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Optimal ordering policies when the supplier provides a progressive interest scheme

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Abstract

In fact, most credit card issuers (or home equity banks) frequently offer cardholders (or customers) a teaser interest rate (say, I_1), which is significantly lower than the regular interest rate of I_2 (with $I_2 > I_1$) for only 6 months or a year (say, M_2) to lure new customers from their competitors. Consequently, the customer faces a progressive interest charge from the bank. If the customer pays the outstanding balance by the grace period (say, M_1 which is generally 25 days), then the bank does not charge any interest. If the outstanding amount is paid after M_1, but by M_2 (with $M_2 > M_1$), then the bank charges the customer the teaser interest rate of I_1 on the unpaid balance. If the customer pays the outstanding amount after M_2, then the bank charges the regular interest rate of I_2. In this paper, we first establish an appropriate EOQ model for a retailer when the bank (or the supplier) offers a progressive interest charge, and then provide an easy-to-use closed-form solution to the problem.

Keywords: Finance; Inventory; EOQ; Progressive interest charge

1. Introduction

In practice, a supplier frequently offers a retailer a delay of a fixed time period (say, 30 days) for settling the amount owed to him. Usually, there is no interest charge if the outstanding amount is paid within the permissible delay period. Note that this credit term in financial management is denoted as "net 30". For example, see Brigham (1995). However, if the payment is not paid in full by the end of the permissible delay period, then interest is charged on the outstanding amount. Therefore, it is clear that a customer will delay the payment up to the last moment of the permissible period allowed by the supplier. The permissible delay in payments produces two benefits to the supplier: (1) it not only encourages customers to order more, but also attracts new customers, and (2) it may be applied as an alternative to price discount because it does not provoke

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competitors to reduce their prices and thus introduce lasting price reductions. On the other hand, the policy of granting credit terms adds not only an additional cost but also an additional dimension of default risk to the supplier.

Goyal (1985) established an EOQ model when the supplier offers the retailer a permissible delay in payments. Aggarwal and Jaggi (1995) then extended Goyal's model for deteriorating items. Jamal et al. (1997) further generalized the model to allow for shortages and deterioration. Hwang and Shinn (1997) developed the optimal pricing and lot sizing for the retailer under the condition of permissible delay in payments. Liao et al. (2000) developed an inventory model for stock-depend demand rate when a delay in payment is permissible. Chang and Dye (2001) extended the model by Jamal et al. (1997) to allow for not only a varying deterioration rate of time but also the backlogging rate to be inversely proportional to the waiting time. All the above models ignored the difference between unit price and unit cost. In contrast, Jamal et al. (2000) and Sharker et al. (2000) amended Goyal's model by considering the difference between unit price and unit cost, and concluded from computational results that the retailer should settle his account relatively sooner as the unit selling price increases relative to the unit cost. Recently, Teng (2002) provided an alternative conclusion from Goyal (1985), and mathematically proved that it makes economic sense for a well-established buyer to order less quantity and take the benefits of the permissible delay more frequently. Chang et al. (2003) then extended Teng's model, and established an EOQ model for deteriorating items in which the supplier provides a permissible delay to the purchaser if the order quantity is greater than or equal to a predetermined quantity. Moreover, Teng et al. (2005b) further developed an algorithm for a retailer to determine its optimal price and lot size simultaneously when the supplier offers a permissible delay in payments. Lately, Huang (2003) extended Goyal's model to develop an EOQ model in which the supplier offers the retailer the permissible delay period M, and the retailer in turn provides the trade credit period N (with $N \leqslant M$) to his/her customers. He then obtained the closed-form optimal solution and two interesting theoretical results. Teng et al. (2005a) further complement the shortcoming of Huang's model by considering the difference between unit price and unit cost.

