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ORIGINAL ARTICLE

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Investment model development for repetitive inspections and measurement equipment in imperfect production systems

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Abstract An investment model can be developed for investment in repetitive inspections and investment in measurement equipment in imperfect production systems. The relevant costs and benefits of the investments are developed to predict the return of the investments. The investment in repetitive inspections affects the number of repetitive inspections. The investment in measurement equipment reduces the measurement error of measurement equipment. They are then linked to the proportion of good units sent to the customer, which can result in an increase in market share and a decrease in product warranty cost. The model includes the investment in inspection, the investment in measurement equipment, failure cost in inspection, failure cost in measurement, manufacturing cost, market share increase, and warranty cost.

Keywords Investment · Measurement error · Repetitive inspections

1 Introduction

Deliman and Feldman [4] considered serial manufacturing systems with imperfect inspection and rework of defective items. Duffuaa [6] determined the optimal number of repeat inspections for a complete repeat inspection plan. Ding et al. [8] considered repetitive inspection strategies for examining as to whether it is better to repetitively test rejected items, or to repetitively test accepted items. Basnet and Case [1] discussed the effect of measurement error on accept/reject probabilities for homogeneous products. Engel and Vries [7] showed that the testing procedure could lead to incorrect decisions due to measurement error. We use the results [1, 6] to develop the effect of repetitive inspections and measurement error on the proportion of conforming units among accepted items sent to the customers. The investment model presented by Lee et al. [9] will be used as the basis for developing these investment models.

In this research, the joint model for investment in inspection and the investment in measurement equipment will be developed. On one hand, investment in inspection depends on the number of repetitive inspections, and hence affects the proportion of conforming units among accepted items sent to the customer. On the other hand, investment in measurement equipment will reduce measurement error, which also impacts the proportion of conforming units among accepted items sent to the customer. Thus, the common link between these two investment opportunities is the proportion of conforming units among accepted items sent to the customer. Investments in inspection and measurement equipment result in the increase of the proportion of conforming units among accepted items sent to the customer, and hence affects warranty cost, market share increase, and relevant costs.

2 Model development

Let's assume that a multistage manufacturing system will provide the framework for developing the analytical models. The two–tuple (i, j) indicates the j^{th} stage of component/subassembly i, where i=1, …, N j=1, …, n (i). The proportion of nonconforming and conforming components/subassemblies i at stage j can be denoted as $p\ (i, j)$ and $1-p\ (i, j)$, respectively.

2.1 The effect of investment in repetitive inspections

At any stage, the inspector may reject or accept a conforming unit if it corresponds to components/subassemblies or products with probabilities P_1 and $1-P_1$ respectively, when that item is inspected. The inspector will accept or reject the

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nonconforming unit at all stages for components/subassemblies or products with the probabilities P_2 and 1– P_2 respectively, each time an item is inspected. The repetitive inspection concept [6] is used as the basis for developing the effect of the investment in inspection.

The proportion of falsely rejected components/subassemblies after ω (*i*, *j*) repetitive inspections, $P_{\mathrm{RG}'\text{ù}}$ (*i*, *j*), at stage *j* for component/subassembly *i* can be written as

$$P_{RG'\omega}(i,j) = [1 - p(i,j)]P_1(1 - P_1)^{\omega(i,j)-1} \tag{1}$$

From Eq. (1), the sum of the proportions of falsely rejected components/subassemblies for all ω (*i*, *j*) repetitive inspections, $SP_{\mathrm{RG}'\text{ù}}$ (*i*, *j*), can be written as

$$SP_{RG'\omega}(i,j) = [1 - p(i,j)](P_1)[1 + \sum_{k=1}^{\omega(i,j)-1} (1 - P_1)^k]\} \tag{2}$$

