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# Retailer's optimal ordering policies with trade credit financing

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In this article, we extended Goyal's model to develop an Economic Order Quantity (EOQ) model in which the supplier offers the retailer the permissible delay period $M$, and the retailer in turn provides the trade credit period $N$ (with $N \leq M$) to his/her customers. In addition, we assume that (1) the retailer's selling price per unit is necessarily higher than its unit cost, and (2) the interest rate charged by a supplier or a bank is not necessarily higher than the retailer's investment return rate. We then establish an appropriate EOQ model with trade credit financing, and provide an easy-to-use closed-form solution to the problem. Furthermore, we find it is possible that a well-established buyer may order a lower quantity and take the benefit of the permissible delay more frequently, which contradicts to the result by the previous researchers. Finally, we perform some sensitivity analyses to illustrate the theoretical results and obtain some managerial results.

*Keywords*: Finance; Inventory; EOQ; Permissible delays

## 1. Introduction

In practice, a supplier frequently offers a retailer a delay of a fixed time period (say, 30 days) for settling the amount owed to him. Usually, there is no interest charge if the outstanding amount is paid within the permissible delay period (note that this credit term in financial management is denoted as 'net 30'). However, if the payment is not paid in full by the end of the permissible delay period, then interest is charged on the outstanding amount. Therefore, it is clear that a customer will delay the payment up to the last moment of the permissible period allowed by the supplier. The permissible delay in payments produces two benefits to the supplier: (1) it attracts new customers who consider it to be a type of price reduction, and (2) it may be applied as an alternative to price discount because it does not provoke competitors to reduce their prices and thus introduce lasting price reductions. On the other hand, the policy of granting credit terms adds not only an additional cost but also an additional dimension of default risk to the supplier.

Goyal (1985) developed an EOQ model under conditions of permissible delay in payments. He ignored the difference between the selling price and the purchase cost. Although Dave (1985) corrected Goyal's model by assuming the fact that the selling price is necessarily higher than its purchase price, his viewpoint did not draw much attention to the recent researchers. Mandal and Phaujdar (1989) studied the EOQ models by considering the interest earned from the sales revenue on the remaining beyond the settlement period. Aggarwal and Jaggi (1995) extended Goyal's model for deteriorating items. Jamal *et al.* (1997) further extended the model to allow for shortages. Hwang and Shinn (1997) added the pricing strategy into the model, and developed the optimal pricing and lot sizing for the

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retailer under the condition of permissible delay in payments. Sarker *et al*. (2000) presented an inventory model with deteriorating items for optimal cycle and payment times for a retailer, when a supplier allows a specified credit period to the retailer for payment without penalty. Teng (2002) amended Goyal's model by considering the difference between unit price and unit cost. Chang *et al*. (2003) developed an EOQ model for deteriorating items under supplier credits linked to ordering quantity. Chung and Huang (2003) developed an Economic Production Quantity (EPQ) inventory model for a retailer when the supplier offers a permissible delay in payments by assuming that the selling price is the same as the purchase cost. Huang (2003) extended Goyal's model to develop an EOQ model in which the supplier offers the retailer the permissible delay period $M$ (i.e., the supplier trade credit), and the retailer in turn provides the trade credit period $N$ (with $N \leq M$) to his/her customers (i.e., the customer trade credit). He then obtained the closed-form optimal solution and two interesting theoretical results. However, he assumed that not only is the unit purchase cost $c$ the same as the selling price per unit $p$, but also the interest rate $I_k$ charged by the supplier is always not lower than the retailer's return rate on investment $I_e$. As we know, the selling price per unit for a retailer is usually significantly higher than the unit cost in order to obtain profit. In addition, the supplier may charge the retailer the prime rate of 4.25% on unpaid balance in today's financial markets. However, the retailer may invest the money into stock markets or to develop new products, and get a return on investment, which is much higher than 4.25%. Many related articles can be found (Jamal *et al*. 2000, Liao *et al*. 2000, Arcelus *et al*. 2003, Biskup *et al*. 2003, Huang 2003, 2004, 2005, Shinn and Hwang 2003, Chang 2004, Chang and Teng 2004, Chung and Liao 2004, 2006, Ouyang *et al*. 2005a, 2005b, 2006, Teng *et al*. 2005, 2007, De and Goswami 2006, Shah 2006, Song and Cai 2006, and their references).