As a matter of fact, most credit card issuers (or banks) frequently offer customers a teaser interest rate (say, I_1), which is significantly lower than the regular interest rate of I_2 (with $I_2 > I_1$) for only 6 months or a year (say, M_2) to lure new customers from their competitors. Consequently, the customer faces a progressive interest charge from the bank. If the customer pays the outstanding balance by the grace period (say, M_1 which is generally 25 days), then the bank does not charge any interest. If the outstanding amount is paid after M_1, but by M_2 (with $M_2 > M_1$), then the bank charges the customer the teaser interest rate of I_1 on the unpaid balance. If the customer pays the outstanding amount after M_2, then the bank charges the regular interest rate of I_2. In this paper, we first establish an appropriate EOQ model for a retailer when the bank (or the supplier) offers a progressive interest charge, and then provide an easy-to-use closed-form solution to the problem. Furthermore, we study the effect of the teaser rate to the retailer. From numerical examples as shown in Tables 1 and 2 below, we conclude that the retailer will order more quantity and pay less total relevant cost per year if the supplier (or the bank) provides a short-term teaser interest rate.

2. Assumptions and notation

The following assumptions are similar to those in Goyal's (1985) EOQ model:

(1) The demand for the one-item is constant with time.
(2) Shortages are not allowed.
(3) Replenishment is instantaneous.
(4) The supplier (or the bank) provides a retailer (or the customer) trade credits as follows: If the retailer pays by M_1, then supplier does not charge the retailer any interest. If the retailer pays after M_1 but before M_2, then the supplier charges the retailer an interest rate of I_1. If the retailer pays after M_2, then supplier charges the retailer an interest rate of I_2, with $I_2 > I_1$.
(5) Time horizon is infinite.

In addition, the following notation is used throughout this paper.

D	the demand rate per year
h	the unit holding cost per year excluding interest charges
p	the selling price per unit
c	the unit purchasing cost, with $c < p$
M_1	the first period of permissible delay in settling account without extra charges
M_2	the second period of permissible delay in settling account with an interest charge of I_1 and $M_2 > M_1$
I_1	the interest charged per \$ in stocks per year by the supplier when the retailer pays after M_1 and before M_2
I_2	the interest charged per \$ in stocks per year by the supplier when the retailer pays after M_2
I_e	the interest earned per \$ per year
S	the ordering cost per order
Q	the order quantity
T	the replenishment time interval
$I(t)$	the level of inventory at time t, $0 \leqslant t \leqslant T$
$Z(T)$	the total relevant cost per year, which consists of (a) cost of placing orders, (b) cost of carrying inventory (excluding interest charges), (c) cost of interest charges for unsold items after the permissible delay M_1 or M_2, and (d) interest earned from sales revenue during the permissible period $[0, M_1]$

3. Mathematical formulation

The level of inventory $I(t)$ gradually decreases mainly to meet demand. Hence, the variation of inventory with respect to time can be described by the following differential equations:

$$\frac{dI(t)}{dt} = -D, \quad 0 \leqslant t \leqslant T, \tag{1}$$

with the boundary conditions: $I(0) = Q$, $I(T) = 0$. Consequently, the solution of (1) is given by

$$I(t) = D(T - t), \quad 0 \leqslant t \leqslant T \tag{2}$$

and the order quantity is $Q = DT$.

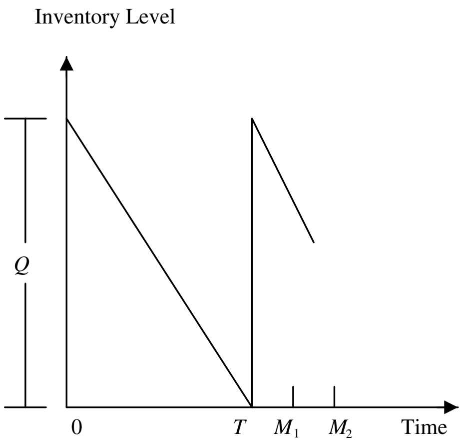

Fig. 1. Graphical representation for Case 1.

