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A simulated annealing approach for curve fitting in automated manufacturing systems

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Abstract

Purpose – This research aims to develop an effective and efficient algorithm for solving the curve fitting problem arising in automated manufacturing systems.

Design/methodology/approach – This paper takes curve fitting as an optimization problem of a set of data points. Expressing the data as a function will be very effective to the data analysis and application. This paper will develop the stochastic optimization method to apply to curve fitting. The proposed method is a combination optimization method based on pattern search (PS) and simulated annealing algorithm (SA).

Findings – The proposed method is used to solve a nonlinear optimization problem and then to implement it to solve three circular arc-fitting problems of curve fitting. Based on the analysis performed in the experimental study, the proposed algorithm has been found to be suitable for curve fitting.

Practical implications – Curve fitting is one of the basic form errors encountered in circular features. The proposed algorithm is tested and implemented by using nonlinear problem and circular data to determine the circular parameters.

Originality/value – The developed machine vision-based approach can be an online tool for measurement of circular components in automated manufacturing systems.

Keywords Automation, Manufacturing systems, Optimization techniques, Algorithmic languages

Paper type Research paper

Introduction

The automatic interpretation and acquisition of the information of product features is an important procedure in automated manufacturing systems. With the increasing demand for manufacturing automation, curve fitting receives considerable attention because it plays a key role in computer vision, reverse engineering, rapid prototyping, computer simulation, etc. (Chan *et al.*, 2002; Luo *et al.*, 2003; Xiyu *et al.*, 2003). The advantages of using such automatic systems include a decrease in the time required for measurement as well as the greater accuracy of measurement and better flexibility than the conventional method (Chen *et al.*, 1999; Tseng, 2006). The machine vision techniques for inspection are gaining recognition as the trend in industry. It can provide a non-contract measurement process of 100 percent inspection for measuring

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a wide class of objects in small-batch and mass production. The study of effective algorithms specific to manufactured parts and form fitting becomes imperative in vision-based inspection.

Curve fitting is the process of provided data set in approaching to a function. Expressing the data as function will be very effective to the data analysis and application. In the automated manufacturing systems, curve fitting is combined with the analysis in common use or engineering technology, such as system simulation, forecast modeling and image analysis, etc. to be applied to the solution of the production-related problem.

The purpose of curve fitting is to select function parameter value to minimize the total error sum of a set of data points that is taken into consideration. Once the function forming and error representation method is determined, curve fitting will be the optimization problem of a set of data points. Although least squares method is usually used in curve fitting, the solution obtained does not often have the least error (Shunmugan, 1986). So this current paper proposes to apply an optimization method to the complicated curve-fitting problem. This method does not need the differential data of the function.

The curve fitting proposed in this current paper is a stochastic optimization method. This method is the combination of pattern search (PS) and simulated annealing algorithm (SA) and is named PSSA. PS is an effective optimization method, and has been successfully applied to the nonlinear programming problem. SA was developed into an effective optimization technology recently, and has been proved to have the capability of jumping out local optimum, but SA needs a considerable quantity of calculation demand. To improve SA efficiency in problem solving, the PSSA proposed in this paper will take PS as the search move function of SA.

The remainder of this paper is organized as follows: next section provides mathematical models of least squares circle for circle fitting; in the following section presents the proposed PSSA optimization method of curve fitting; fourth section shows a test example and an actual example of circle fitting; and final section gives the conclusion.

Least squares circle

Form fitting algorithms have become increasingly important in modern dimensional measurement systems. This is particular true for vision-based inspection systems (Chen *et al.*, 1999; Hopp, 1993). The algorithms for form fitting which convert measured data to the reference geometry can be major source of error in a measurement system. Form fitting can be viewed as an optimization problem in finding the parameters of reference geometry that minimizes a particular fitting objective for a set of points (Yue *et al.*, 1999). The traditional instruments for circle fitting generally apply the least squares technique to evaluate the fitting errors from measured points.