The major assumptions used in the related previous articles are summarized in table 1. It is clear from table 1 that only a few previous studies took the following two important facts into consideration: (1) the selling price per unit is significantly higher than the unit cost, and (2) the retailer receives the supplier trade credit and

Table 1. Summary of related literature for trade credits.

| Author(s) and year | EOQ or EPQ | Supplier trade credit | Customer trade credit | Assuming $p=c$ | Assuming $I_k \geq I_e$ |
|---|---|---|---|---|---|
| Aggarwal and Jaggi (1995) | EOQ | Yes | No | Yes | Yes |
| Arcelus *et al*. (2003) | EOQ | Yes | Yes | No | $I_k = I_e$ |
| Biskup *et al*. (2003) | EPQ | Yes | Yes | Yes | $I_k = I_e$ |
| Chang (2004) | EOQ | Yes | No | No | No |
| Chang *et al*. (2003) | EOQ | Yes | No | No | No |
| Chang and Teng (2004) | EOQ | Yes | No | No | No |
| Chung and Huang (2003) | EPQ | Yes | No | Yes | Yes |
| Chung and Liao (2004) | EOQ | Yes | No | Yes | Yes |
| Chung and Liao (2006) | EOQ | Yes | No | Yes | No discussion |
| Dave (1985) | EOQ | Yes | No | No | No |
| De and Goswami (2006) | EOQ | Yes | No | Yes | Yes |
| Goyal (1985) | EOQ | Yes | No | Yes | Yes |
| Huang (2003) | EOQ | Yes | Yes | Yes | Yes |
| Huang (2004) | EPQ | Yes | No | No | Yes |
| Huang (2005) | EOQ | Yes | No | Yes | Yes |
| Hwang and Shinn (1997) | EOQ | Yes | No | Yes | Yes |
| Liao *et al*. (2000) | EOQ | Yes | No | Yes | Yes |
| Jamal *et al*. (2000) | EOQ | Yes | No | Yes | Yes |
| Jamal *et al*. (2000) | EOQ | Yes | No | No | Yes |
| Ouyang *et al*. (2005a) | EOQ | Yes | No | No | No |
| Ouyang *et al*. (2005b) | EOQ | Yes | No | No | No |
| Ouyang *et al*. (2006) | EOQ | Yes | No | No | No |
| Sarker *et al*. (2000) | EOQ | Yes | No | Yes | Yes |
| Shah (2006) | EOQ | Yes | No | No | Yes |
| Shinn and Hwang (2003) | EOQ | Yes | No | No | Yes |
| Song and Cai (2006) | EOQ | Yes | No | No | Yes |
| Teng (2002) | EOQ | Yes | No | No | No |
| Teng *et al*. (2005) | EOQ | Yes | No | No | No |
| Present article | EOQ | Yes | Yes | No | No |

provides the customer trade credit simultaneously. So far, based on our knowledge, it seems no one has taken both facts into consideration. As a result, in this article, we complement the shortcoming of Huang's model by considering the difference between unit price and unit cost, and relaxing the unnecessary condition of $I_k \geq I_e$. Then we use a simple technique to establish the characteristics of the optimal solution, and hence develop four theoretical results. Furthermore, all previous publications under the assumption of indifference between unit price and unit cost concluded that the economic order quantity generally increases marginally under the permissible delay in payments. In contrast, by considering the fact that the unit price is higher than the unit cost, we are able to find that the economic order quantity may decrease under the permissible delay in payments. Finally, we provide two close to real-world examples to illustrate the problem.

## 2. Mathematical model

The following notations are used throughout the article.

- $D$ the annual demand rate.
- $A$ the ordering cost per order.
- $p$ the selling price per unit.
- $c$ the unit cost.
- $h$ the annual inventory holding cost per unit excluding interest charges.
- $I_k$ the annual interest charged per \$ in stocks by the supplier (or the bank).
- $I_e$ the annual interest earned or return on investment per \$.
- $M$ the retailer's trade credit period offered by the supplier in years.
- $N$ the customer's trade credit period offered by the retailer in years; we assume that $N \leq M$.
- $T$ the replenishment cycle time in years, which is a decision variable.
- $TVC(T)$ the annual total relevant cost, which is the sum of the annual ordering cost, annual inventory holding cost (excluding interest charges), annual interest payable, and minus annual interest earned.
- $T^*$ the optimal replenishment cycle time of $TVC(T)$.