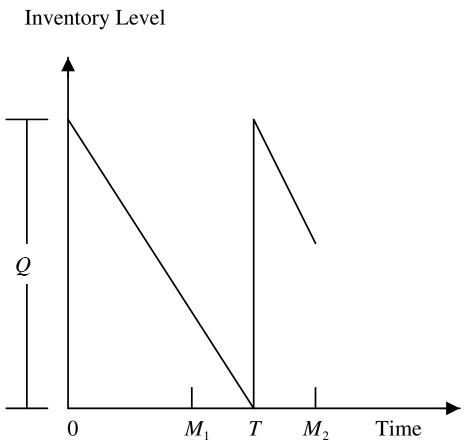

Fig. 2. Graphical representation for Cases 2.1 and 2.2.

The total relevant cost per year consists of the following elements:

(a) Cost of placing orders $= S/T$, (3)

(b) Cost of carrying inventory $= h\int_0^T I(t)\mathrm{d}t/T = \frac{hD}{2}T$. (4)

Regarding interests charged and earned (i.e., costs of (c) and (d)), based on the length of the replenishment cycle T, we have three possible cases: (1) $T \leqslant M_1$, (2) $M_1 < T < M_2$, and (3) $T \geqslant M_2$ (see Figs. 1–3).

Case 1. $T \leqslant M_1$

In this case, the retailer sells DT units in total at time T, and has cDT dollars to pay the supplier in full at time M_1. Consequently, there is no interest payable. However, during $[0, T]$ period, the retailer sells products and deposits the revenue into an account that earns I_e per dollar per year. In the period $[T, M_1]$, the retailer only deposits the total revenue into an account that earns I_e per dollar per year. Therefore, the interest earned per year is

$$pI_e\left[\int_0^T Dt\,\mathrm{d}t + DT(M_1 - T)\right] \Big/ T = pI_eD(M_1 - T/2). \tag{5}$$

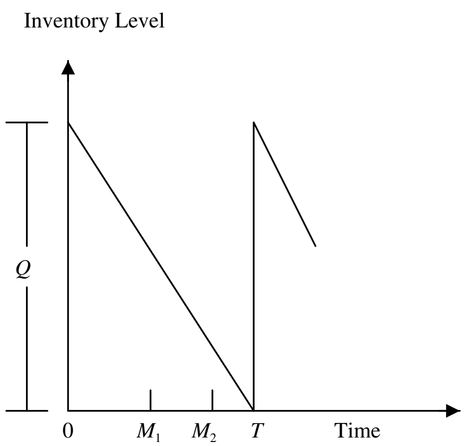

Fig. 3. Graphical representation for Cases 3.1–3.3.

From (3)–(5), we have the total relevant cost per year $Z_1(T)$ is

$$Z_1(T) = \frac{S}{T} + \frac{hD}{2}T - pI_eD\left(M_1 - \frac{T}{2}\right). \tag{6}$$

Case 2. $M_1 < T < M_2$

During $[0, M_1]$ period, the retailer sells products and deposits the revenue into an account that earns I_e per dollar per year. Therefore, the interest earned during this period is $pI_e \int_0^{M_1} Dt\,dt = pI_eDM_1^2/2$. Additionally, the retailer buys DT units at time 0, and owes cDT dollars to the supplier. At time M_1, the retailer sells (DM_1) units in total and has pDM_1 dollars plus interest earned $(pI_eDM_1^2/2)$ dollars to pay the supplier. From the difference between the total purchase cost cDT and the total amount of money in the account $pDM_1 + pI_eDM_1^2/2$, there are two possible sub-cases: (2.1) $pDM_1 + pI_eDM_1^2/2 \geqslant cDT$, and (2.2) $pDM_1 + pI_eDM_1^2/2 < cDT$.