The proportion of falsely accepted components/subassemblies after ω (*i*, *j*) repetitive inspections is

$$P_{AB'\omega}(i,j) = p(i,j)P_2^{\omega(i,j)} \tag{3}$$

From Eq. (3), the sum of the proportions of falsely accepted components/subassemblies for all ω (*i*, *j*) repetitive inspections, $SP_{\mathrm{AB}'\text{ù}}$ (*i*, *j*), can be written as

$$SP_{AB'\omega}(i,j) = p(i,j)[\sum_{k=1}^{\omega(i,j)} (P_2)^k] \tag{4}$$

The proportion of correctly accepted components/subassemblies after ω (*i*, *j*) repetitive inspections, $\mathrm{P}_{\mathrm{AG}'\text{ù}}$ (*i*, *j*), at stage *j* for component/subassembly *i* is

$$P_{AG'\omega}(i,j) = [1 - p(i,j)](1 - P_1)^{\omega(i,j)} \tag{5}$$

From Eq. (5), the sum of the proportions of correctly accepted components/subassemblies for all ω (*i*, *j*) repetitive inspections, $SP_{\mathrm{AG}'\text{ù}}$ (*i*, *j*), can be written as

$$SP_{AG'\omega}(i,j) = [1 - p(i,j)][\sum_{k=1}^{\omega(i,j)} (1 - P_1)^k] \tag{6}$$

The proportion of correctly rejected components/subassemblies after ω (*i*, *j*) repetitive inspections, $P_{\mathrm{RB}'\text{ù}}$ (*i*, *j*), at stage *j* for component/subassembly *i* is

$$P_{RB'\omega}(i,j) = p(i,j)(1 - P_2)P_2^{\omega(i,j)-1} \tag{7}$$

From Eq. (7), the sum of the proportions of correctly rejected components/subassemblies for all ω (*i*, *j*) repetitive inspections, $SP_{\mathrm{RB}'\text{ù}}$ (*i*, *j*), can be written as

$$SP_{RB'\omega}(i,j) = p(i,j)(1 - P_2)[1 + \sum_{k=1}^{\omega(i,j)-1} (P_2)^k] \tag{8}$$

By adding Eq. (3) to Eq. (5), the proportion of accepted components/subassemblies after ω (*i*, *j*) repetitive inspections, $\eta'_{\text{ù}}$ (*i*, *j*), is

$$\eta'_{\omega}(i,j) = p(i,j)P_2^{\omega(i,j)} + [1 - \mathrm{p}(i,j)](1 - \mathrm{P}_1)^{\omega(i,j)} \tag{9}$$

By dividing Eq. (5) by Eq. (9), the proportion of conforming components/subassemblies among accepted components/subassemblies after ù (*i*, *j*) repetitive inspections, $\rho'_{\text{ù}}$ (*i*, *j*), is

$$\rho\prime_{\omega}(\mathrm{i, j}) = [1 - \mathrm{p}(i,j)](1 - \mathrm{P}_1)^{\omega(i,j)} \Big/ \left\{p(i,j)P_2^{\omega(i,j)} + [1 - \mathrm{p}(i,j)](1 - \mathrm{P}_1)^{\omega(i,j)}\right\} \tag{10}$$

From Eq. (9), the proportion of accepted components/subassemblies after ω (*i*, *j*) −1 repetitive inspections, ç'$_{\text{ù}-1}$ (*i*, *j*), for the ω (*i*, *j*)$^{\text{th}}$ repetitive inspection is

$$\eta'_{\omega-1}(i,j) = p(i,j)P_2^{\omega(i,j)-1} + [1 - \mathrm{p}(i,j)](1 - \mathrm{P}_1)^{\omega(i,j)-1} \tag{11}$$

Only the accepted components/subassemblies after each inspection are inspected again at a given stage. The proportion of produced components/subassemblies for the first inspection is 1. Then, from Eq. (11), the sum of the proportions of accepted components/subassemblies for all ω (*i*, *j*)$^{\text{th}}$ repetitive inspections, $\psi'_{\text{ù}}$ (*i*, *j*), can be written as

$$\varphi'_{\omega}(i,j) = 1 + \sum_{k=1}^{\omega(i,j)-1} \Big[p(i,j)P_2^k + [1 - \mathrm{p}(i,j)](1 - \mathrm{P}_1)^k \tag{12}$$

The total of proportions of rejection for all repetitive inspections at stage *j* for component/subassembly *i* is the sum of the total of proportions of falsely rejected components/subassemblies for all repetitive inspections and the total of proportions of correctly rejected components/subassemblies for all repetitive inspections, and can be written as

$$\tau_{\omega}(i,j) = SP_{RG'_{\omega}}(i,j) + SP_{RB'_{\omega}}(i,j) \tag{13}$$

