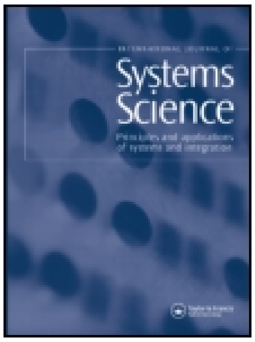

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Integrated vendor–buyer inventory system with sublot sampling inspection policy and controllable lead time

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This article investigates the impact of inspection policy and lead time reduction on an integrated vendor–buyer inventory system. We assume that an arriving order contains some defective items. The buyer adopts a sublot sampled inspection policy to inspect selected items. The number of defective items in the sublot sampling is a random variable. The buyer's lead time is assumed reducible by adding crash cost. Two integrated inventory models with backorders and lost sales are derived. We first assume that the lead time demand follows a normal distribution, and then relax the assumption about the lead time demand distribution function and apply the minimax distribution-free procedure to solve the problem. Consequently, the order quantity, reorder point, lead time and the number of shipments per lot from the vendor to the buyer are decision variables. Iterative procedures are developed to obtain the optimal strategy.

Keywords: Integrated inventory model; Defective items; Sub-lot sampling; Lead time reduction; Minimax distribution-free procedure

1. Introduction

A vendor–buyer channel coordinates to achieve better joint profit by optimizing the integrated inventory policy. This vendor–buyer coordination policy has received significant attention among researchers over the past two decades. Goyal (1976) first developed an integrated inventory model for a single supplier–single customer problem. More interesting and relevant papers related to integrated inventory models have been asserted such as Banerjee (1986), Goyal (1988), Ha and Kim (1997), Hill (1999), Goyal and Nebebe (2000) and Kelle *et al.* (2003). These researches focused on the production shipment schedule in terms of the number and batch sizes transferred between both parties under perfect quality.

As a result of imperfect vendor production, careless handling and/or damage in transit, an arriving order lot often contains some defective items. These defective items will influence the on-hand inventory level, service level and the frequency of orders in the inventory system. Therefore, to adjust the assumption of imperfect quality, many researchers proposed inventory models involving defective items. Porteus (1986) and Rosenblatt and Lee (1986) were the first two who introduced the concept and developed inventory models to discuss the relationship between an imperfect production process and an optimal lot size. Some similar problems related to quality and lot size have been discussed by several authors such as Schwaller (1988), Paknjad *et al.* (1995), Ouyang *et al.* (1999b), Salameh and Jaber (2000), Wu and Ouyang (2000, 2001), Chang (2003), Balkhi (2004), Hou and Lin (2004) and Papachristos and Konstantaras (2006). The above models tackled defective items

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focusing on lot sizing under an EOQ/EPQ model. In a recent paper, Huang (2002) developed an integrated vendor–buyer cooperative inventory model for items with imperfect quality and assumed that the number of defective items followed a given probability density function. However, both shortages and lead time reduction were not considered in Huang's (2002) model. Later, Khouja (2003) used the assumptions of Porteus (1986) and Rosenblatt and Lee (1986) to formulate producer–retailer supply chain models in which the proportion of defective products increases with increased production lot sizes. Still, lead time reduction was not considered in Khouja's (2003) model.

Recently, inventory models that considered lead time as a decision variable have been developed. Lead time usually consists of the following components: setup time, process time, wait time, move time and queue time (Tersine 1994). In many practical situations, lead time can be reduced by adding an additional crash cost. In other words, lead time is controllable. By shortening the lead time, we can lower the safety stock level, reduce the stock-out loss and improve the customer service level to gain competitive advantages. Liao and Shyu (1991) first presented a probability inventory model in which lead time was a unique decision variable. Later, Ben-Daya and Raouf (1994) extended Liao and Shyu's (1991) model by considering both lead time and ordering quantity as decision variables where shortages are not allowed. Ouyang *et al.* (1996) generalized Ben-Daya and Raouf's (1994) model by allowing shortages. Some similar problems related to lead time reduction have been discussed by Ouyang and Chuang (2000), Pan and Hsiao (2001), Ouyang and Chang (2002), Pan *et al.* (2004) and Yang *et al.* (2005). The above inventory models focused on determining the optimal policy in controllable lead time for the buyer only. Pan and Yang (2002) proposed an integrated inventory model with controllable lead time. However, both shortages and imperfect quality were not taken into account, and the reorder point of the buyer is given in their model.

Motivated by the need for the optimal policies that coordinate the operations of both partners (vendor and buyer), we present in this article an analysis of the vendor–buyer integrated inventory model with imperfect quality. We assumed that the vendor delivers the order quantity to the buyer in several equal-sized shipments and each lot contains some defective items. When the arrival quantity is large or the inspection process is time consuming, the buyer adopts a sublot sampling inspection policy to inspect the selected items. The inspection process is assumed to be nondestructive and error-free. The defective items found are discarded. We also assumed that uninspected defective items are not replaceable, but will cause a treatment procedure. Therefore, our models adopt an extra cost for the inspection of each lot and a treatment cost for uninspected defective items. The inventory is continuously reviewed and whenever the inventory level falls to the reorder point a successive delivery is scheduled to arrive. Consequently, we consider integrated inventory models with a mixture of backorders and lost sales in which the order quantity, reorder point, lead time and number of shipments from the vendor to the buyer are decision variables. We first assumed that the lead time demand follows a normal distribution. Next, we relaxed this assumption and merely assumed that the first and second moments of the lead time demand probability distribution are known and finite. The second model is solved using the minimax distribution-free approach which is to find the most unfavorable distribution for each decision variable and then minimizing over the decision variables. The minimax distribution-free approach was the original work by Scarf (1958) and later expanded by many authors such as Gallego and Moon (1993), Ouyang and Wu (1998), Moon and Silver (2000) and Silver and Moon (2001). Finally, sensitivity analysis and numerical examples are provided to illustrate the results.