The least squares circle is the circle chosen so that the sum of the squares of the radial distance of all data points from the fitting circle is a minimum. Given a set of data points in two dimensions $P = \{p_i = (x_i, y_i), i = 1, 2, \ldots n\}$ which represents the profile of a workpiece, it is possible to find explicitly the circle parameters by minimizing the least square errors between the given set of data points and the curve (Thomas and Chan, 1989). The least squares circle is the most widely used reference circle for the assessment of form errors (Coope, 1993; Gander *et al.*, 1994; Thomas and Chan, 1989) due to its computational simplicity.

Let (x_i, y_i), $i = 1, 2, \ldots n$, be the coordinate measurements of n points from a circle of center (x_c, y_c) and radius r, because of measurement errors and other factors, the requirement of circle fitting is to find the best estimate of x_c, y_c and r from (x_i, y_i). Suppose two dimension circle placed in the xy-plane may be written as:

$$f(x,y) = (x - x_c)^2 + (y - y_c)^2 - r^2 = 0. \tag{1}$$

The normal deviation error (e_i) between a data point (x_i, y_i) and the circle of radius r and center (x_c, y_c) is given by:

$$e_i = [(x_i - x_c)^2 + (y_i - y_c)^2]^{1/2} - r. \tag{2}$$

Most literatures about the least squares fitting of circle (Chan *et al.*, 2002, 2000; Chan and Thomas, 1997; Kim and Kim, 1996) for selecting the circle parameters x_c, y_c, r have been concerned with the squares sum of geometric distances:

$$M_s = \sum_{i=1}^{n} \{[(x_i - x_c)^2 + (y_i - y_c)^2]^{1/2} - r\}^2. \tag{3}$$

Several iterative solutions are available in the literature (Ahn *et al.*, 2001; Chan *et al.*, 2002, 2000; Joseph, 1994; Thomas and Chan, 1989) so as to minimize the function:

$$M_c = \sum_{i=1}^{n} \{(x_i - x_c)^2 + (y_i - y_c)^2 - r^2\}^2. \tag{4}$$

The result is a closed form solution for the parameters x_c, y_c, r. Another linear estimator (Chan *et al.*, 2000; Coope, 1993; Kim and Kim, 1996; Thomas and Chan, 1989) comes from the rearrangement of Equation (4) to give:

$$M_l = \sum_{i=1}^{n} \left[x_i^2 + y_i^2 - \left(2x_ix_c + 2y_iy_c + r^2 - x_c^2 - y_c^2\right)\right]^2. \tag{5}$$

By treating x_c, y_c and $r^2 - x_c^2 - y_c^2$ as the three unknowns, a simple least square minimizing of Equation (5) gives a closed form estimation of the circle parameters.

Determining the circle of best fit to a set of point in the plane is an important problem (Coope, 1993; Kim and Kim, 1996). An unbiased iterative method for obtaining a least squares fit of a circular arc has been described (Chan *et al.*, 2000; Joseph, 1994). Chan and Thomas (1997) have proposed an approximate maximum likelihood linear estimator of circle parameters. Chan *et al.* (2000) presented two algorithms, one iterative and another has a closed form solution, providing unbiased and reliable estimates of the circle parameters for noisy measurements on arcs. Chan *et al.* (2002) proposed an estimation scheme for the circle parameters by first computing different centers from all combinations.

Optimization method of curve fitting

In this section, the solving method of curve fitting optimization model is developed. The optimization method proposed in this paper combines the SA and PS method. SA is able to effectively jump out local optimum and try to search global optimum, but if the search frequency is taken into consideration, SA needs very large amount of search frequency and causes the lack in efficiency. PS is very effective in solving nonlinear programming problem (Ignizio, 1976), but PS also tends to have convergence at the local optimum. This PSSA optimization method makes SA more effective in problem solving. Therefore, PS is used as the search move function to avoid PS converging at the inferior local optimum and to let PSSA become the stochastic optimization method.