Case 2.1. $pDM_1 + pI_eDM_1^2/2 \geqslant cDT$

In this sub-case, the retailer has enough money in his/her account to pay off the total purchase cost at time M_1. Hence, the total purchase cost is paid at M_1 and there is no interest charge. The interest earned per year is $pI_e \int_0^{M_1} Dt\,dt/T = pI_eDM_1^2/2T$. Therefore, the total relevant cost per year $Z_{2.1}(T)$ is

$$Z_{2.1}(T) = \frac{S}{T} + \frac{hD}{2}T - \frac{pI_eDM_1^2}{2T}. \tag{7}$$

Case 2.2. $pDM_1 + pI_eDM_1^2/2 < cDT$

If $pDM_1 + pI_eDM_1^2/2 < cDT$, then the supplier starts to charge the retailer the unpaid balance $L_1 = cDT - [pDM_1 + pI_eDM_1^2/2]$ with interest rate I_1 at time M_1. Thereafter, the retailer gradually reduces the amount of the loan due to constant sales and revenue received. As a result, the interest payable per year is

$$I_1L_1[L_1/(pD)]/(2T) = \frac{I_1}{2pDT}[cDT - pDM_1(1 + I_eM_1/2)]^2. \tag{8}$$

The interest earned per year is $pI_e \int_0^{M_1} Dt\,dt/T = pI_eDM_1^2/2T$. Therefore, the total relevant cost per year $Z_{2.2}(T)$ is

$$Z_{2.2}(T) = \frac{S}{T} + \frac{hD}{2}T - \frac{pI_eD}{2T}M_1^2 + \frac{I_1}{2pDT}[cDT - pDM_1(1 + I_eM_1/2)]^2. \tag{9}$$

Case 3. $T \geqslant M_2$

This case is similar to Case 2. Based on the total purchase cost cDT, the total amount of money in the account at M_1, $pDM_1 + pI_eDM_1^2/2$, and the total amount of money in the account at M_2, $pDM_2 + pI_eDM_2^2/2$, there are three possible sub-cases: (3.1) $pDM_1 + pI_eDM_1^2/2 \geqslant cDT$, (3.2) $pDM_1 + pI_eDM_1^2/2 < cDT$ but $[pD(M_2 - M_1) + pI_eD(M_2 - M_1)^2/2] \geqslant [cDT - pDM_1 - pI_eDM_1^2/2]$ and (3.3) $pDM_2 + pI_eDM_2^2/2 < cDT$ and $[pD(M_2 - M_1) + pI_eD(M_2 - M_1)^2/2] < [cDT - pDM_1 - pI_eDM_1^2/2]$.

Case 3.1. $pDM_1 + pI_eDM_1^2/2 \geqslant cDT$

This sub-case is the same as Case 2.1. Hence, the retailer will pays the total purchase cost at M_1 and there is no interest charge. The total relevant cost per year $Z_{3.1}(T)$ is

$$Z_{3.1}(T) = \frac{S}{T} + \frac{hD}{2}T - \frac{pI_eDM_1^2}{2T}. \tag{10}$$

Case 3.2. $pDM_1 + pI_eDM_1^2/2 < cDT$ but $[pD(M_2 - M_1) + pI_eD(M_2 - M_1)^2/2] \geqslant [cDT - pDM_1 - pI_eDM_1^2/2]$

In this sub-case, the retailer has not enough money in his/her account to pay off the total purchase cost at time M_1, but he/she can pay off the total purchase cost before or on M_2. Hence, retailer only pays

$pDM_1 + pI_eDM_1^2/2$ at M_1 and the supplier start to charge the retailer the unpaid balance $cDT - [pDM_1 + pI_eDM_1^2/2]$ with interest rate I_1 at time M_1. Therefore, the sub-case is the same as Case 2.2 and the total relevant cost per year $Z_{3.2}(T)$ as follows:

$$Z_{3.2}(T) = \frac{S}{T} + \frac{hD}{2}T - \frac{pI_eD}{2T}M_1^2 + \frac{I_1}{2pDT}[cDT - pDM_1(1 + I_eM_1/2)]^2. \tag{11}$$

Case 3.3. $pDM_2 + pI_eDM_2^2/2 < cDT$ and $[pD(M_2 - M_1) + pI_eD(M_2 - M_1)^2/2] < [cDT - pDM_1 - pI_eDM_1^2/2]$