2. Notations and assumptions

To develop the proposed models, we adopt the following notations and assumptions:

Notations:

- D Expected demand per unit time on the buyer.
- P Production rate of the vendor.
- A_b Buyer's ordering cost per order.
- A_v Vendor's set-up cost per set-up.
- F Transportation cost per delivery.
- h_v Vendor's holding cost per item per unit time.
- h_b Buyer's non-defective (including uninspected defective items) holding cost per item per unit time.
- π Buyer's shortage cost per unit short.
- π_0 Buyer's profit per unit.
- y Buyer's unit inspection cost.
- w Buyer's unit treatment cost for uninspected defective items.
- β Fraction of the demand during the stock-out period will be backordered, $\beta \in [0, 1]$.
- ρ Defective rate in an order lot (independent of lot size) which is a random variable and has a probability density function (p.d.f.) $g(\rho)$, $0 < \rho < 1$, with finite mean $M\rho$.
- Q Order quantity of the buyer (decision variable).
- δ Buyer's proportion of quantity inspected per shipment, $0 < \delta \leq 1$.

m The number of shipments delivered from the vendor to the buyer in one production cycle, a positive integer (decision variable).

Z Number of defective items among the inspected $\delta Q/m$ units, a random variable.

r Reorder point of the buyer (decision variable).

L Length of lead time for the buyer (decision variable).

X The lead time demand which has a p.d.f. $f(x)$ with finite mean DL and standard deviation $\sigma\sqrt{L}$, where σ denotes the standard deviation of the demand per unit time.

$E(\cdot)$ Mathematical expectation.

x^+ Maximum value of x and 0, i.e. $x^+ = \max\{x, 0\}$.

Assumptions:

(1) There is a single-vendor and single-buyer for a single product in this model.

(2) The vendor's production rate of expected non-defective items is finite and greater than the buyer's demand rate, i.e., $(1 - M_\rho)P > D$.

(3) The reorder point r = the expected demand during lead time + safety stock (SS), and $SS = k \times$ (standard deviation of lead time demand), that is,

$$r = DL + k\sigma\sqrt{L}, \tag{1}$$

where k is a safety factor.

(4) The buyer orders a lot of size Q. The vendor produces quantity Q at one setup and delivers in quantity Q/m in each shipment. That is, the buyer will receive the order quantity in m equal-size shipments.

(5) The inventory is continuously reviewed. The successive deliveries are arranged to arrive when the buyer's on hand inventory reaches the reorder point r.

(6) An arrival lot may contain some defective items and the proportion defective as in ρ. Upon arrival, $\delta Q/m$ units will be inspected and Z defective units will be discovered and discarded. After the inspection process the remaining $(Q/m) - Z$ items enter the inventory to meet the demand, which will contain $(pQ/m) - Z$ uninspected defective items. The uninspected defective items are not replaceable but will cause a treatment cost.

(7) Inspection is non-destructive and error-free. Because the sublot sampled inspection process is considered a speedy action, the length of the inspection period is neglected.

(8) The lead time L has n mutually independent components. The i-th component has minimum duration a_i, normal duration b_i, and a crash cost per unit time c_i. For further convenience, we rearrange c_i such that $c_1 \le c_2 \le \cdots \le c_n$.

(9) The lead time components are crashed one at a time starting with component 1 (because it has the minimum unit crash cost), and then component 2, and so on.

(10) The extra costs incurred by the vendor will be fully transferred to the buyer if shortened lead time is requested.

3. Basic model

In this section we establish an integrated inventory model in which an arrival lot contains some defective items. The defective rate, ρ, is a random variable. In our model, shortages are allowed and a mixture of back-orders and lost sales is considered. The lead time is reducible by adding a crash cost. The buyer orders Q units and the vendor delivers the order quantity to the buyer over equal sized shipments. From each arriving shipment, $\delta Q/m$ items will be immediately inspected. On average $E(Z)$ defective items will be discovered and discarded. The remaining $(Q/m) - E(Z)$ units will be entered into the inventory to meet customer demand. In general, if ρ is the defect rate in a lot, then $E(pQ/m) - E(Z)$ uninspected defective items will be entered into the buyer's inventory system.

3.1 *The buyer's expected total cost per unit time*

Because the inventory system is continuously reviewed by the buyer, successive shipments will arrive when the inventory level drops to the reorder point r. As mentioned earlier, we assumed that shortages are allowed and the lead time demand X has a finite mean DL and standard deviation $\sigma\sqrt{L}$. The reorder point is $r = DL + k\sigma\sqrt{L}$, where k is the safety factor. The expected shortage at the end of a shipping cycle is given by $E(X-r)^+$. Thus, the expected number of backorders and lost sales per shipping cycle are $\beta E(X-r)^+$ and $(1-\beta)E(X-r)^+$, respectively. For each shipping cycle, the fixed shortage cost is $\pi E(X-r)^+$ and the lost sales is $\pi_0(1-\beta)E(X-r)^+$. Hence, the stock-out cost per shipping cycle is $[\pi + \pi_0(1-\beta)]E(X-r)^+$.

Using the same approach in Montgomery *et al.* (1973), the expected net inventory level just before receipt of a shipment is $r - DL + (1-\beta)E(X-r)^+$ and the expected inventory level at the beginning of a shipping cycle, given that there are $(Q/m) - E(Z)$ items entering into buyer's inventory system, is $(Q/m) - E(Z) + r - DL + (1-\beta)E(X-r)^+$. Therefore, the average inventory level over the shipping cycle can be approximated by $(1/2)[(Q/m)-E(Z)]+r-DL+(1-\beta)E(X-r)^+$. Since the expected shipping cycle length is $[(Q/m)-E(Z)]/D$, the holding cost per shipping cycle is $h_b[(1/2)(Q/m)-E(Z))+r-DL+(1-\beta)E(X-r)^+]\times[(Q/m)-E(Z)]/D$.