Simulated annealing algorithm

SA (Kirkpatrick *et al.*, 1983; Metropolis *et al.*, 1953) is a stochastic search technique. It is designed to lead jumping out local optimum during the search process. SA is a very effective combinatorial optimization method, which is successfully applied to VLSI design, schedule, plant layout and, etc. and the combinatorial optimization problem related to production (Collins *et al.*, 1988) while extending its application to the continuous variable optimization problem (Corana *et al.*, 1987).

Similar to statistics mechanism, the search process of SA is also operated in accordance with transition probability. This transition probability depends on control temperature and the changing amount of objective function. Since, SA possesses stochastic search policy, it can be "uphill" move by using the solution of a larger objective function as the present solution. Moving the present solution to an inferior solution under a controlled probability may allow SA jump out local optimum and may be a better "downhill" path will be found, which further obtains a better solution. The "uphill" move is controlled carefully by the temperature. When temperature is too high, "uphill" move probability will increase; when temperature decreases gradually, "uphill" move probability will decrease accordingly. Kirkpatrick *et al.* (1983) demonstrated that SA is capable of obtaining the real optimum under a very long search time. In actual execution, this real optimum is not obtainable due to the restriction of calculation time, but an approximation of the real optimum can be obtained.

When SA is applied to one problem, four basic components should be defined (Rutenbar, 1989). These are:

- *Configuration.* represents the possible solution of problem.
- *Move set.* Is an allowable move, which can let us achieve all feasible configuration, this set have to be easy for calculation.
- *Cost function.* Is used to measure the quality of configuration.
- *Cooling schedule.* Sets the initial temperature and the cooling regulation to determine when and how many degrees to decrease the present temperature, and when to end the annealing.

The SA method that solves the continuous function problem proposed by Corana *et al.* (1987) faces a tough problem that needs long copious calculations. The curve fitting optimization model mentioned in the previous section is immense and complicated. The method mentioned by Corana *et al.* is not suitable to apply to the optimization model solving. So to decrease the SA calculation demand, the PSSA optimization method proposed in this current paper uses PS as the move generating function to accelerate search process.

Pattern search

The PSSA optimization method recommended in this paper uses PS (Hooke and Jeeves, 1961) as move generation mechanism. The operation of PS includes two kinds of move: exploratory move and pattern move. Exploratory move examines the local behavior of function, and finds out the direction of "downhill" path. Pattern move utilizes the information generated by exploratory move to reach the valley rapidly.

Exploratory move starts searching from initial point and moves along the axial direction of each variable according to step size, and decides whether neighboring move is set as present solution in accordance with objective function. If setting neighboring solution as present solution is unacceptable (no improvement in objective function), it will search the opposite direction. During exploratory move, each move only refers to one variable, and explores each variable in proper order to check whether there is improvement in the objective function. If there is an improvement, it means that a direction of "downhill" path (or pattern direction) exists. After an exploratory move ends, it will check whether a pattern direction exists. If it does exist, make the pattern move according to this pattern direction, and this move can rapidly reach the minimum of the function (the valley) and continue executing the pattern move until no objective function improves anymore.

Changing step size or not depends on pattern move existence after each exploratory move ends. If a pattern direction does not exist, then change the step size. PS uses exploratory move and pattern move repeatedly until the result condition is satisfied.

PSSA optimization algorithm

As mentioned above, when the SA approach is applied to the optimization problems with continuous variables, it encounters a serious problem of huge computational requirements. Therefore, the PS algorithm is incorporated into the SA algorithm as the move generation mechanism to improve the efficiency of optimization procedure. An application of the SA approach needs first to define four basic components of the algorithm. The four basic components in the proposed PSSA optimization algorithm are stated as follows:

(1) *Configuration.* A legal configuration is the parameter combination of curve fitting optimization model S.
(2) *Move set.* All curve parameter combination generated by PS are the element of move set. The search move set of PSSA optimization method proposed in this paper is determined by PS, and PS also decides and adjusts the step size of optimization process and search direction.
(3) *Cost function.* Objective function is the expression of curve fitting error.
(4) *Cooling schedule.* The geometric cooling schedule (Collins *et al.*, 1988) is adopted here.