Since the retailer has not enough money in his/her account to pay off the total purchase cost at time M_2, he/she only pays $[pDM_1 + pI_eDM_1^2/2]$ at M_1 and $[pD(M_2 - M_1) + pI_eD(M_2 - M_1)^2/2]$ at M_2. Hence, the supplier starts to charge the retailer the unpaid balance $L_1 = cDT - [pDM_1 + pI_eDM_1^2/2]$ with interest rate I_1 during $[M_1, M_2]$ and $L_2 = cDT - [pDM_1 + pI_eDM_1^2/2] - [pD(M_2 - M_1) + pI_eD(M_2 - M_1)^2/2]$ with interest rate I_2 at time M_2. Therefore, the interest payable per year is

$$\begin{aligned} &I_1[cDT - pDM_1 - pI_eDM_1^2/2](M_2 - M_1)/T + I_2L_2[L_2/(pD)]/(2T) \\ &= \frac{I_1(M_2 - M_1)D}{T}[cT - pM_1(1 + I_eM_1/2)] + \frac{I_2D}{2pT}\left[cT - pM_2 - \frac{pI_e}{2}\left[M_1^2 + (M_2 - M_1)^2\right]\right]^2. \end{aligned} \tag{12}$$

The total relevant cost per year $Z_{3.3}(T)$ is

$$Z_{3.3}(T) = \frac{S}{T} + \frac{hD}{2}T - \frac{pI_eD}{2T}M_1^2 + \frac{I_1(M_2 - M_1)D}{T}[cT - pM_1(1 + I_eM_1/2)] + \frac{I_2D}{2pT}\left[cT - pM_2 - \frac{pI_e}{2}\left[M_1^2 + (M_2 - M_1)^2\right]\right]^2. \tag{13}$$

4. Theoretical results

The first-order condition for $Z_1(T)$ in (6) to be minimized is $\mathrm{d}Z_1(T)/\mathrm{d}T = 0$, which leads to

$$2S = D(h + pI_e)T^2 \tag{14}$$

and thus the optimal value of T for Case 1 is

$$T_1 = \sqrt{2S/[D(h + pI_e)]}. \tag{15}$$

The second-order condition as

$$\frac{\mathrm{d}^2Z_1(T)}{\mathrm{d}T^2} = \frac{2S}{T^3} > 0. \tag{16}$$

Substituting (15) into inequality $T_1 \leqslant M_1$, we know that

$$\text{if and only if } 2S \leqslant D(h + pI_e)M_1^2, \text{ then } T_1 \leqslant M_1. \tag{17}$$

Likewise, the first-order condition for Case 2.1 is $\mathrm{d}Z_{2.1}(T)/\mathrm{d}T = 0$, which leads us to

$$(2S - pI_eDM_1^2) = hDT^2. \tag{18}$$

Consequently, we obtain the optimal value of T for Case 2.1 is

$$T_{2.1} = \sqrt{(2S - pI_eDM_1^2)/(hD)}. \tag{19}$$

For the second-order condition, we get

$$\frac{\mathrm{d}^2Z_{2.1}(T)}{\mathrm{d}T^2} = \frac{2S - pI_eDM_1^2}{T^3} > 0. \tag{20}$$

To ensure $M_1 < T_{2.1} < M_2$ and $pDM_1 + pI_eDM_1^2/2 \geqslant cDT$, we substitute (19) into both inequalities and obtain that

$$pI_eDM_1^2 + hD\Delta_1 > 2S > (h + pI_e)DM_1^2, \tag{21}$$

where $\Delta_1 = \min\{M_2^2, [(pM_1/c)(1 + I_eM_1/2)]^2\} > M_1^2$.