Let L_i be the length of lead time with components $1,2,\ldots,i$ crashed to their minimum duration, then L_i can be expressed as $L_i=\sum_{j=1}^{n}b_j-\sum_{j=1}^{i}(b_j-a_j)$, $i=1,2,\ldots,n$. Furthermore, for convenience, we let $L_0=\sum_{j=1}^{n}b_j$. Thus, for $L\in[L_i,L_{i-1}]$, the lead time crash cost per shipping cycle $R(L)$ is given by

$$R(L)=c_i(L_{i-1}-L)+\sum_{j=1}^{i-1}c_j(b_j-a_j). \tag{2}$$

Therefore, the expected total cost per shipping cycle given that there are $(Q/m)-E(Z)$ units entering into inventory is the sum of the ordering cost, transportation cost, stock-out cost, inspecting cost, uninspected defective treatment cost, holding cost, and lead time crash cost. The buyer's expected total cost per shipping cycle can be expressed as:

$$\begin{aligned}C_b=&\frac{A_b}{m}+F+[\pi+\pi_0(1-\beta)]E(X-r)^+\\&+y\frac{\delta Q}{m}+w\left[E\left(\frac{\rho Q}{m}\right)-E(Z)\right]\\&+h_b\left[\frac{1}{2}\left(\frac{Q}{m}-E(Z)\right)+r-DL+(1-\beta)E(X-r)^+\right]\\&\times\frac{(Q/m)-E(Z)}{D}+R(L).\end{aligned} \tag{3}$$

Hence, the expected total cost per unit time is

$$\begin{aligned}&ETC_b(Q,r,L,m)\\&=C_b\cdot\frac{D}{(Q/m)-E(Z)}\\&=\frac{D}{(Q/m)-E(Z)}\left\{\frac{A_b}{m}+F+[\pi+\pi_0(1-\beta)]E(X-r)^+\right.\\&\left.\quad+y\frac{\delta Q}{m}+R(L)\right\}+\frac{Dw[E(\rho Q/m)-E(Z)]}{(Q/m)-E(Z)}\\&\quad+h_b\left[\frac{1}{2}\left(\frac{Q}{m}-E(Z)\right)+r-DL+(1-\beta)E(X-r)^+\right].\end{aligned} \tag{4}$$

For a given defective rate ρ in the entire lot, the number of defective units in the sublot sampled is a random variable, Z, which has a hypergeometric distribution with parameters Q, δ and ρ. That is, for given ρ, Z has a hypergeometric probability mass function (p.m.f.):

$$\Pr(Z=z|\rho)=\frac{C_z^{\rho Q/m}C_{(\delta Q/m)-z}^{(Q/m)-(\rho Q/m)}}{C_{\delta Q/m}^{Q/m}} \tag{5}$$

where $0\le z\le\min\{\delta Q/m,\rho Q/m\}$.

In this case,

$$E(Z|\rho)=\rho(\delta Q/m). \tag{6}$$

Hence, the expected number of defective units is

$$E(Z)=\int_0^1E(Z|\rho)g(\rho)\mathrm{d}\rho=\frac{\delta M_\rho Q}{m}, \tag{7}$$

where $M_\rho=E(\rho)$. Substituting equation (7) into equation (4), we get

$$\begin{aligned}ETC_b(Q,r,L,m)=&\frac{D}{(Q/m)(1-\delta M_\rho)}\\&\times\left\{\frac{A_b}{m}+F+\bar{\pi}E(X-r)^++R(L)\right\}\\&+\frac{Dy\delta}{1-\delta M_\rho}+\frac{Dw(1-\delta)M_\rho}{1-\delta M_\rho}\\&+h_b\left[r-DL+(1-\beta)E(X-r)^+\right]\\&+\frac{h_bQ}{2m}\left(1-\delta M_\rho\right),\end{aligned} \tag{8}$$

where $\bar{\pi}=\pi+\pi_0(1-\beta)$.

3.2 *The vendor's expected total cost per unit time*

When the first Q/m units have been produced, the vendor delivers them to the buyer. After that the vendor makes the deliveries on average every $[Q/m-E(Z)]/D$ units of time until the inventory level falls to zero (figure 1). Consequently, the expected total inventory per production cycle for the vendor is

$$\begin{aligned}&\left[Q\left(\frac{Q}{mP}+(m-1)\frac{Q/m-E(Z)}{D}\right)-\frac{Q^2}{2P}\right]\\&\quad-\left\{\frac{Q[Q/m-E(Z)]}{mD}[1+2+\cdots+(m-1)]\right\}\\&=\frac{Q^2}{mP}+\frac{(m-1)Q[Q-mE(Z)]}{2mD}-\frac{Q^2}{2P}.\end{aligned} \tag{9}$$

The vendor's expected total cost per production cycle is the sum of the set-up cost and inventory holding cost.

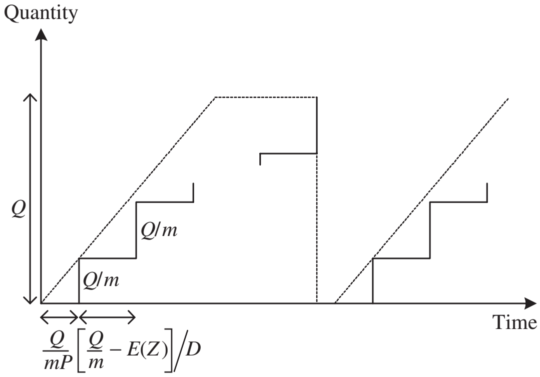

Figure 1. Vendor's inventory level.

Hence, the vendor's expected total cost per production cycle can be expressed as

$$C_v = A_v + h_v\left[\frac{Q^2}{mP} + \frac{(m-1)Q[Q-mE(Z)]}{2mD} - \frac{Q^2}{2P}\right]. \quad (10)$$

The expected length of production cycle is $[Q-mE(Z)]/D$, hence the expected total cost per unit time for the vendor is

$$\begin{aligned} ETC_v(Q,m) &= C_v \cdot \frac{D}{Q-mE(Z)} \\ &= \frac{A_vD}{Q-mE(Z)} + \frac{h_vDQ^2}{Q-mE(Z)}\left(\frac{1}{mP}-\frac{1}{2P}\right) \\ &\quad + \frac{h_v(m-1)Q}{2m}. \end{aligned} \quad (11)$$

Substituting equation (7) into (11), we get

$$\begin{aligned} ETC_v(Q,m) &= \frac{A_vD}{Q(1-\delta M_\rho)} + \frac{h_vDQ}{1-\delta M_\rho}\left(\frac{1}{mP}-\frac{1}{2P}\right) \\ &\quad + \frac{h_v(m-1)Q}{2m} \\ &= \frac{A_vD}{Q(1-\delta M_\rho)} + \frac{h_vQ}{2m(1-\delta M_\rho)} \\ &\quad \times \left[\frac{D}{P} + (m-1)\left(1-\delta M_\rho - \frac{D}{P}\right)\right]. \end{aligned} \quad (12)$$