2.2 The effect of investment in measurement equipment

The measurement variance is caused by the variation due to the measuring gauge (repeatability) and the operators (reproducibility). Let $X\,(i, j)$ and $Y\,(i, j)$ denote the true value of the quality characteristic, and the measured value of the quality characteristic at stage j for component/subassembly i, respectively. The measurement error associated with the measurement system is $V\,(i, j)$, and $X\,(i, j)$ and $V\,(i, j)$ are assumed to be independent. The true value of X (i, j) is normally distributed with a mean $\mu\,(i, j)$ and variance $\sigma^2\,(i, j)$. The measurement error is normally distributed with a mean of 0 and variance $\sigma^2{}_M\,(i, j)$. The measured value of the quality characteristic is $Y\,(i, j) = X\,(i, j) + V\,(i, j)$, which is normally distributed with a mean $\mu\,(i, j)$ and variance $\sigma^2\,(i, j) + \sigma^2{}_M\,(i, j)$. Let USL (i, j) and LSL (i, j) be the upper specification limit and lower specification limit. We use the results from Basnet and Case [1] to develop the effect of measurement error.

The proportion of correctly accepted good units considering measurement error, is $P_{AG}(i, j)=P[\mathrm{LSL}(i, j)\leq Y(i, j)\leq \mathrm{USL}(i, j)$ (accepted) when $\mathrm{LSL}(i, j)\leq X(i, j) \leq \mathrm{USL}(i, j)$ (good units)]

$$=\int_{LSL(i,j)}^{USL(i,j)}\left\{\Phi\left[\frac{USL(i,j)-x(i,j)}{\sigma_M(i,j)}\right]-\Phi\left[\frac{LSL(i,j)-x(i,j)}{\sigma_M(i,j)}\right]\right\}f(x(i,j))dx(i,j) \tag{14}$$

The proportion of falsely rejected good units considering measurement error is $P_{RG}\,(i, j) = P\,[Y\,(i, j) <\mathrm{LSL}\,(i, j)$, or $Y\,(i, j) >\mathrm{USL}\,(i, j)$ (rejected) when $\mathrm{LSL}\,(i, j) \leq X\,(i, j) \leq \mathrm{USL}\,(i, j)$ (good units)]

$$=\int_{LSL(i,j)}^{USL(i,j)} f(x(i,j)dx(i,j)-P_{AG}(i,j) \tag{15}$$

The proportion of falsely accepted bad units considering measurement error is $P_{AB}\,(i, j) = P\,[\mathrm{LSL}\,(i, j) \leq Y\,(i, j) \leq \mathrm{USL}\,(i, j)$ (accepted) when or $X\,(i, j) < \mathrm{LSL}\,(i, j)$, $X\,(i, j) > \mathrm{USL}\,(i, j)$ (bad units)]

$$\begin{aligned}&=\int_{-\infty}^{LSL(i,j)}\left\{\Phi\left[\frac{USL(i,j)-x(i,j)}{\sigma_M(i,j)}\right]-\Phi\left[\frac{LSL(i,j)-x(i,j)}{\sigma_M(i,j)}\right]\right\}f(x(i,j))dx(i,j)\\&\quad+\int_{USL(i,j)}^{\infty}\left\{\Phi\left[\frac{USL(i,j)-x(i,j)}{\sigma_M(i,j)}\right]-\Phi\left[\frac{LSL(i,j)-x(i,j)}{\sigma_M(i,j)}\right]\right\}f(x(i,j))dx(i,j)\end{aligned} \tag{16}$$

The proportion of correctly rejecting bad units considering measurement error is $P_{RB}\,(i, j) = P\,[Y\,(i, j) < \mathrm{LSL}\,(i, j)$, or $Y\,(i, j) > \mathrm{USL}\,(i, j)$ (rejected) when $X\,(i, j) < \mathrm{LSL}\,(i, j)$, or $X\,(i, j) > \mathrm{USL}\,(i, j)$ (bad units)]

$$\begin{aligned}&=\left[\int_{-\infty}^{LSL(i,j)} f(x(i,j))dx(i,j)+\int_{USL(i,j)}^{\infty} f(x(i,j))dx(i,j)\right]\\&\quad-P_{AB}(i,j)\end{aligned} \tag{17}$$