Likewise, the first-order condition for Case 2.2 is $\mathrm{d}Z_{2.2}(T)/\mathrm{d}T = 0$, which leads us to

$$\left(hD + \frac{c^2I_1D}{p}\right)T^2 = 2S - pI_eDM_1^2 + \frac{I_1D}{p}[pM_1(1 + I_eM_1/2)]^2 \tag{22}$$

and thus the optimal value of T for Case 2.2 is

$$T_{2.2} = \sqrt{\frac{2S - pI_eDM_1^2 + (I_1D/p)[pM_1(1 + I_eM_1/2)]^2}{hD + (c^2I_1D/p)}}. \tag{23}$$

For the second-order condition, we get

$$\frac{\mathrm{d}^2Z_{2.2}(T)}{\mathrm{d}T^2} = \frac{D}{T}\left(h + \frac{c^2I_1}{p}\right) > 0. \tag{24}$$

To ensure $pDM_1 + pI_eDM_1^2/2 < cDT$ and $M_1 < T_{2.2} < M_2$, we substitute (23) into both inequalities and obtain that

$$pI_eDM_1^2 + hD[(pM_1/c)(1 + I_eM_1/2)]^2 < 2S < pI_eDM_1^2 - \frac{I_1D}{p}[pM_1(1 + I_eM_1/2)]^2 + M_2^2\left(hD + \frac{c^2I_1D}{p}\right). \tag{25}$$

By using an analogous argument, we can obtain the first-order condition for Case 3.1 is $\mathrm{d}Z_{3.1}(T)/\mathrm{d}T = 0$, which leads us to

$$\left(2S - pI_eDM_1^2\right) = hDT^2. \tag{26}$$

Consequently, we obtain the optimal value of T for Case 3.1 is

$$T_{3.1} = \sqrt{(2S - pI_eDM_1^2)/hD}. \tag{27}$$

For the second-order condition, we get

$$\frac{\mathrm{d}^2Z_{3.1}(T)}{\mathrm{d}T^2} = \frac{2S - pI_eDM_1^2}{T^3} > 0. \tag{28}$$

From $pDM_1 + pI_eDM_1^2/2 \geqslant cDT$ and $T_{3.1} \geqslant M_2$, we substitute (27) into both inequalities and obtain that

$$pI_eDM_1^2 + hDM_2^2 \leqslant 2S \leqslant pI_eDM_1^2 + hD[(pM_1/c)(1 + I_eM_1/2)]^2. \tag{29}$$

For Case 3.2, we can obtain the first-order condition $\mathrm{d}Z_{3.2}(T)/\mathrm{d}T = 0$, which leads us to

$$\left(hD + \frac{c^2I_1D}{p}\right)T^2 = 2S - pI_eDM_1^2 + \frac{I_1D}{p}[pM_1(1 + I_eM_1/2)]^2 \tag{30}$$

and thus the optimal value of T for Case 3.2 is

$$T_{3.2} = \sqrt{\frac{2S - pI_eDM_1^2 + (I_1D/p)[pM_1(1 + I_eM_1/2)]^2}{hD + (c^2I_1D/p)}}. \tag{31}$$

The second-order condition

$$\frac{\mathrm{d}^2Z_{3.2}(T)}{\mathrm{d}T^2} = \frac{D}{T}\left(h + \frac{c^2I_1}{p}\right) > 0. \tag{32}$$

From $pDM_1 + pI_eDM_1^2/2 < cDT$ but $[pD(M_2 - M_1) + pI_eD(M_2 - M_1)^2/2] \geqslant [cDT - pDM_1 - pI_eDM_1^2/2]$ and $T_{3.2} \geqslant M_2$, we substitute (31) into three inequalities and obtain that

$$pI_eDM_1^2 - \frac{I_1D}{p}[pM_1(1+I_eM_1/2)]^2 + \left(hD + \frac{c^2I_1D}{p}\right)\Delta_2^2 < 2S < pI_eDM_1^2 - \frac{I_1D}{p}[pM_1(1+I_eM_1/2)]^2 + \left(hD + \frac{c^2I_1D}{p}\right)\Delta_3^2, \tag{33}$$

where $\Delta_2 = \max\{M_2, [(pM_1/c)(1 + I_eM_1/2)]\}$ and $\Delta_3 = \left\{\frac{pM_2}{c} + \frac{pI_e}{2c}[(M_2 - M_1)^2 + M_1^2]\right\} > M_2$.