3.3 *The joint expected total cost per unit time*

Once the vendor and buyer have established a long-term strategic partnership and contracted to commit the relationship, they will jointly determine the best policy for the integrated inventory system. The joint expected total cost per unit time can be obtained as the sum of the buyer's and vendor's expected total cost per unit time. Furthermore, the nondefective items in each shipment from vendor to buyer should meet the lead time demand and safety stock requirement. That is, $(1-M_\rho)Q/m \geq DL + k\sigma\sqrt{L}$. Hence, our problem becomes

$$\begin{aligned} &\text{minimize } JETC(Q,r,L,m) \\ &\quad = ETC_b(Q,r,L,m) + ETC_v(Q,m) \\ &\quad = \frac{D}{Q(1-\delta M_\rho)}\left\{A_b + A_v + m\left[F + \bar{\pi}E(X-r)^+ + R(L)\right]\right\} \\ &\quad + \frac{Dy\delta}{1-\delta M_\rho} + \frac{Dw(1-\delta)M_\rho}{1-\delta M_\rho} \\ &\quad + h_b\left[r - DL + (1-\beta)E(X-r)^+\right] + \frac{h_bQ}{2m}(1-\delta M_\rho) \\ &\quad + \frac{h_vQ}{2m(1-\delta M_\rho)}\left[\frac{D}{P} + (m-1)\left(1-\delta M_\rho - \frac{D}{P}\right)\right], \end{aligned}$$

subject to

$$\frac{(1-M_\rho)Q}{m} \geq DL + k\sigma\sqrt{L}. \quad (13)$$

4. Normal distribution model

In this section, we assumed that the lead time demand X has a normal p.d.f. $f(x)$ with mean DL and standard deviation $\sigma\sqrt{L}$. By Assumption 3, the reorder point $r = DL + k\sigma\sqrt{L}$, hence the expected demand short at the end of shipping cycle is given by

$$E(X-r)^+ = \int_r^\infty (x-r)f(x)\mathrm{d}x = \sigma\sqrt{L}\psi(k),$$

where $\psi(k) \equiv \phi(k) - k[1-\Phi(k)]$, and $\phi, \mathbf{\Phi}$ are the standard normal p.d.f. and distribution function (d.f.), respectively. Furthermore, we can also consider the safety factor k as a decision variable instead of r because $r = DL + k\sigma\sqrt{L}$. Therefore, model (13) can be written as

$$\begin{aligned} &\text{minimize } JETC_N(Q,k,L,m) \\ &\quad = \frac{D}{Q(1-\delta M_\rho)} \\ &\quad \times \left\{A_b + A_v + m\left[F + \bar{\pi}\sigma\sqrt{L}\psi(k) + R(L)\right]\right\} \\ &\quad + \frac{Dy\delta}{1-\delta M_\rho} + \left(\frac{Dw(1-\delta)M_\rho}{1-\delta M_\rho}\right) \\ &\quad + h_b\sigma\sqrt{L}[k + (1-\beta)\psi(k)] \\ &\quad + \left(\frac{h_bQ}{2m}\right)(1-\delta M_\rho) \\ &\quad + \left(\frac{h_vQ}{2m(1-\delta M_\rho)}\right)\left[\frac{D}{P} + (m-1)\left(1-\delta M_\rho - \left(\frac{D}{P}\right)\right)\right], \end{aligned}$$

subject to

$$\frac{(1-M_\rho)Q}{m} \geq DL + k\sigma\sqrt{L}, \quad (14)$$

where the subscript N in $JETC$ denotes the normal distribution case.

To solve this nonlinear programming problem, we first ignore the restriction of $(1-M_\rho)Q/m \geq DL + k\sigma\sqrt{L}$ and try to find the optimal value of (Q,k,L,m) such that $JETC_N(Q,k,L,m)$ has a minimum value. For fixed Q, k, and m, $JETC_N(Q,k,L,m)$ is a concave function of $L \in [L_i, L_{i-1}]$, because

$$\frac{\partial^2 JETC_N(Q,k,L,m)}{\partial L^2} = \frac{-mD\bar{\pi}\sigma\psi(k)}{4Q[1-\delta M_\rho]}L^{-3/2} - \frac{h_b\sigma}{4}[k+(1-\beta)\psi(k)]L^{-3/2} < 0. \tag{15}$$

Thus, for fixed (Q,k,m), the minimum joint expected total cost per unit time will occur at the end points of the interval $[L_i, L_{i-1}]$. Furthermore, we can also prove that $JETC_N(Q,k,L,m)$ is a convex function of m for fixed Q, k, and $L \in [L_i, L_{i-1}]$ (see appendix A).

Next, it can also be shown that for fixed m and $L \in [L_i, L_{i-1}]$, $JETC_N(Q,k,L,m)$ is a convex function in (Q,k) (see appendix B). Thus, for fixed m and $L \in [L_i, L_{i-1}]$, the minimum value of $JETC_N(Q,k,L,m)$ will occur at the point (Q,k) which satisfies $\partial JETC_N(Q,k,L,m)/\partial Q = 0$ and $\partial JETC_N(Q,k,L,m)/\partial k = 0$, simultaneously. Solving these two equations gives

$$Q = \sqrt{\frac{2mD\{A_b + A_v + m[F + \bar{\pi}\sigma\sqrt{L}\psi(k) + R(L)]\}}{h_b(1-\delta M_\rho)^2 + h_v[(D/P) + (m-1)(1-\delta M_\rho - (D/P))]}}, \tag{16}$$

and

$$\Phi(k) = 1 - \frac{h_b Q(1-\delta M_\rho)}{mD\bar{\pi} + h_b(1-\beta)Q(1-\delta M_\rho)}. \tag{17}$$

The optimal solutions Q and k for given m and L can be obtained by solving equations (16) and (17) iteratively until Q and k converges. This convergence can be shown by adopting a graphical technique similar to that used in Hadley and Whitin (1963).