This method uses $\Gamma = c \times \Gamma$ mode to cool down after M times of move, where Γ is the present control temperature, c is the cooling ratio, and $0 < c < 1$, that is one cycle. When there is still no present optimum change condition in K times of cycle continuously, the annealing is regarded as frozen and at this moment, the search will stop. There are many cooling schedules discussed in the literature (Van Laarhoven and Aarts, 1987), and geometric cooling schedule is very fast and very effective (Jeffcoat and Bulfin, 1993). Therefore, this study adopts geometric cooling schedule to decrease the calculation demand.

The definition of symbol used in PSSA optimization method developed in this paper is listed below:

- M: number of variables
- $S = \{S_1, S_2, \ldots S_m\}$: parameter combination
- $S^0 = \left\{S_1^0, S_2^0, \ldots, S_m^0\right\}$: initial solution

- S': neighboring solution of S
- $U = \{u_1, u_2, \ldots u_m\}$: step size vector
- $U^0 = \left\{U_1^0, U_2^0, \ldots, U_m^0\right\}$: initial step size vector
- $\Delta S = \{\Delta s_1, \Delta s_2, \ldots \Delta s_m\}$: pattern direction vector
- $F(S)$: objective function value
- $\Delta E = F(S')\text{-}F(S)$: change quantity of objective function value
- n_{I}: a counter for number of step size increment
- r_{I}: step size increasing rate, $r_{\mathrm{I}} > 1$
- I: maximum number of step size increment allowed
- n_{D}: a counter for number of step size decrement
- r_{D}: step size decreasing rate, $0 < r_{\mathrm{D}} < 1$
- n_M: a counter for number of search points at a temperature level
- M: specified number of search points at a temperature level
- n_k: a counter for checking frozen state achieved
- K: specified maximum number of n_k
- Γ^o: the initial control temperature
- Γ: the control temperature
- C: cooling ratio, $0 < c < 1$

The algorithm is listed below:

Step 1. (Initialize the search procedure)

(a) Get an initial solution S^O, an initial control temperature Γ^o and initial step sizes U^O.

(b) set $S = S^O$, $U = U^O$, $\Gamma = \Gamma^o$, $n_k = 0$, $n_M = 0$, $n_{\mathrm{I}} = 0$, $n_{\mathrm{D}} = 0$, imp $= 0$, frozen $= 0$, evaluate $F(S)$.

Step 2. (Exploratory move) Set $\Delta S = 0$, $S' = S$,

For $j = 1$ to m

(a) set $s'_j = s_j + u_j$, $\Delta E = F(S') - F(S)$

If $\Delta E < 0$ (downhill move), set $S = S'$ $\Delta s_j = u_j$, imp $= 1$.

(b) If $\Delta E \geq 0$ (uphill move), set $S = S'$ $\Delta s_j = u_j$,

imp $= 1$ with probability $e^{-\Delta E/\Gamma}$

(c) Perform sub-procedure CHECK.

If frozen $= 1$, go to Step 5.

(d) If $\Delta s_j = 0$, set $s'_j = s_j - u_j$, $\Delta E = F(S') - F(S)$.

Otherwise, return to (a).

(e) If $\Delta E < 0$ (downhill move), set $S = S'$ $\Delta s_j = -u_j$, imp $= 1$.

(f) If $\Delta E \geq 0$ (uphill move), set $S = S'$ $\Delta s_j = -u_j$,

imp $= 1$ with probability $e^{-\Delta E/\Gamma}$.

