s(g_1p^{-1}\beta-I_{\rm d})]=0. \quad (35)$$

Next, we need to check the second-order condition for concavity. That is

$$\frac{d^2AZ_1(p)}{dp^2}=D\{-p^{-2}\beta c+(2g_1)^{-3/2}s^{1/2} \times [g_1^2p^{-2}\beta(\beta-2)+I_d^2]\}<0. \quad (36)$$

Likewise, from (30), we obtain the first-order condition for $AZ_2(p)$ as

$$\begin{aligned}\frac{{\rm d}AZ_2(p)}{{\rm d}p}&=D[(1-\beta)+p^{-1}\beta c(1-I_{\rm c}m) - g_2(2g_2Ds_1)^{-1/2}(-\beta p^{-1}s_1+s'_1)]\\ &=D\{(1-\beta)+p^{-1}\beta c(1-I_{\rm c}m) - g_2(2g_2Ds_1)^{-1/2}(-\beta p^{-1}s_1 + \frac{Dm^2}{2}(I_{\rm d}(\beta-1)-p^{-1}\beta cI_{\rm c})]\}=0.\end{aligned} \quad (37)$$

The second-order condition for concavity is

$$\begin{aligned}\frac{{\rm d}^2AZ_2(p)}{{\rm d}p^2}&=D\{-p^{-2}\beta c(1-I_{\rm c}m) + (2s_1)^{-3/2}(g_2)^{1/2}(D)^{-1/2}[p^{-2}s_1^2\beta^2+s'^2_1 - 2\beta p^{-1}s_1s'_1-2s_1(p^{-2}s_1\beta-2p^{-1}\beta s'_1 + \frac{Dm^2}{2}p^{-1}\beta cI_{\rm c})]\}\\ &=D\{-p^{-2}\beta c(1-I_{\rm c}m) + (2s_1)^{-3/2}(g_2)^{1/2}(D)^{-1/2}[(\beta p^{-1}s_1+s'_1)^2 - 2p^{-1}s_1\beta(s_1+\frac{Dm^2}{2}cI_{\rm c})]\}<0.\end{aligned} \quad (38)$$

6. An algorithm

Based on the above discussion, we develop the following solution algorithm to determine an optimal solution for the approximate model.

Step 1: Determine p_0 by solving (33).
Step 2: If there exists a p_1 such that $p_1\leqslant p_0$, and p_1 satisfies both the first-order condition as in (35) and the second-order condition for concavity as in (36), then we determine $T_1(p_1)$ by (20) and $AZ_1(T_1(p_1),p_1)$ by (24). Otherwise, we set $AZ_1(T_1(p_1),p_1)=0$.
Step 3: If there exists a p_2 such that $p_2\geqslant p_0$, and p_2 satisfies both the first-order condition as in (37) and the second-order condition for concavity as in (38), then we calculate $T_2(p_2)$ by (26) and $AZ_2(T_2(p_2),p_2)$ by (30). Otherwise, we set $AZ_2(T_2(p_2),p_2)=0$.
Step 4: If $AZ_1(T_1(p_1),p_1)\geqslant AZ_2(T_2(p_2),p_2)$, then the optimal total annual profit is $AZ^*(T^*(p^*),p^*)=AZ_1(T_1(p_1),p_1)$, and stop. Otherwise, $AZ^*(T^*(p^*),p^*)= AZ_2(T_2(p_2),p_2)$, and stop.

7. Numerical examples

Example 1. For generality, we use the following example in which $c\ I_{\rm c}<p^*I_{\rm d}$. Given $h=\$0.5$/unit/year, $I_{\rm c}=0.09$/\$/year, $I_{\rm d}=0.06$/\$/year, $c=\$4.5$ per unit, $\theta=0.05$, $s=\$10$/per order, $\alpha=100000$ and $\beta=1.5$. Using the above solution algorithm, we

Table 1
Optimal solutions for different trade credit period m

m(days)	p_0	p^*	T^*	$Q(T^*)$	AZ^*	Z^*
5	0.8076	$p_2=13.6432$	$T_2=0.094077$	187.1249	17943.671	17943.529
10	2.1765	$p_2=13.6258$	$T_2=0.092887$	185.1077	17957.162	17957.039
15	4.0594	$p_2=13.6079$	$T_2=0.090938$	181.5699	17972.387	17972.280
20	6.5569	$p_2=13.5897$	$T_2=0.088184$	176.4136	17989.461	17989.361
25	9.8363	$p_2=13.5712$	$T_2=0.084555$	169.4845	18008.552	18008.467
30	14.1329	$p_1=13.5535$	$T_1=0.080547$	161.1747	18029.911	18030.279
40	27.1234	$p_1=13.5312$	$T_1=0.080482$	162.0156	18074.581	18075.204
50	49.1789	$p_1=13.5090$	$T_1=0.080418$	162.2895	18119.287	18120.214
60	85.5835	$p_1=13.4869$	$T_1=0.080354$	162.5574	18164.029	18164.276
70	143.1669	$p_1=13.4648$	$T_1=0.080290$	162.8307	18208.809	18209.456

obtain the computational results for various values of m as shown in Table 1. Table 1 reveals that (1) the difference between AZ^* and Z^* is negligible, (2) a higher value of m causes a higher value of Z^*, but lower values of p^* and T^*, (3) if $T^*\geqslant m$, then a higher value of m causes a lower value of $Q(T^*)$, (4) if $T^*\leqslant m$, then a higher value of m causes a higher value of $Q(T^*)$, and (5) we know from Eq. (31) that the classical optimal EOQ $Q^*=187.6761$, which confirms the result in Part (a) of Theorem 2 (i.e., $Q^*(T_1)$ and $Q^*(T_2)<Q^*$, if $c\ I_c<p^*I_d$).

8. Conclusions and future research

In this paper, we develop an appropriate pricing and lot-sizing model for a retailer when the supplier provides a permissible delay in payments. We then establish the necessary and sufficient conditions for the unique optimal replenishment interval, and use Taylor's series approximation to obtain the explicit closed-form optimal solution. Next, we derive the first and second-order conditions for finding the optimal price, and then develop an algorithm to solve the problem. Furthermore, we establish Theorem 1, which provides us a simple way to obtain the optimal replenishment interval by examining the explicit conditions. We then compare the classical EOQ with the proposed model here, and obtain Theorem 2. Finally, our numerical example reveals that a higher value of the permissible delay m causes a higher value of the unit profit Z^*, but lower values of the selling price p^* and the replenishment cycle time T^*.

The model proposed in this paper can be extended in several ways. For instance, we may extend the constant deterioration rate to a two-parameter Weibull distribution. Also, we could consider the demand as a function of quality as well as time varying. Finally, we could generalize the model to allow for shortages, quantity discounts, discount and inflation rates, and others.

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