In this article, we adopt the same assumptions as in Goyal (1985), such as constant demand rate, instantaneous replenishment, and shortages prohibited. However, for correctness, we assume that neither $p = c$ nor $I_k \geq I_e$. Note that if $I_k < I_e$, then the interest earned is higher than the interest charged, and a risk-taking retailer may not want to return money to the supplier (or the bank). However, in reality, no bank will provide a loan to anyone without demanding monthly payments. As a matter of fact, most banks need the inventory items as collateral to offer the retailer a low-interest security loan. As a result, the retailer must pay the bank a certain amount of money when the inventory items are sold. Consequently, in this article, we assume that the retailer pays off debt and invests profits when the inventory items are sold. On the other hand, if $I_k > I_e$, then we assume that the retailer acts like many householders who pay a mortgage rate $I_k$, while earning a passbook savings rate ($I_e < I_k$). Logically, those householders should return all of their savings to the mortgage bank. However, in reality, most of those householders only pay the monthly mortgage to the mortgage bank. It is because they may need the money for an emergency or other use. Consequently, the retailer has the following two possible ways to pay off the loan. First of all, we assume that the retailer pays off the amount owed to the supplier whenever he/she has money obtained from sales. We then discuss the other case in which the retailer keeps his/her profits for developing new products or other investment use.

### 2.1 *The retailer pays off loan whenever he/she has money*

From possible values of $T$, $N$ and $M$, we have the following three possible cases.

**Case 1.1** $T > M$**:** During $[0, M]$ period, the retailer sells products and deposits the revenue into an account that earns $I_e$ per dollar per year. Therefore, the interest earned per cycle is

$$pI_e \int_N^M Dt\,\mathrm{d}t = \frac{pI_eD}{2}(M^2 - N^2). \tag{1}$$

Hence, the retailer has $pDM + pI_eD(M^2 - N^2)/2$ in the account at time $M$. Since the retailer buys $DT$ units at time 0, the retailer owes the supplier $cDT$ at time $M$. From the difference between the purchase cost and the money in the account, we have the following two cases: $pDM + pI_eD\,(M^2 - N^2)/2 < cDT$, or $pDM + pI_eD(M^2 - N^2)/2 \geq cDT$.

**Sub-case 1.1.1** $pDM + pI_eD\,(M^2 - N^2)/2 < cDT$**:** If the money in the account $pDM + pI_eD(M^2 - N^2)/2$ at time $M$ is less than the purchase cost $cDT$, then the retailer needs to finance the difference $L = cDT - [pDM + pI_eD(M^2 - N^2)/2]$ (at interest rate $I_k$) at time $M$, and pay the supplier in full in order to get the permissible delay. Thereafter, the retailer gradually reduces the amount of financed loan from constant sales and revenue received. Hence, we obtain the interest payable per cycle as

$$I_k L[L/(pD)]/2 = \frac{I_k}{2pD}\left\{cDT - pD\left[M + \frac{I_e(M^2 - N^2)}{2}\right]\right\}^2. \quad (2)$$

$$TVC(T) = \begin{cases} TVC_{1-1}(T) & \text{if } T > M \text{ and the retailer pays off the supplier after } M \\ TVC_{1-2}(T) & \text{if } T > M \text{ and the retailer pays the supplier in full by } M \\ TVC_2(T) & \text{if } N \le T \le M \\ TVC_3(T) & \text{if } 0 < T \le N. \end{cases} \quad (8)$$

Hence, we have the annual total relevant cost as follows:

$$TVC_{1-1}(T) = \frac{A}{T} + \frac{DTh}{2} + \frac{I_k}{2pDT}\left\{cDT - pD\left[M + \frac{I_e\left(M^2 - N^2\right)}{2}\right]\right\}^2 - \frac{pI_eD}{2T}\left(M^2 - N^2\right). \quad (3)$$

**Sub-case 1.1.2** $pDM + pI_eD(M^2 - N^2)/2 \ge cDT$**:** If the money in the account $pDM + pI_eD(M^2 - N^2)/2$ at time $M$ is greater than or equal to the purchase cost $cDT$, then there is no interest payable, and the annual total relevant cost is as follows

$$TVC_{1-2}(T) = \frac{A}{T} + \frac{DTh}{2} - \frac{pI_eD}{2T}\left(M^2 - N^2\right). \quad (4)$$

**Case 1.2** $N \le T \le M$**:** Since $T \le M$, we know that the replenishment cycle $T$ is less than or equal to the permissible delay $M$. As a result, the retailer pays no interest charge while the interest earned per cycle is

$$pI_e\left[\int_N^T Dt\,\mathrm{d}t + DT(M - T)\right] = \frac{pI_eD}{2}(N + T)(T - N) + pI_eDT(M - T). \quad (5)$$

The annual total relevant cost in this case is

$$TVC_2(T) = \frac{A}{T} + \frac{DTh}{2} - \frac{pI_eD\left(2MT - N^2 - T^2\right)}{2T}. \quad (6)$$

**Case 1.3** $0 < T \le N$**:** In this case, the retailer pays no interest charge while the interest earned per cycle is $pI_eDT(M - N)$. Hence, we have the annual total relevant cost as

$$TVC_3(T) = \frac{A}{T} + \frac{DTh}{2} - pI_eD(M - N). \quad (7)$$

Let the annual total relevant cost be as follows

Note that $TVC_3(N) = TVC_2(N)$ and $TVC_2(M) = TVC_{1-2}(M)$. However, $TVC_2(M) \ne TVC_{1-1}(M)$. Therefore, $TVC(T)$ is continuous if $pDM + pI_eD(M^2 - N^2)/2 \ge cDT$. Otherwise, $TVC(T)$ is not continuous at time $M$.