(g) Perform sub-procedure CHECK.
If frozen $= 1$, go to Step 5.
Next j.
Step 3. (Check pattern direction found and adjust step sizes)
(a) If $\Delta S \neq 0$, go to Step 4.
(b) If $n_{\mathrm{I}} \leqq I$, set $n_{\mathrm{I}} = n_{\mathrm{I}} + 1$,
set $U = (r_{\mathrm{I}})^{n_{\mathrm{I}}} \times U^0$ (increase step sizes).
Go to Step 2.
Otherwise, set $n_{\mathrm{D}} = n_{\mathrm{D}} + 1$,
set $U = (r_{\mathrm{D}})^{n_{\mathrm{D}}} \times U^0$ (decrease step sizes).
Go to Step 2.
Step 4. (Pattern move)
(a) set $S' = S + \Delta S$, $\Delta E = F(S') - F(S)$
(b) If $\Delta E < 0$ (downhill move), set $S = S'$ imp $= 1$, go to (d).
(c) If$\Delta E \geq 0$ (uphill move), set $S = S'$,
imp $= 1$ with probability $e^{-\Delta E/\Gamma}$
(d) Perform sub-procedure CHECK.
(e) If frozen $= 1$, go to Step 5.
(f) If $S = S'$ return to (a) (continue pattern move).
Otherwise, return to Step 2 with S.
Step 5. (Termination)
Return S and terminate search.
Sub-procedure check (Check improvement and lower control temperature)
Step 1. Set $n_{\mathrm{M}} = n_{\mathrm{M}} + 1$
If $n_{\mathrm{M}} = M$, go to Step 2.
Otherwise, go to Step 4.
Step 2. (Check improvement during M moves) Set $n_M = 0$.
If imp $= 1$, (current best solution improved)
set $n_k = 0$.
Otherwise, set $n_k = n_k + 1$,
If $n_k = K$, (frozen state achieved)
set frozen $= 1$.
Otherwise, set frozen $= 0$.
Step 3. (Lower control temperature)
Set $\Gamma = c \times \Gamma$.
Step 4. Return

Generally, the setting method of SA set parameter differs in accordance with different application problem. In the PSSA search process, present optimum will move up and down, so algorithm will remember the search point with the minimal objective function. This PSSA optimization method possesses the following characteristics: the ability to jump out local optimum; easy transformation into computer program; management of curve fitting problem without strict assumption of the objective function and parameter.

The proposed PSSA algorithm has several desirable characteristics, including the ability to jump out local optimum, ease of implementation algorithmically, robustness for dealing with complicated nonlinear problems, and no restrictive assumptions about objective function, constraint set and parameter set. Based on the survey of Koulamas *et al.* (1994), simulated annealing is a viable optimization method and is less sensitive to the problem size. Generally, a large number of iterations yield the solution with a higher probability of convergence to the optimum one. From the primary experiments, the proposed algorithm can efficiently approach to the vicinity of the optimal solution. In practice, if the PSSA process is not converged, it may restart the search procedure by using a new random number sequence. This process can be repeated until the objective is achieved.

Implementation and results

PSSA solving ability test

First, the proposed method PSSA was used to solve a nonlinear optimization problem with many local optimums to test and prove its solving ability. This nonlinear optimization problem is as follows:

$$\mathrm{Min}\left(x_1^2+x_2^2\right)/2-\cos(20\pi x_1)\cos(20\pi x_2)+2 \quad -10\leq X_1\leq 10 \quad -10\leq X_2\leq 10 \quad (6)$$

This optimization problem has 40,000 local optimums, and its real optimum is on (0, 0), the optimal value is 1. Yip and Pao (1995) had applied two kinds of genetic algorithm (Goldberg, 1989) to solve this problem. One of them is simulated evolution (SE), while the other method is the combination of genetic algorithm and SA, which is called guided evolutionary simulated annealing (GESA) (Yip and Pao, 1994). Yip and Pao used SE and GESA, respectively, to run the problem for 50 times. In each result, 66 percent of SE obtains the real optimum, 34 percent obtains approximated optimum (function value 1.0025), 100 percent of GESA obtains the real optimum, but, SE and GESA calculation demand is very large. The solution search points of each run are 800,000 points.