The proportion of acceptance due to measurement error at stage j for component/subassembly i, $\eta_M\,(i, j)$, is

$$\eta_M(i,j)=p_M(i,j)P_{AB}(i,j)+[1-p_M(i,j)]P_{AG}(i,j) \tag{18}$$

where $P_{AB}(i, j)$ and $P_{AG}(i, j)$ are given in Eqs. (16) and (14), respectively, and $p_M(i, j)$ and $1–p_M(i, j)$ are the proportion of nonconforming and conforming components/subassemblies without measurement error respectively.

From Eq. (18), the proportion of conforming units among accepted items considering measurement error, $\rho_M\,(i, j)$, is

$$\begin{aligned}\eta_M(i,j)&=[1-p_M(i,j)]P_{AG}(i,j)\\&\quad/\{p_M(i,j)P_{AB}(i,j)+[1-p_M(i,j)]P_{AG}(i,j)\}\end{aligned} \tag{19}$$

2.3 The joint link between the investment in repetitive inspections and measurement equipment

The proportion of conforming units among accepted items can be the common link to integrate the investments in inspection and measurement equipment. By multiplying Eq. (10) by Eq. (19), the proportion of conforming units among accepted items sent to the customer as the result of

ω (*i*, *j*) repetitive inspections and measurement error at stage *j* for component/subassembly *i* is

$$\rho(i,j) = \left[\rho'_{\omega}(i,j)\right]\left[\rho_M(i,j)\right] \tag{20}$$

By multiplying the proportion of accepted units as the result of ω (*i*, *j*) repetitive inspections η'_{ω} (*i*, *j*) in Eq. (9) and the proportion of accepted units due to measurement error η_{M} (*i*, *j*) in Eq. (18), the proportion of accepted units, η (*i*, *j*), as the result of repetitive inspections and measurement errors at stage *j* for component/subassembly *i* can be written as

$$\eta(i,j) = \left\{\left[\eta'_{\omega}(i,j)\right]\left[\eta_M(i,j)\right]\right\} \tag{21}$$

Let *Q* indicate the batch quantity and the annual demand *D* for the final products be assumed to be satisfied completely. The production quantity per batch due to repetitive inspections and measurement error, *M* (*i*, *j*), at stage *j* of component/subassembly *i* can be written as

$$M(i,j) = \frac{Q}{\prod\limits_{g=j}^{n(i)}\left[\eta'_{\omega}(i,g)\eta_M(i,g)\right]} \tag{22}$$

The relevant costs in the joint model for investment in inspection and test equipment will be derived as follows:

(1) Total manufacturing cost related to repetitive inspections and measurement error: Let the manufacturing cost per unit be *C* (*i*, *j*). The total manufacturing cost per year is

$$TC_M = \frac{D}{Q}\sum_{i=1}^{N}\sum_{j=1}^{n(i)} C(i,j)M(i,j) \tag{23}$$

(2) Total inspection cost: Let the inspection cost per unit be denoted as $C_{I}(i,j)$. The total inspection cost per year related to repetitive inspections and measurement error for all stages is

$$TC_{INS} = \frac{D}{Q}\sum_{i=1}^{N}\sum_{j=1}^{n(i)} C_I(i,j)M(i,j)\psi'_{\omega}(i,j) \tag{24}$$

(3) Investment in measurement equipment: The investment in measurement equipment is a function of measurement variance σ^2_{M} (*i*, *j*). The total investment in measurement equipment per year for all stages is

$$TC_{ME} = \sum_{i=1}^{N}\sum_{j=1}^{n(i)} f\left(\sigma^2_M(i,j)\right) \tag{25}$$

(4) Total profit loss: The total of proportions of rejection due to repetitive inspections and measurement error is

$$\mathrm{TP_R}(i,j) = \tau_{\omega}(i,j) + \eta'_{\omega}(i,j)[(1 - p_M(i,j))P_{RG}(i,j) + p_M(i,j)P_{RB}(i,j)] \tag{26}$$