We now consider the constraint $(1-M_\rho)Q/m \geq DL + k\sigma\sqrt{L}$. If $(1-M_\rho)Q/m \geq DL + k\sigma\sqrt{L}$ holds, then (Q,k) is an interior optimal solution for given m and $L \in [L_i, L_{i-1}]$. However, if $(1-M_\rho)Q/m < DL + k\sigma\sqrt{L}$, we set the order quantity, Q, equal to $m(DL + k\sigma\sqrt{L})/(1-M_\rho)$, then use equation (17) to find safety factor k. Note that the closed-form solution of (Q,k) cannot be obtained from equations (16) and (17). The following iterative algorithm is established to obtain the optimal solution of (Q,k,L,m).

Algorithm 1

Step 1: For each L_i, $i = 0, 1, \ldots, n$, perform (i) to Step 4.

(i) Start with $m_I = 1$.
(ii) Set $k_{i1} = 0$ (implies $\psi(k_{i1}) = 0.39894$).
(iii) Substitute $\psi(k_{i1})$ into equation (16) to evaluate Q_{i1}.
(iv) Utilizing Q_{i1} to determine $\Phi(k_{i2})$ from equation (17), then finds k_{i2} by checking the standard normal table, and hence $\psi(k_{i2})$.
(v) Repeat (iii)–(iv) until no change occurs in the values of Q_i and k_i.
(vi) Utilizing Q_i and k_i to determine new m_I from equation (A2).
(vii) Repeat (ii)–(vi) until no change occurs in the values of Q_i, k_i and m_I.

Step 2: Set $m_{i1} = \lfloor m_I \rfloor$ and $m_{i2} = \lfloor m_I \rfloor + 1$ where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

(i) For m_{i1}, repeat Step 1 (ii)–(v) until no change occurs to find Q_{i1} and k_{i1}.
(ii) For m_{i2}, repeat Step 1 (ii)–(v) until no change occurs to find Q_{i2} and k_{i2}.

Step 3: Compare $(1-M_\rho)Q_{ij}/m_{ij}$ and $DL_i + k_{ij}\sigma\sqrt{L_i}$, $j = 1, 2$.

If $(1-M_\rho)Q_{ij}/m_{ij} \geq DL_i + k_{ij}\sigma\sqrt{L_i}$, go to Step 4.

If $(1-M_\rho)Q_{ij}/m_{ij} < DL_i + k_{ij}\sigma\sqrt{L_i}$, let $Q_{ij} = m_{ij}(DL_i + k_{ij}\sigma\sqrt{L_i})/(1-M_\rho)$ and then determine new k_{ij} from equation (17).

Step 4: Compute the corresponding $JETC_N(Q_{ij}, k_{ij}, L_i, m_{ij})$, $j = 1, 2$.

Let $JETC_N(Q_i, k_i, L_i, m_i) = \min_{j=1,2} JETC_N(Q_{ij}, k_{ij}, L_i, m_{ij})$.

Step 5: Find $\min_{i=0,1,\ldots,n} JETC_N(Q_i, k_i, L_i, m_i)$.

Let $JETC_N(Q^*, k^*, L^*, m^*) = \min_{i=0,1,\ldots,n} JETC_N(Q_i, k_i, L_i, m_i)$, then (Q^*, k^*, L^*, m^*) is the optimal solution. And hence, the optimal reorder point $r^* = DL^* + k^*\sigma\sqrt{L^*}$ follows.

5. Distribution-free model

In some situations, the information about the probability distribution of the lead time demand is limited. Hence, in this section, we relaxed the restriction about the form of the probability distribution of the lead time demand and only assumed that it has given finite first and second moments (and hence, mean and variance are also known and finite); i.e. the p.d.f. $f(x)$ of X belongs to the class $\Re$ of p.d.f.s with finite mean DL and variance $\sigma^2 L$. Since the probability distribution of X is unknown, we cannot find the exact value of the expected shortage quantity $E(X-r)^+$. Hence, we use the minimax distribution-free approach to solve this problem. The minimax distribution-free approach is to find the most unfavorable p.d.f. $f(x)$ in $\Re$ for each (Q, r, L, m) and then minimize over (Q, r, L, m). That is, our problem is to solve

$$\min_{Q,r,L,m} \max_{f(x)\in\Re} JETC(Q, r, L, m),$$
$$\text{subject to } \frac{(1-M_\rho)Q}{m} \geq DL + k\sigma\sqrt{L}. \tag{18}$$

To this end, we need the following proposition that was asserted by Gallego and Moon (1993).

Proposition 1: *For any $f(x) \in \Re$,*

$$E(X-r)^+ \leq \frac{1}{2}\left[\sqrt{\sigma^2 L + (r-DL)^2} - (r-DL)\right]. \tag{19}$$

Furthermore, the upper bound of the above equation is tight.

Because we have $r = DL + k\sigma\sqrt{L}$, and for any probability distribution of the lead time demand X, the above inequality always holds. Then, using model (13) and inequality (19), and considering the safety factor k as a decision variable instead of r, model (18) is reduced to

minimize $JETC_U(Q, k, L, m)$

$$= \frac{D}{Q(1-\delta M_\rho)} \times \left\{A_b + A_v + m\left[F + \frac{\bar{\pi}\sigma\sqrt{L}(\sqrt{1+k^2}-k)}{2} + R(L)\right]\right\} + \frac{Dy\delta}{1-\delta M_\rho} + \frac{Dw(1-\delta)M_\rho}{1-\delta M_\rho} + h_b\sigma\sqrt{L}\left[k + (1-\beta)\frac{\sqrt{1+k^2}-k}{2}\right] + \frac{h_b Q}{2m}(1-\delta M_\rho) + \frac{h_v Q}{2m(1-\delta M_\rho)} \times \left[\frac{D}{P} + (m-1)\left(1-\delta M_\rho - \frac{D}{P}\right)\right],$$

subject to

$$\frac{(1-M_\rho)Q}{m} \geq DL + k\sigma\sqrt{L}, \tag{20}$$

where the subscript U in $JETC$ denotes the distribution-free case.