For Case 3.3, we obtain the first-order condition as

$$\left(hD + \frac{c^2I_2D}{p}\right)T^2 = 2S - pI_eDM_1^2 + \frac{I_2D}{p}\left\{pM_2 + \frac{pI_e}{2}\left[(M_2 - M_1)^2 + M_1^2\right]\right\}^2 - 2I_1(M_2 - M_1)DpM_1(1 + I_eM_1/2). \tag{34}$$

Consequently, we obtain the optimal value of T for Case 3.3 is

$$T_{3.3} = \sqrt{\frac{2S - pI_eDM_1^2 + (I_2D/p)\Delta_4^2 - 2I_1(M_2 - M_1)DpM_1(1 + I_eM_1/2)}{hD + (c^2I_2D/p)}}, \tag{35}$$

where $\Delta_4 = \left\{pM_2 + \frac{pI_e}{2}[(M_2 - M_1)^2 + M_1^2]\right\}$.

The second-order condition

$$\frac{d^2Z_{3.3}(T)}{dT^2} = \frac{D}{T}\left(h + \frac{c^2I_2}{p}\right) > 0. \tag{36}$$

From $pDM_1 + pI_eDM_1^2/2 < cDT$ and $[pD(M_2 - M_1) + pI_eD(M_2 - M_1)^2/2] < [cDT - pDM_1 - pI_eDM_1^2/2]$, we obtain than

$$2S > pI_eDM_1^2 + 2I_1(M_2 - M_1)DpM_1(1 + I_eM_1/2) - \frac{I_2D}{p}\Delta_4^2 + \left(hD + \frac{c^2I_2D}{p}\right)\Delta_3^2. \tag{37}$$

Theorem 1. *When $M_2 < [(pM_1/c)(1 + I_eM_1/2)]$, we have the following results:*

(1) *If $2S \leqslant D(h + pI_e)M_1^2$, then $T^* = T_1$.*
(2) *If $D(h + pI_e)M_1^2 < 2S < pI_eDM_1^2 - \frac{I_1D}{p}[pM_1(1 + I_eM_1/2)]^2 + M_2^2\left(hD + \frac{c^2I_1D}{p}\right)$, then $T^* = T_{2.1}$.*
(3) *If $pI_eDM_1^2 - \frac{I_1D}{p}[pM_1(1 + I_eM_1/2)]^2 + M_2^2\left(hD + \frac{c^2I_1D}{p}\right) \leqslant 2S \leqslant pI_eDM_1^2 + hDM_2^2$, then we know:*
   (a) *If $Z_{2.1}(T_{2.1}) \leqslant Z_{3.2}(T_{3.2})$ then $T^* = T_{2.1}$.*
   (b) *Otherwise, $T^* = T_{3.2}$.*
(4) *If $pI_eDM_1^2 + hDM_2^2 < 2S < pI_eDM_1^2 + hD[(pM_1/c)(1 + I_eM_1/2)]^2$, then we know:*
   (a) *If $Z_{3.1}(T_{3.1}) \leqslant Z_{3.2}(T_{3.2})$ then $T^* = T_{3.1}$.*
   (b) *Otherwise, $T^* = T_{3.2}$.*
(5) *If $pI_eDM_1^2 + hD[(pM_1/c)(1 + I_eM_1/2)]^2 < 2S < pI_eDM_1^2 - \frac{I_1D}{p}[pM_1(1 + I_eM_1/2)]^2 + \left(hD + \frac{c^2I_1D}{p}\right)\Delta_3^2$ then $T^* = T_{3.2}$.*
(6) *If $2S > pI_eDM_1^2 + 2I_1(M_2 - M_1)DpM_1(1 + I_eM_1/2) - \frac{I_2D}{p}\Delta_4^2 + \left(hD + \frac{c^2I_2D}{p}\right)\Delta_3^2$, then $T^* = T_{3.3}$.*