The test example is solved on an IBM PC 586 compatible computer using the C programming language. The initial solution is randomly selected within the variable bounds. The user-specified parameters for this ability test problem and actual example of curve fitting are listed in Table I. The developed PSSA is used to run this problem for 50 times. The resulting 87 percent obtains the real optimum, 13 percent the approximated optimum (1.0025); the average solution search point of each run is 27,118 points, which is only 3.4 percent of SE and GESA. Although PSSA solution quality is inferior to GESA, its solution efficiency is far better than SE and GESA (Table II). Johnson *et al.* (1989) had executed an experiment on SA with figure division problem and found that not only the number of solution point was very large, but also the solution obtained varied in large range. In this example, the solution variation obtained

Symbol	Value
I	5
r_I	1.47
R_D	0.77
M	30
K	50
Γ^o	1000.0
C	0.95
U^0 (test example)	{0.1, 0.1}
U^0 (actual example)	{10.0, 10.0, 10.0}

Table I.
The setting values of PSSA

	PSSA	SE	GESA
Run times	50	50	50
Real optimum obtained (percent)	87	66	100
Approximated optimum	1.0025	1.0025	1.0025
Solution search points	27,118	800,000	800,000

Table II.
Comparison of PSSA to SE and GESA

by PSSA is very small and is not affected by the different initial solution. From this result, the solution efficiency of PSSA and the result reliability can be proved.

Actual example of curve fitting

As mentioned aforesaid, once the function form and error expression method are decided, the curve fitting becomes the optimization problem of a set of data points. This current study uses three arc functions to produce three groups of data points and uses these three groups of data points to make the curve fitting of complete circle, 1/2 circular arc and 1/4 circular arc of different interval, and to prove the feasibility and reliability of PSSA application on curve fitting problem. In this paper, error expression method is the least square error. This optimization problem of least squares circle can be expressed as:

$$\text{Min}\, F(s) = \sum_{i=1}^{n} \left[(x_i - x_c)^2 + (y_i - y_c)^2 - r^2 \right]^2 \quad x_L \leq x_c \leq x_U \quad y_L \leq y_c \leq y_U \quad 0 \leq r \leq r_U \tag{7}$$

where $S = \{x_c, y_c, r\}$ is the arc parameter, the decision variable of this problem, (x_c, y_c) is the coordinate value of the center of circle, r is radius. $F(S)$ is the objective function of PSSA, (x_i, y_i) is the coordinate value of the i th point, n is the data point, x_U, y_U, r_U are the upper bound of x_c, y_c, r,, respectively, x_L, y_L, 0 are the lower bound of x_c, y_c, r,, respectively. In the actual application, arc parameters often present in a certain interval, so the upper and lower bound are set. In this paper, the upper bound and lower bound of the center coordinate of circle are set at 500 and 500 separately, and the largest value of radius is 500.

Three circular arcs are circle 1: center of circle (100, -20) and radius 10, circle 2: center of circle (0, 0) and radius 100, circle 3: center of circle (-10, -12) and radius 200, respectively. Each group of data points is 1,000 points. The results of three arcs curve fitting are listed in Tables III-V, respectively. All initial points are randomly selected