The first-order and second-order conditions for $TVC_3(T)$ in equation (7) to be minimized are:

$$TVC_3'(T) = -\frac{A}{T^2} + \frac{Dh}{2} = 0, \quad (9)$$

and

$$TVC_3''(T) = \frac{2A}{T^3} > 0. \quad (10)$$

It is clear that $TVC_3(T)$ is a strictly convex function on $T$. Consequently, the corresponding unique optimal solution $T_3^*$ is

$$T_3^* = \sqrt{\frac{2A}{Dh}}. \quad (11)$$

Therefore, the optimal order quantity $Q_3^*$ is

$$Q_3^* = DT_3^* = \sqrt{\frac{2AD}{h}}. \quad (12)$$

To ensure $T_3^* \le N$, we substitute equation (11) into inequality $T \le N$, and obtain that

$$\text{if and only if } \Delta_3 = DN^2h - 2A \ge 0, \quad \text{then } T_3^* \le N. \quad (13)$$

Similarly, we can easily obtain the unique optimal $T_2^*$ as

$$T_2^* = \sqrt{\frac{2A + DN^2pI_e}{D(h + pI_e)}}, \quad (14)$$

and the optimal order quantity $Q_2^*$ as

$$Q_2^* = DT_2^* = \sqrt{\frac{D(2A + DN^2 pI_e)}{h + pI_e}}. \tag{15}$$

Substituting (14) into inequality $N \le T \le M$, and simplifying terms, we have

if and only if $\Delta_3 \le 0$ and $\Delta_2 = DM^2(h + pI_e) - 2A - DN^2 pI_e \ge 0$, then $N \le T_2^* \le M$. (16)

It is obvious that $\Delta_2 > \Delta_3$. From (13) and (16), we know that $T_1 > M$ only if $\Delta_2 < 0$.

The first-order condition for $TVC_{1-1}(T)$ in equation (3) to be minimized is $TVC'_{1-1}(T) = 0$, which leads to

$$\frac{hD + (I_k Dc^2/p)}{2} T^2 = A + \frac{pD}{2}\left\{ I_k\left[ M + \frac{I_e(M^2 - N^2)}{2}\right]^2 - I_e(M^2 - N^2)\right\}. \tag{17}$$

Consequently, the corresponding optimal solution $T_{1-1}^*$ is

$$T_{1-1}^* = \sqrt{\frac{2A + pD\{I_k[M + I_e(M^2 - N^2)/2]^2 - I_e(M^2 - N^2)\}}{hD + (I_k Dc^2/p)}}. \tag{18}$$

The optimal order quantity $Q_{1-1}^*$, hence, is

$$Q_{1-1}^* = DT_{1-1}^* = \sqrt{\frac{2AD + pD^2\{I_k[M + I_e(M^2 - N^2)/2]^2 - I_e(M^2 - N^2)\}}{h + (I_k c^2/p)}}. \tag{19}$$

Since $\Delta_2 = DM^2(h + pI_e) - 2A - DN^2 pI_e < 0$ in this case, we obtain the second-order condition as

$$\begin{aligned} TVC''_{1-1}(T) &= \frac{-2}{T^3}\left\{ \frac{hD + (I_k Dc^2/p)}{2} T^2 - A - \frac{pD}{2}\left\{ I_k\left[\frac{M + I_e(M^2 - N^2)}{2}\right]^2 . - I_e(M^2 - N^2)\right\}\right\} + \frac{1}{T}(hD + I_k Dc^2) \\ &> \frac{D}{T^3}\left\{ pI_k\left[ M + \frac{I_e(M^2 - N^2)}{2}\right]^2 + hM^2\right\} + \frac{1}{T} I_k Dc^2\left(1 - \frac{1}{p}\right) > 0, \end{aligned} \tag{20}$$

if $p \ge 1$. Consequently, if $p \ge 1$, then $TVC_{1-1}(T)$ is a strictly convex function on $T$, and the optimal solution $T_{1-1}^*$ is unique. From the condition of $pDM + pI_e D(M^2 - N^2)/2 < cDT$, we obtain $T > (p/c) \times [M + I_e (M^2 - N^2)/2] > M$. To ensure $T_{1-1}^* \ge (p/c)[M + I_e (M^2 - N^2)/2]$, we substitute equation (18) into inequality $T > (p/c)[M + I_e (M^2 - N^2)/2]$, and obtain that

if and only if $\Delta_1 = pD\left\{ I_e(M^2 - N^2) + \left(\frac{hp}{c^2}\right)\left[ M + \frac{I_e(M^2 - N^2)}{2}\right]^2\right\} - 2A < 0$, then $T_{1-1}^* > \left(\frac{p}{c}\right)\left[\frac{M + I_e(M^2 - N^2)}{2}\right] > M$. (21)