The profit loss per rejected unit due to rejection is the difference between the value per unit, *V* (*i*, *j*), and the manufacturing cost per unit, *C* (*i*, *j*). The profit loss per batch is

$$PL(i,j) = [V(i,j) - C(i,j)]M(i,j)TP_R(i,j) \tag{27}$$

The profit loss related to repetitive inspections and measurement error per year for all stages is

$$TC_{PL} = \frac{D}{Q}\sum_{i=1}^{N}\sum_{j=1}^{n(h)} PL(i,j) \tag{28}$$

(5) Total false rejection cost related to repetitive inspections and measurement error: The total of proportions of false rejection due to repetitive inspections and measurement error at stage *j* for component/subassembly *i* is

$$\begin{aligned} T\mathrm{P_{RG}}(i,j) &= SP_{RG'\omega}(i,j) \\ &\quad + \eta'_{\omega}(i,j)[1 - p_M(i,j)]P_{RG}(\mathrm{i,j}) \end{aligned} \tag{29}$$

Let the false rejection penalty per unit be denoted as C_{RG} (*i*, *j*). The false rejection cost per batch related to repetitive inspections and measurement error at stage *j* for component/subassembly *i* is

$$FR(i,j) = C_{RG}(i,j)M(i,j)TP_{RG}(i,j) \tag{30}$$

The total false rejection cost related to repetitive inspections and measurement error per year for all stages is

$$TC_{FR} = \frac{D}{Q}\sum_{i=1}^{N}\sum_{j=1}^{n(h)} FR(i,j) \tag{31}$$

(6) Total false acceptance cost related to repetitive inspections and measurement error: The total of proportions of false acceptance due to repetitive inspections and measurement error at stage *j* for component/subassembly *i* is

$$T\mathrm{P_{AB}}(i,j) = SP_{AB'\omega}(i,j) + \eta'_{\omega}(i,j)p_M(i,j)P_{AB}(i,j) \tag{32}$$

Let the false acceptance penalty per unit be C_{AB} (*i*, *j*). The false acceptance cost per batch at stage *j* for component/subassembly *i* is

$$FA(i,j) = C_{AB}M(i,j)TP_{AB}(i,j) \tag{33}$$

The total false acceptance cost due to repetitive inspections and measurement error per year for all stages is

$$TC_{FA} = \frac{D}{Q}\sum_{i=1}^{N}\sum_{j=1}^{n(i)} FA(i,j) \tag{34}$$

(7) Warranty cost: For a conforming item and a nonconforming item, the expected number of repairs within the warranty period W can be $\int_0^W \varphi_1(t)\,dt$ and $\int_0^W \varphi_2(t)\,dt$ respectively [3], where $\varphi_1(t)$ and $\varphi_2(t)$ are the failure rate for a conforming item and a nonconforming item respectively. Let the repair cost per unit be C_R. Let $\rho(q, n(q))$ be the proportion of conforming units of the final product sent to the customers. The warranty cost is

$$\begin{aligned} TC_W = C_R(D)\{\rho(q,n(q)) \int_0^W \varphi_1(t)dt \\ + [1-\rho(q,n(q))] \int_0^W \varphi_2(t)dt \end{aligned} \tag{35}$$

(8) Market share increase: Let the value of the final product per unit be V and the annual demand of the final products sent to the customers be D. The market share increase is a function of $\rho(q, n(q))$, and total market share increase per year can be written as

$$MS = f[\rho(q,n(q))]V(D) \tag{36}$$

3 Conclusion

The total cost after the investment in inspection and measurement equipment can be obtained by adding Eqs. (23), (24), (25), (28), (31), (34), and (35) and subtracting Eq. (36). The investments in repetitive inspections and measurement equipment will affect the number of repetitive inspections and reduce measurement error of measurement equipment respectively, and therefore are linked to the proportion of good units sent to the customer, which will result in the increase in market share and product warranty cost. The investment in repetitive inspections with respect to the number of repetitive inspections, $\omega(i,j)$, and the investment in measurement equipment with respect to measurement error, $\sigma_M(i, j)$, can be obtained. The proportion of defective components/subassemblies, $p(i, j)$, and batch quantity, Q, can be also obtained. The model can include the investment in inspection, the investment in measurement equipment, failure cost in inspection, failure cost in measurement, manufacturing cost, market share increase and warranty cost, and can be then used to predict the benefits of investment.

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