Table III. Curve fitting result of circle 1

Curve fitting interval		N	S^0	S^*	$F(S^*)$	Search frequency
Complete circle	$[0, 2\pi]$	1000	{2.44, 61.62, 25.00}	{100.000, −20.000,10.000}	0.000	15,601
1/2 circle	$[0, \pi]$	501	{−95.30, 73.40, 8.00}	{100.000, −20.000,10.000}	0.000	14,221
	$[\pi/4, 5\pi/4]$		{85.80, −38.00, 91.00}	{100.000, −20.000,10.000}	0.000	14,101
	$[\pi/2, 3\pi/2]$		{−93.42, −37.22, 97.00}	{100.000, −20.000,10.000}	0.000	14,101
	$[-\pi/2, \pi/2]$		{65.82, −59.02, 14.00}	{100.000, −20.000,10.000}	0.000	14,791
1/4 circle	$[0, \pi/2]$	251	{10.32, −37.58, 23.00}	{100.000, −20.000,10.000}	0.000	14,211
	$[\pi/4, 3\pi/4]$		{−72.28, 11.36, 41.00}	{100.000, −20.000,10.000}	0.000	25,201
	$[\pi/2, \pi]$		{24.66, −50.40, 82.00}	{100.000, −20.000,10.000}	0.000	15,691
	$[3\pi/4, 5\pi/4]$		{71.38, −17.00, 79.00}	{100.000, −20.000,10.000}	0.000	17,311
	$[\pi\ \pi/2]$		{53.02, −75.24, 39.00}	{100.000, −20.000,10.000}	0.000	16,231
	$[\pi\ 3\pi/2]$		{93.28, 34.72, 21.00}	{100.000, −20.008, 9.992}	0.009	11,941
	$[5\pi/4, 7\pi/4]$		{35.38, 86.10, 41.00}	{100.000, −20.000,10.000}	0.000	14,491
	$[-\pi/4, \pi/4]$		{51.76, 21.46, 99.00}	{100.000, −20.000,10.000}	0.000	15,001

Table IV.
Curve fitting result of circle 2

Curve fitting interval		N	S^0	S^*	$F(S^*)$	Search frequency
Complete circle	$[0, 2\pi]$	1000	{63.16, − 78.80, 82.00}	{0.000, 0.000,100.000}	0.000	12,541
1/2 circle	$[0, \pi]$	501	{53.38, 2.74, 66.00}	{0.000, 0.000,100.000}	0.000	14,251
	$[\pi/4, 5\pi/4]$		{15.84, − 66.86, 95.00}	{0.000, 0.000,100.000}	0.000	12,541
	$[\pi/2, \pi/2]$		{− 81.24, 28.26, 96.00}	{0.000, 0.000,100.000}	0.000	12,241
	$[-\pi/2, \pi/2]$		{72.90, 72.82, 7.00}	{0.000, 0.000,100.000}	0.000	12,541
1/4 circle	$[0, \pi/2]$	251	{68.92, 65.04, 84.00}	{0.000, 0.000,100.000}	0.000	12,511
	$[\pi/4, 3\pi/4]$		{− 5.86, − 64.64, 92.00}	{0.000, − 0.003,100.003}	0.124	14,521
	$[\pi/2, \pi]$		{46.52, − 42.08, 51.00}	{0.000, 0.000,100.000}	0.000	12,871
	[3π/4, 5π?/4		{8.32, − 63.62, 85.00}	{0.000, 0.000,100.000}	0.000	12,901
	[π 3π/2]		{− 68.46, 44.38, 29.00}	{0.000, 0.000,100.000}	0.000	12,871
	[π 3π?/2]		{67.84, 38.74, 62.00}	{0.000, 0.003,100.003}	0.107	14,521
	$[5\pi/4, 7\pi/4]$		{− 42.92, 11.36, 79.00}	{0.000, 0.000,100.000}	0.000	12,811
	$[-\pi/4, \pi/4]$		{60.90, − 61.82, 9.00}	{0.000, 0.000,100.000}	0.000	12,961