Likewise, the first-order condition for $TVC_{1-2}(T)$ in (4) to be minimized is $TVC'_{1-2}(T) = 0$. Consequently, we obtain the optimal solution $T_{1-2}^*$ as

$$T_{1-2}^* = \sqrt{\frac{2A - pDI_e(M^2 - N^2)}{hD}}. \tag{22}$$

The optimal order quantity $Q_{1-2}^*$, hence, is

$$Q_{1-2}^* = DT_{1-2}^* = \sqrt{\frac{2AD - pD^2 I_e(M^2 - N^2)}{h}}. \tag{23}$$

The second-order conditions for $TVC_{1-2}(T)$ in (4) to be minimized is:

$$TVC''_{1-2}(T) = \frac{-1}{T^3}\left[-2A + pDI_e(M^2 - N^2)\right] > 0, \quad \text{since } \Delta_2 \le 0. \tag{24}$$

As a result, $TVC_{1-2}(T)$ is a strictly convex function on $T$, and the optimal solution $T_{1-2}^*$ is unique. Similarly, to ensure $T_{1-2}^* \le (p/c)[M + I_e(M^2 - N^2)/2]$, we substitute equation (22) into inequality $T \le (p/c)[M + I_e(M^2 - N^2)/2]$, and obtain that

if and only if $\Delta_2 \le 0$ and $\Delta_1 \ge 0$, then $T_{1-2}^* > \left(\frac{p}{c}\right)\left[ M + \frac{I_e(M^2 - N^2)}{2}\right] > M$. (25)

Since

$$\Delta_1 = pD\left\{I_e\left(M^2-N^2\right)+\left(\frac{hp}{c^2}\right)\left[M+\frac{I_e(M^2-N^2)}{2}\right]^2\right\}-2A$$
$$=pDI_e\left(M^2-N^2\right)+\frac{p^2}{c^2}Dh\left[M+\frac{I_e(M^2-N^2)}{2}\right]^2-2A$$
$$> DM^2(h+pI_e)-2A-DN^2pI_e(=\Delta_2)$$
$$> DN^2h-2A(=\Delta_3), \tag{26}$$

we obtain the following theorem.

**Theorem 1**

(A) *If* $\Delta_3 \ge 0$ (*consequently*, $\Delta_2 > 0$ *and* $\Delta_1 > 0$), *then* $TVC(T^*) = TVC(T_3^*)$ *and* $T^* = T_3^*$.
(B) *If* $\Delta_3 \le 0$, *and* $\Delta_2 \ge 0$ (*consequently*, $\Delta_1 > 0$), *then* $TVC(T^*) = TVC(T_2^*)$ *and* $T^* = T_2^*$.
(C) *If* $\Delta_2 \le 0$, *and* $\Delta_1 \ge 0$ (*consequently*, $\Delta_3 < 0$), *then* $TVC(T^*) = TVC(T_{1-2}^*)$ *and* $T^* = T_{1-2}^*$.
(D) *If* $\Delta_1 < 0$ (*consequently*, $\Delta_2 < 0$, *and* $\Delta_3 < 0$), *and* $p \ge 1$, *then* $TVC(T^*) = TVC(T_{1-1}^*)$ *and* $T^* = T_{1-1}^*$.

**Proof:** It immediately follows from equations (13), (16), (21) and (25). □

In the classical economic order quantity model, the supplier must be paid for the items as soon as the retailer receives them. Therefore, it is a special case of Case 1.1 with $p = c$ and $M = 0$ (so is $N = 0$), and its optimal order quantity is

$$Q_{\mathrm{EOQ}}^* = \sqrt{\frac{2AD}{h+cI_k}}. \tag{27}$$

Consequently, we have the following theorem.

**Theorem 2**

(A) $Q_3^* > Q_{\mathrm{EOQ}}^*$.
(B) *If* $cI_k \ge pI_e$, *then* $Q_2^* > Q_{\mathrm{EOQ}}^*$.

**Proof:** It is obvious from (12), (15), and (27). □

Note that if $cI_k < pI_e$, then the result is not clear.