By analogous arguments to the normal distribution demand case, it can be verified that for fixed (Q, k, m), $JETC_U(Q, k, L, m)$ is a concave function in $L \in [L_i, L_{i-1}]$. Hence, for fixed (Q, k, m), the minimum value of the joint expected total cost will occur at the end points of the interval $[L_i, L_{i-1}]$. $JETC_U(Q, k, L, m)$ can also be proven to be a convex function of m for fixed Q, k and $L \in [L_i, L_{i-1}]$, then the local minimum of $JETC_U(Q, k, L, m)$ turns out to be the global minimum of $JETC_U(Q, k, L, m)$. Furthermore, for fixed m and $L \in [L_i, L_{i-1}]$, it can also be shown that $JETC_U(Q, k, L, m)$ is a convex function in Q and k. Therefore, for fixed m and $L \in [L_i, L_{i-1}]$, the minimum value of $JETC_U(Q, k, L, m)$ will occur at the point (Q, k) which simultaneously satisfies $\partial JETC_U(Q, k, L, m)/\partial Q = 0$ and $\partial JETC_U(Q, k, L, m)/\partial k = 0$. The resulting solutions are

$$Q = \sqrt{\frac{2mD\{A_b + A_v + m[F + (\bar{\pi}\sigma\sqrt{L}(\sqrt{1+k^2}-k)/2) + R(L)]\}}{h_b(1-\delta M_\rho)^2 + h_v[(D/P) + (m-1)(1-\delta M_\rho - (D/P))]}}, \tag{21}$$

and

$$\frac{k}{\sqrt{1+k^2}} = 1 - \frac{2h_b Q(1-\delta M_\rho)}{mD\bar{\pi} + h_b(1-\beta)Q(1-\delta M_\rho)}. \tag{22}$$

An algorithm procedure similar to that proposed in section 4 can be used to obtain the optimal solution $(Q^{**}, k^{**}, L^{**}, m^{**})$.

6. Effects of the parameters

(1) From model (14), we can see that the objective function has a minimum value when $\beta = 1$ (i.e. complete backordered case) and a maximum value when $\beta = 0$ (i.e. complete lost sales case). Hence, for $0 < \beta < 1$, $JETC_{\beta=1} < JETC_\beta < JETC_{\beta=0}$. The distribution-free model has the same result.
(2) Note that $\bar{\pi} = \pi + \pi_0(1-\beta)$. When $\beta = 1$, equation (16) becomes

$$Q_{\beta=1} = \sqrt{\frac{2mD\{A_b + A_v + m[F + \pi\sigma\sqrt{L}\psi(k) + R(L)]\}}{h_b(1-\delta M_\rho)^2 + h_v[(D/P) + (m-1)(1-\delta M_\rho - (D/P))]}}. \tag{23}$$

When $\beta = 0$, equation (16) becomes

$$Q_{\beta=0} = \sqrt{\frac{2mD\{A_b + A_v + m[F + (\pi + \pi_0)\sigma\sqrt{L}\psi(k) + R(L)]\}}{h_b(1 - \delta M_\rho)^2 + h_v[(D/P) + (m-1)(1 - \delta M_\rho - (D/P))]}}. \quad (24)$$

Hence, for fixed k, m and $L \in [L_i, L_{i-1}]$, comparing equations (16), (23) and (24), we get $Q_{\beta=1} < Q_\beta < Q_{\beta=0}$. That is, the order quantity Q decreases as the backorder rate β increases for fixed k, m and $L \in [L_i, L_{i-1}]$. The distribution-free model has the same result.

(3) Note that $\bar{\pi} = \pi + \pi_0(1 - \beta)$. When $\beta = 1$, equation (17) becomes

$$\Phi(k)_{\beta=1} = 1 - \frac{h_b Q(1 - \delta M_\rho)}{mD\pi}. \quad (25)$$

When $\beta = 0$, equation (17) becomes

$$\Phi(k)_{\beta=0} = 1 - \frac{h_b Q(1 - \delta M_\rho)}{mD(\pi + \pi_0) + h_b Q(1 - \delta M_\rho)}. \quad (26)$$

Hence, for fixed Q, m and $L \in [L_i, L_{i-1}]$, comparing equations (17), (25) and (26), we get $\Phi(k)_{\beta=1} < \Phi(k)_\beta < \Phi(k)_{\beta=0}$ which implies $k_{\beta=1} < k_\beta < k_{\beta=0}$, or equivalently, $r_{\beta=1} < r_\beta < r_{\beta=0}$. That is, the reorder point r decreases as the backorder rate β increases for fixed Q, m and $L \in [L_i, L_{i-1}]$.

(4) Under the assumption that the lead time demand follows normal distribution, for fixed β, m and $L \in [L_i, L_{i-1}]$, taking the derivative of equation (16) with respect to k, we have

$$\frac{\mathrm{d}Q}{\mathrm{d}k} = \frac{1}{Q} \cdot \frac{m^2 D\bar{\pi}\sigma\sqrt{L}[\Phi(k) - 1]}{h_b(1 - \delta M_\rho)^2 + h_v[(D/P) + (m-1)(1 - \delta M_\rho - (D/P))]}. \quad (27)$$

Note that $\Phi(k) - 1 < 0$, hence $\mathrm{d}Q/\mathrm{d}k < 0$. That is, safety factor k (or reorder point r) and order quantity Q have negative relative relation. Decreasing safety factor (or reorder point) will increase the order quantity. Also, the distribution-free model has the same result.

7. Numerical examples

Example 1: Consider an inventory system with normally distributed demand during the lead time and the following parameter values: $D = 1000$ units/year, $P = 3200$ units/year, $A_b = \$25$/order, $A_v = \$400$/set-up, $h_b = \$5$/unit/year, $h_v = \$4$/unit/year, $\sigma = 7$units/week, and the lead time has three components with data shown in table 1 (Banerjee 1986, Goyal 1988, Ouyang *et al.* 1999a, Pan and Yang 2002). It is assumed 1 year = 52 weeks and 1 week = 7 days here. Besides, we take $y = \$1.6$/unit, $w = \$10$/defective unit, $\delta = 0.1$, $\pi = \$10$/unit, $\pi_0 = \$20$/unit, and $F = \$15$/shipment. The defective rate ρ has a Beta distribution with parameters $s = 1$ and $t = 9$; that is, the p.d.f. of ρ is given by $g(\rho) = 9(1-\rho)^8$, $0 < \rho < 1$. Hence, $M_\rho = s/(s+t) = 0.1$.