Table V. Curve fitting result of circle 3

Curve fitting interval		N	S^0	S^*	$F(S^*)$	Search frequency
Complete circle	$[0, 2\pi]$	1,000	$\{-95.74, -96.96, 21.00\}$	$\{-10.000, -12.000, 200.000\}$	0.000	15,601
1/2 circle	$[0, \pi]$	501	$\{34.32, 11.02, 9.00\}$	$\{-10.000, -12.000, 200.000\}$	0.000	9,421
	$[\pi/4, 5\pi/4]$		$\{1.82, -37.22, 15.00\}$	$\{-10.000, -12.000, 200.000\}$	0.000	12,391
	$[\pi/2, 3\pi/4]$		$\{-27.08, 63.38, 11.00\}$	$\{-10.000, -12.000, 200.000\}$	0.071	7,651
	$[-\pi/2, \pi/2]$		$\{-33.10, -49.78, 7.00\}$	$\{-10.000, -12.000, 200.000\}$	0.000	11,191
1/4 circle	$[0, \pi/2]$	251	$\{-16.16, -70.42, 97.00\}$	$\{-10.000, -12.000, 200.000\}$	0.000	11,581
	$[\pi/4, 3\pi/4]$		$\{-1.88, -29.86, 47.00\}$	$\{-10.000, -11.997, 199.997\}$	0.549	11,581
	$[\pi/2, \pi]$		$\{-50.94, -12.46, 75.00\}$	$\{-10.000, -12.000, 200.000\}$	0.000	11,611
	$[3\pi/4, 5\pi/4]$		$\{88.94, -83.04, 3.00\}$	$\{-10.000, -12.000, 200.000\}$	0.000	12,511
	$[\pi\ 3\pi/2]$		$\{23.78, 31.12, 44.00\}$	$\{-10.000, -12.000, 200.000\}$	0.000	10,651
	$[\pi\ 3\pi/2]$		$\{30.00, 86.82, 96.00\}$	$\{-10.000, -11.998, 200.002\}$	0.289	11,401
	$[5\pi/4, 7\pi/4]$		$\{-46.76, -58.92, 9.00\}$	$\{-10.000, -12.000, 200.000\}$	0.000	10,831
	$[-\pi/4, \pi/4]$		$\{-7.38, -0.50, 84.00\}$	$\{-10.000, -12.000, 200.000\}$	0.000	12,091

within the variable bounds. The user-specified parameters for this actual example of curve fitting are listed in Table I. The average CPU time of each group only 0.3 seconds.

Table III shows that the 1/4 circular arc of interval [π, $3\pi/2$] obtains the approximated optimum (0.009) and the others obtain the real optimum. Table IV shows that the 1/4 circular arc of interval [$\pi/4$, $3\pi/4$] and [π, $3\pi/2$] obtain the approximated optimum (0.124 and 0.107) and the others obtain the real optimum. Table V shows that the 1/2 circular arc of interval [$\pi/2$, $3\pi/2$] obtain the approximated optimum (0.071) and two intervals of 1/4 circular arc obtain the approximated optimum and the others obtain the real optimum. Calculation result shows that the solution obtained by PSSA are excellent.

The result shows that PSSA can be effectively applied to the curve fitting of image data. This PSSA is able to obtain the approximated optimum through reasonable search frequency and has the ability to jump out local optimum. This method has consistency and the solution obtained is not sensitive to the initial solution. This PSSA method is the optimization method in common use, and can be easily modified to apply to different curve fitting optimization model, such as model with different error expression.

Conclusions

This study has successfully presented an optimization algorithm based on the PS and the simulated annealing algorithm (PSSA), which is applicable to complicated curve fitting problem. Curve fitting is one of the basic form errors encountered in circular features. In this paper, the mathematical programming models for least squares circle and a stochastic optimization method have been demonstrated to determine the circular parameters. The proposed PSSA algorithm is tested and implemented by using nonlinear problem and circular data. Based on the analysis performed in the experimental study, the proposed PSSA algorithm has been found to be suitable for curve fitting. The developed machine vision-based approach can be an online tool for measurement of circular components in automated manufacturing systems.

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