## 2.2 *The retailer keeps profit for other use than payoff loan*

Similar to the previous discussion, we have three possible annual total relevant costs for the retailer who keeps profit for other use than pay off the loan as follows:

**Case 2.1:** If $T \ge M$, then

$$TVC_4(T) = \frac{A}{T}+\frac{DTh}{2}+\frac{cI_kD(T-M)^2}{2T}-\frac{pI_eD(M^2-N^2)}{2T}. \tag{28}$$

**Case 2.2:** If $N \le T \le M$, then

$$TVC_2(T) = \frac{A}{T}+\frac{DTh}{2}-\frac{pI_eD(2MT-N^2-T^2)}{2T}. \tag{29}$$

**Case 2.3:** If $0 < T \le N$, then

$$TVC_3(T) = \frac{A}{T}+\frac{DTh}{2}-pI_eD(M-N). \tag{30}$$

Let the annual total relevant cost be as follows

$$TVC(T) = \begin{cases} TVC_4(T) & \text{if } T \ge M \\ TVC_2(T) & \text{if } N \le T \le M \\ TVC_3(T) & \text{if } 0 < T \le N. \end{cases} \tag{31}$$

Since $TVC_4(M) = TVC_2(M)$, and $TVC_2(N) = TVC_3(N)$, we know that $TVC(T)$ is continuous. Using arguments similar to those in the previous discussion, we know that $T_4 > M$ only if $\Delta_2 < 0$. If $\Delta_2 < 0$, then the second-order condition

$$TVC_4''(T) = \left\{\frac{2A+D\left[M^2(cI_k-pI_e)+N^2pI_e\right]}{T^3}\right\}$$
$$> DM^2(cI_k-pI_e)+DM^2(h+pI_e)$$
$$= DM^2(cI_k+h) > 0. \tag{32}$$

Hence, $TVC_4(T)$ is a strictly convex function on $T$. Consequently, the corresponding unique optimal solution $T_4^*$ is

$$T_4^* = \sqrt{\frac{2A+DM^2(cI_k-pI_e)+N^2DpI_e}{D(h+cI_k)}}, \tag{33}$$

the optimal order quantity $Q_4^*$ as

$$Q_4^* = DT_4^* = \sqrt{\frac{D[2A+DM^2(cI_k-pI_e)+N^2DpI_e]}{h+cI_k}}, \tag{34}$$

and the fact that

$$\text{if and only if } \Delta_2 \le 0, \quad \text{then } T_4^* \ge M. \tag{35}$$

From (13), (16), and (35), we obtain the following two theorems.

**Theorem 3**

(A) *If* $\Delta_3 \ge 0$ (*consequently* $\Delta_2 \ge 0$), *then* $TVC(T^*) = TVC(T_3^*)$ *and* $T^* = T_3^*$.

(B) *If* $\Delta_2 \geq 0$, *and* $\Delta_3 \leq 0$, *then* $TVC(T^*) = TVC(T_2^*)$ *and* $T^* = T_2^*$.

(C) *If* $\Delta_2 \leq 0$ (*consequently* $\Delta_3 \leq 0$), *then* $TVC(T^*) = TVC(T_4^*)$ *and* $T^* = T_4^*$.

**Proof:** It immediately follows from equations (13), (16), and (35). □

Note that Theorem 3 is a general form of the corresponding Theorem 1 in Huang (2003), in which it requires $I_k \geq I_e$ and $p = c$. If $N = 0$, then Theorem 3 is reduced to the corresponding Theorem 1 in Teng (2002). As a matter of fact, the Theorem 1 in Teng (2002) is a general form of the corresponding Theorem 2 in Huang (2003), in which it assumes $I_k \geq I_e$ and $p = c$.

**Theorem 4**

(A) $Q_3^* > Q_{\mathrm{EOQ}}^*$.

(B) *If* $cI_k > pI_e$, *then* $Q_4^* > Q_{\mathrm{EOQ}}^*$ *and* $Q_2^* > Q_{\mathrm{EOQ}}^*$.

(C) *If* $cI_k = pI_e$, *then* $Q_4^* = Q_2^* > Q_{\mathrm{EOQ}}^*$.

**Proof:** It is obvious from (12), (15), (27), and (34). □

Note that if $cI_k < pI_e$, then the result is not clear. We know from Theorem 4 that if a supplier wants to reduce his/her large level of inventory, then he/she should charge an excessive interest rate $I_k$ such that $cI_k > pI_e$. Under this specific condition, a retailer will order to buy a higher quantity than the classical economic order quantity, $Q_{\mathrm{EOQ}}^*$.