Applying Algorithm 1 procedure yields the results shown in table 2 for $\beta = 0$, 0.5, 0.8 and 1. From table 2, the optimal inventory policy can be found by comparing $JETC_N(Q_i, k_i, L_i, m_i)$, and a summary is presented in table 3. Table 3 shows that the reorder point (r*), safety factor (k*), the joint expected total cost per unit time ($JETC_N$) decrease, whereas the order quantity (Q*) increases as the backorder rate (β) increases. This implies that the backorder rate and reorder point have a relatively negative relationship. The reorder point and order quantity also have a relatively negative relationship. Furthermore, we can see that the joint expected total cost per unit time has the minimum value at the complete backordered case (i.e. $\beta = 1$) and the maximum value at the complete lost sales case (i.e. $\beta = 0$).

Example 2: In this example, we want to confirm the effects of taking the reorder point r as a decision variable. The data and assumptions are the same as in Example 1, except that we assume the arrival lot contains no defective items. Therefore, here we take $y = \$0$ and $M_\rho = 0$. The computational results are presented in table 4 which shows that our model

Table 1. Lead time data.

Lead time component i	Normal duration b_i (days)	Minimum duration a_i (days)	Unit crash cost c_i (\$/day)
1	20	6	0.1
2	20	6	1.2
3	16	9	5.0

Table 2. The results of solution procedure in Example 1.

β	i	L_i	Q_i	r_i	k_i	m_i	$JETC_N$ (Q_i, k_i, L_i, m_i)
0.0	0	8	553	195	2.10	5	\$3176.68
	1	6	555	151	2.10	5	\$3156.82
	2	4	566	105	2.00	4	\$3252.78
	3	3	577	80	1.87	3	\$3433.73
0.5	0	8	553	192	1.93	5	\$3161.60
	1	6	556	148	1.92	5	\$3143.74
	2	4	567	102	1.82	4	\$3241.73
	3	3	578	78	1.68	3	\$3423.70
0.8	0	8	554	189	1.76	5	\$3147.54
	1	6	556	146	1.76	5	\$3131.56
	2	4	567	100	1.65	4	\$3231.38
	3	3	578	76	1.50	3	\$3414.23
1.0	0	8	555	186	1.60	5	\$3133.47
	1	6	557	143	1.60	5	\$3119.37
	2	4	568	98	1.47	4	\$3220.97
	3	3	579	74	1.31	3	\$3404.61

L_i in weeks.

Table 3. Optimal solutions of Example 1.

β	m^*	Q^*	r^*	k^*	L^*	$JETC_N$ (Q^*, k^*, L^*, m^*)
0.0	5	555	151	2.10	6	\$3156.82
0.5	5	556	148	1.92	6	\$3143.74
0.8	5	556	146	1.76	6	\$3131.56
1.0	5	557	143	1.60	6	\$3119.37

L^* in weeks.

provides lower cost compared with that in Pan and Yang (2002). These results reveal that taking the reorder point as a decision variable will improve the joint total expected cost significantly.

Example 3: The data are as in Example 1 except the probability distribution of the lead time demand is free. A summary of the optimal results is presented in table 5, which shows that the safety factor and the joint expected total cost per unit time decrease as the backorder rate increases. Similar to Example 1, table 5 reveals that the joint expected total cost per unit time has the minimum value at the complete backordered case (i.e. $\beta = 1$) and the maximum value at the complete lost sales case (i.e. $\beta = 0$).

From Examples 1 and 3, we can compare the results of the worst case distribution against the normal distribution. The joint expected total cost

Table 4. Summary of the comparisons.

	β	m^*	Q^*	k^*	L^*	$JETC_N$ (Q^*, k^*, L^*, m^*)
Pan and Yang's model	–	4	132	2.33 (fixed)	6	\$2114.33
This model (with $\gamma = 0$ and $M_\rho = 0$)	0.0	5	551	2.10	6	\$2081.40
	0.5	5	552	1.92	6	\$2068.33
	0.8	5	552	1.76	6	\$2056.14
	1.0	5	553	1.60	6	\$2043.94

L^* in weeks.

Table 5. Optimal solutions of Example 3.

β	m^{**}	Q^{**}	r^{**}	k^{**}	L^{**}	$JETC_N$ $(Q^{**}, k^{**}, L^{**}, m^{**})$
0.0	3	563	162	2.73	6	\$3505.37
0.5	3	551	153	2.20	6	\$3410.82
0.8	3	542	146	1.80	6	\$3340.64
1.0	4	573	144	1.67	6	\$3279.05

L^{**} in weeks.

Table 6. Comparison of the two procedures.

β	$JETC_N$ $(Q^{**}, k^{**}, L^{**}, m^{**})$	$JETC_N$ (Q^*, k^*, L^*, m^*)	$EVAI$
0.0	\$3254.94	\$3156.82	98.12
0.5	\$3211.34	\$3143.74	67.60
0.8	\$3183.93	\$3131.56	52.37
1.0	\$3135.68	\$3119.37	16.31

$JETC_N(Q^{**}, k^{**}, L^{**}, m^{**})$ can be obtained by substituting $(Q^{**}, k^{**}, L^{**}, m^{**})$ in table 5 into model (14), in which the lead time demand is normally distributed. The expected value of additional information (EVAI) is the largest amount for a buyer that he/she is willing to pay for the information about the form of the lead time demand distribution which is equal to $JETC_N(Q^{**}, k^{**}, L^{**}, m^{**}) - JETC_N(Q^*, k^*, L^*, m^*)$. The EVAI concept in the distribution free approach is the original work of Gallego and Moon (1993) and has been expanded by many researchers. The comparatively small EVAI value reveals that the minimax decision criterion is a good approach. Computation results are shown in table 6 which reveals that the EVAI decreases as the backorder rate β increases.