## 3. Numerical examples

A one-dollar store (i.e., $p = \$1$) buys nail cutters from a supplier at $c = \$0.50$ a piece. The supplier offers a permissible delay if the payment is made within 60 days (i.e., $M = 1/6$). This credit term in finance management is usually denoted as 'net 60' (e.g., see Brigham 1995). However, if the payment is not made in full by the end of 60 days, then 4% interest (i.e., $I_k = 0.04$) is charged on the outstanding amount.

**Example 1:** Suppose $D = 3600$ units, $N = 30$ days (i.e., $N = 1/12$), $h = \$0.5$/unit/year, $A = \$10$ per order, and $I_e = 1$ or 2% if the store deposits its revenue into a money-market account; or $I_e = 10\%$ if it invests its revenue into a mutual fund account.

Since $\Delta_2 = DM^2(h + pI_e) - 2A - DN^2 pI_e > 0$, for $I_e = 1$, 2 or 10%, and $\Delta_3 = DN^2 h - 2A = -7.50$, we know from Theorems 1 and 3 that the optimal replenishment interval is $T_2^*$. Substituting the numerical values into (15), we obtain the economic order quantity $Q_2^*$ as follows.

$$Q_2^* = \begin{cases} 378.01, & \text{if } I_e = 1\%, \\ 376.60, & \text{if } I_e = 2\%, \\ 366.87, & \text{if } I_e = 10\%. \end{cases} \tag{36}$$

From (27), we have the classical optimal economic order quantity

$$Q_{\mathrm{EOQ}}^* = \sqrt{\frac{2AD}{h + cI_k}} = 372.10. \tag{37}$$

The computational result reveals that if $pI_e \leq cI_k$, then the one-dollar store orders a higher quantity than the classical EOQ, and *vice versa*.

Note that Goyal (1985) assumed that $p = c$ and $I_e \leq I_k$, and concluded that the economic replenishment interval and order quantity generally increases marginally under the permissible delay in payment. Similarly, Biskup *et al.* (2003) distinguished neither $p > c$ nor $I_e \neq I_k$, and hence concluded that the inclusion of financial aspects into the EOQ model leads to an order quantity very close to the classical EOQ. In this article, we provide a proper alternative to a well-established retailer (i.e., $pI_e > cI_k$) who orders a lower quantity and take the benefit of the supplier trade credit more frequently. This alternative had been shown in Teng (2002), too.

**Example 2:** Suppose $D = 1500$ units, $A = \$45$ per order, and other parameters are the same as Example 1. Since $\Delta_1 = pD\{I_e(M^2 - N^2) + (hp/c^2)[M + I_e(M^2 - N^2)/2]^2\} - 2A < 0$ (consequently, $\Delta_2 < 0$), for $I_e = 1$, 2 or 10%, we know from Theorems 1 and 3 that the optimal replenishment interval is either $T_{1-1}^*$ or $T_4^*$, which depends on the retailer's different payment schemes. If the retailer pays off the loan whenever he/she has money, then the optimal solution is $T_{1-1}^*$. Otherwise, the optimal solution is $T_4^*$. Substituting the numerical values into (3), (18), (28) and (33), we obtain the computational results as shown in table 2.

Table 2. The optimal solution for different payment schemes.

| $I_e$ | $T_{1-1}^*$ | $TVC(T_{1-1}^*)$ | $T_4^*$ | $TVC(T_4^*)$ |
|---|---|---|---|---|
| 0.01 | 0.3456 | 259.3594 | 0.3407 | 260.7184 |
| 0.02 | 0.3450 | 258.9067 | 0.3401 | 260.2593 |
| 0.10 | 0.3402 | 255.2577 | 0.3353 | 256.5578 |

Table 3. Sensitivity analysis.

| | $T^*_{1-1}$ | $TVC(T^*_{1-1})$ | $T^*_4$ | $TVC(T^*_4)$ |
|---|---|---|---|---|
| $I_e$ ($N=60$ days) | | | | |
| 0.01 | 0.3462 | 259.8113 | 0.3413 | 261.1766 |
| 0.02 | 0.3462 | 259.8113 | 0.3413 | 261.1766 |
| 0.10 | 0.3462 | 259.8113 | 0.3413 | 261.1766 |
| $I_e$ ($N=15$ days) | | | | |
| 0.01 | 0.3454 | 259.2463 | 0.3405 | 260.6037 |
| 0.02 | 0.3447 | 258.6801 | 0.3398 | 260.0295 |
| 0.10 | 0.3388 | 254.1071 | 0.3338 | 255.3903 |
| $N$ | | | | |
| 60 | 0.3462 | 259.8113 | 0.3413 | 261.1766 |
| 45 | 0.3455 | 259.2840 | 0.3406 | 260.6419 |
| 30 | 0.3450 | 258.9067 | 0.3401 | 260.2593 |
| 15 | 0.3447 | 258.6801 | 0.3398 | 260.0295 |
| $I_k$ | | | | |
| 0.01 | 0.3451 | 258.9046 | 0.3439 | 259.2483 |
| 0.04 | 0.3450 | 258.9067 | 0.3401 | 260.2593 |
| 0.06 | 0.3449 | 258.9081 | 0.3376 | 260.9155 |
| 0.10 | 0.3447 | 258.9107 | 0.3330 | 262.1873 |
| $c$ | | | | |
| 0.5 | 0.3450 | 258.9067 | 0.3401 | 260.2593 |
| 0.7 | 0.3418 | 259.3765 | 0.3381 | 260.7854 |
| 0.9 | 0.3376 | 260.6352 | 0.3362 | 261.3026 |
| $h$ | | | | |
| 0.3 | 0.4425 | 200.7517 | 0.4335 | 203.0865 |
| 0.4 | 0.3848 | 231.6226 | 0.3784 | 233.3931 |
| 0.5 | 0.3450 | 258.9067 | 0.3401 | 260.2593 |

Table 2 reveals that $T^*_{1-1}$ is longer than $T^*_4$ while $TVC(T^*_{1-1})$ is always less than $TVC(T^*_4)$. A simple economic interpretation is as follows. If the retailer pays off the loan whenever he/she has money, then he/she pays less interest payable (so is the annual total relevant cost) and, thus, can afford to order more quantity.

**Example 3:** Suppose $D=1500$ units, $p=\$1$, $c=\$0.50$, $M=60$ days (i.e., $M=1/6$), $N=30$ days (i.e., $N=1/12$), $h=\$0.5$/unit/year, $A=\$45$ per order, $I_k=4\%$ and $I_e=2\%$. Since $\Delta_1=pD\{I_e(M^2-N^2)+(hp/c^2)[M+I_e(M^2-N^2)/2]^2\}-2\quad A<0$ (consequently, $\Delta_2<0$), for $N=60$, 45, 30 or 15 days (i.e., 1/6, 1/8, 1/12 or 1/24), using Theorems 1 and 3, we know that the optimal replenishment interval is either $T^*_{1-1}$ or $T^*_4$, which depends on the retailer's different payment schemes. We then perform sensitivity analyses, and obtain the numerical results as shown in table 3.

Based on the computational results as show in table 3, we obtain the following managerial phenomena:

(1) $T^*_{1-1}$ is longer than $T^*_4$ while $TVC(T^*_{1-1})$ is always less than $TVC(T^*_4)$.
(2) The value of interest rate earned $I_e$ does not impact on the values of $T^*_{1-1}$, $T^*_4$, $TVC(T^*_{1-1})$ and $TVC(T^*_4)$ when $N=M$ (i.e., the customer's trade credit period is equal to the retailer's trade credit period).
(3) A higher value of the interest rate earned $I_e$ causes lower values of $T^*_{1-1}$, $T^*_4$, $TVC(T^*_{1-1})$ and $TVC(T^*_4)$ when $N<M$ (i.e., the customer's trade credit period is less than the retailer's trade credit period).
(4) A higher value of the customer's trade credit period $N$ causes higher values of $T^*_{1-1}$, $T^*_4$, $TVC(T^*_{1-1})$ and $TVC(T^*_4)$.
(5) A higher value of the interest rate charged $I_k$ causes lower values of $T^*_{1-1}$ and $T^*_4$, but higher values of $TVC(T^*_{1-1})$ and $TVC(T^*_4)$.
(6) A higher value of the unit cost $c$ causes lower values of $T^*_{1-1}$ and $T^*_4$, but higher values of $TVC(T^*_{1-1})$ and $TVC(T^*_4)$.
(7) A higher value of the holding cost $h$ causes lower values of $T^*_{1-1}$ and $T^*_4$, but higher values of $TVC(T^*_{1-1})$ and $TVC(T^*_4)$.

## 4. Conclusions

In this article, we establish an appropriate EOQ model with trade credit financing, in which the unit price is significantly higher than the unit cost, and the benefit of the permissible delay is measured by the interest earned from sales revenue during the permissible delay. We then provide two different ways for the retailer to payoff the purchase cost. One is that the retailer pays the supplier whenever he/she has money. The other is that the retailer pays the purchase cost only when the item is sold. Furthermore, we derive an easy-to-use, closed-form solution to the problem. Consequently, the results are simple to understand for academicians, and easy to apply for practitioners. Moreover, we find that it is possible that a well-established buyer may order a lower quantity and take the benefit of the permissible delay more frequently, which contradicts to most of the previous finding [such as in Goyal (1985) and others] that the buyer always buys slightly more when there is a permissible delay. Finally, we obtain several managerial results by using sensitivity analyses.

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