8. Concluding remarks

This paper examined the effects of a sublot sampled inspection policy and controllable lead time on integrated vendor–buyer inventory models with defective items. We first assumed that the lead time demand followed a normal distribution. We then relaxed the assumption about the probability distributional form for the lead time demand and applied the minimax distribution-free procedure to solve the problem. We minimize the joint expected total cost per unit time by simultaneously optimizing the order quantity, reorder point, lead time and the number of shipments delivered from the vendor to the buyer.

The sensitivity analysis and numerical example results indicated that the safety factor and the joint expected total cost per unit time decrease as the backorder rate increase regardless of the distribution form of lead-time demand. The results also showed that the joint expected total cost per unit time is the smallest in the complete backordered case and the largest in the complete lost sales case.

The proposed models focused on the basic integrated inventory model, in which the joint expected total cost is simply the sum of the vendor's and buyer's costs. It is interesting to consider different economic equilibrium solutions in forming the joint expected total cost, such as Stackelberg (equilibrium), Pareto efficient scheme or Nash bargaining approach to determine the best cooperative program.

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Appendix

A: For fixed Q, k, and L, $JETC_N(Q,k,L,m)$ is convex in m

To simplify the proof, m is treated as a real number. Taking the partial derivatives of $JETC_N(Q,k,L,m)$ with respect to m, we have

$$\frac{\partial JETC_N(Q,k,L,m)}{\partial m} = \frac{D}{Q(1-\delta M_\rho)}\left[F+\bar{\pi}\sigma\sqrt{L}\psi(k)+R(L)\right] - \frac{h_b Q}{2m^2}(1-\delta M_\rho) + \frac{h_v Q}{2m^2}\left(1-\frac{2D}{P(1-\delta M_\rho)}\right). \quad \text{(A1)}$$

Set $\partial JTEC_N(Q,k,L,m)/\partial m = 0$, we can obtain

$$m_I = \left\{\frac{h_b Q^2(1-\delta M_\rho)^2 - h_v Q^2(1-\delta M_\rho)[1-2D/(P(1-\delta M_\rho))]}{2D[F+\bar{\pi}\sigma\sqrt{L}\psi(k)+R(L)]}\right\}^{1/2}. \quad \text{(A2)}$$

Because

$$\left.\frac{\partial^2 JETC_N(Q,k,L,m)}{\partial m^2}\right|_{m=m_I} = \frac{2D}{m_I Q(1-\delta M_\rho)}\left[F+\bar{\pi}\sigma\sqrt{L}\psi(k)+R(L)\right] > 0, \quad \text{(A3)}$$

$JETC_N(Q,k,L,m)$ is convex in m for fixed Q, k, and L.

B: To proof $JETC_N(Q,k,L,m)$ is a convex function in (Q,k), for fixed m and $L \in [L_i, L_{i-1}]$

When the lead time demand follows a normal distribution, the joint expected total cost per unit time is

$$\begin{aligned} JETC_N(Q,k,L,m) &= \frac{D(A_b+A_v)}{Q(1-\delta M_\rho)} + \frac{mD}{Q(1-\delta M_\rho)}\left\{F+\bar{\pi}\sigma\sqrt{L}\psi(k)+R(L)\right\} \\ &+ \frac{Dy\delta}{1-\delta M_\rho} + \frac{Dw(1-\delta)M_\rho}{1-\delta M_\rho} + h_b\sigma\sqrt{L}[k+(1-\beta)\psi(k)] \\ &+ \frac{h_b Q}{2m}(1-\delta M_\rho) + \frac{h_v Q}{2m(1-\delta M_\rho)} \\ &\times \left[\frac{D}{P} + (m-1)\left(1-\delta M_\rho - \frac{D}{P}\right)\right]. \end{aligned}$$

For fixed m and $L \in [L_i, L_{i-1}]$, taking the second partial derivatives of $JETC_N(Q,k,L,m)$ with respect to Q and k yields

$$\frac{\partial^2 JETC_N(Q,k,L,m)}{\partial Q^2} = \frac{2D(A_b+A_v)}{Q^3(1-\delta M_\rho)} + \frac{2mD\{F+\bar{\pi}\sigma\sqrt{L}\psi(k)+R(L)\}}{Q^3(1-\delta M_\rho)} > 0, \quad \text{(B1)}$$

$$\frac{\partial^2 JETC_N(Q,k,L,m)}{\partial k^2} = \frac{mD\bar{\pi}\sigma\sqrt{L}}{Q(1-\delta M_\rho)}\phi(k) + h_b\sigma\sqrt{L}(1-\beta)\phi(k) > 0, \quad \text{(B2)}$$

and

$$\frac{\partial^2 JETC_N(Q,k,L,m)}{\partial k \partial Q} = \frac{\partial^2 JETC_N(Q,k,L,m)}{\partial Q \partial k} = \frac{-mD\bar{\pi}\sigma\sqrt{L}}{Q^2(1-\delta M_\rho)}[\Phi(k)-1]. \quad \text{(B3)}$$

From equations (B1), (B2) and (B3), we obtain

$$\frac{\partial^2 JETC_N(Q,k,L,m)}{\partial Q^2}\cdot\frac{\partial^2 JETC_N(Q,k,L,m)}{\partial k^2} - \left[\frac{\partial^2 JETC_N(Q,k,L,m)}{\partial k \partial Q}\right]^2 = \frac{2D^2 m\bar{\pi}\sigma\sqrt{L}\phi(k)(A_b+A_v+mF+mR(L))}{Q^4(1-\delta M_\rho)^2} + \frac{2Dh_b\sigma\sqrt{L}(1-\beta)\phi(k)}{Q^3(1-\delta M_\rho)} \times \left\{A_b + A_v + m\left[F + \bar{\pi}\sigma\sqrt{L}\psi(k) + R(L)\right]\right\} + \frac{(Dm\bar{\pi}\sigma\sqrt{L})^2}{Q^4(1-\delta M_\rho)^2}\left\{2\phi(k)\psi(k) - [\Phi(k)-1]^2\right\} > 0,$$

because $\phi(k) > 0$, $\psi(k) > 0$ and $2\phi(k)\psi(k) - [\Phi(k)-1]^2 > 0$, for all $k > 0$ (the proof see Ouyang *et al.* 1999a). Therefore, for fixed m and $L \in [L_i, L_{i-1}]$, $JETC_N(Q,k,L,m)$ is a convex function in (Q,k).

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