Proof. It immediately follows from (17), (21), (25), (29), (33) and (37). □

Theorem 2. *When $M_2 > [(pM_1/c)(1 + I_eM_1/2)]$, we have the following results:*

(1) *If $2S \leqslant D(h + pI_e)M_1^2$, then $T^* = T_1$.*
(2) *If $D(h + pI_e)M_1^2 < 2S < pI_eDM_1^2 + hD[(pM_1/c)(1 + I_eM_1/2)]^2$, then $T^* = T_{2.1}$.*

(3) *If* $pI_eDM_1^2 + hD[(pM_1/c)(1 + I_eM_1/2)]^2 < 2S < pI_eDM_1^2 - \frac{I_1D}{p}[pM_1(1 + I_eM_1/2)]^2 + M_2^2\left(hD + \frac{c^2I_1D}{p}\right)$, *then* $T^* = T_{2.2}$.

(4) *If* $pI_eDM_1^2 - \frac{I_1D}{p}[pM_1(1 + I_eM_1/2)]^2 + M_2^2\left(hD + \frac{c^2I_1D}{p}\right) < 2S < pI_eDM_1^2 - \frac{I_1D}{p}[pM_1(1 + I_eM_1/2)]^2 + (hD + \frac{c^2I_1D}{p})\Delta_3^2$ *then* $T^* = T_{3.2}$.

(5) *If* $2S > pI_eDM_1^2 + 2I_1(M_2 - M_1)DpM_1(1 + I_eM_1/2) - \frac{I_2D}{p}\Delta_4^2 + \left(hD + \frac{c^2I_2D}{p}\right)\Delta_3^2$, *then* $T^* = T_{3.3}$.

Proof. It immediately follows from (17), (21), (25), (29), (33) and (37). □

5. Numerical examples

Example 1. Given $D = 1000$ units/year, $h = \$4$/unit/year, $I_1 = 0.03$/year, $I_2 = 0.12$/year, $I_e = 0.04$/year, $c = \$25$ per unit, $p = \$35$ per unit, $M_1 = 30$ days $= 30/365$ years, and $M_2 = 90$ days $= 90/365$ (or 0.246575) years, we obtain $M_2 > [(pM_1/c)(1 + I_dM_1/2)] = 0.115258$. Using the results in Theorem 2, we obtain the following computational results as shown in Table 1 when $S = 15$, 30, 50, 60, 100, 150, 200, 250, 400, 500 and 600.

Table 1 reveals that the higher the ordering cost S, the higher the ordering quantity Q^*, the replenishment cycle T^*, and the total relevant cost per year $Z(T^*)$.

Example 2. From Example 1, we know that the retailer will take the teaser rate when $S = 50$, 60, 100, 150, 200 and 250. In this example, we want to study the effect of the teaser rate. By using the same parameters as shown in Example 1, if the bank (or the supplier) does not offer the teaser rate, then we obtain the following computational results as shown in Table 2 when $S = 50$, 60, 100, 150, 200 and 250.

By comparing Tables 1 and 2, we know that the retailer will order more quantity Q^* and pay less total relevant cost per year $Z(T^*)$ when the supplier (or the bank) offers a teaser rate.

Table 1
Optimal solutions of Example 1

Ordering cost S	Replenishment cycle T^*	Order quantity Q^*	Total relevant cost per year $Z(T^*)$
15	$T_1 = 0.074536$	74.5356	$Z_1(T_1) = 287.4237$
30	$T_{2.1} = 0.112408$	112.4080	$Z_{2.1}(T_{2.1}) = 449.6324$
50	$T_{2.2} = 0.146735$	146.7348	$Z_{2.2}(T_{2.2}) = 603.8019$
60	$T_{2.2} = 0.161061$	161.0607	$Z_{2.2}(T_{2.2}) = 668.7801$
100	$T_{2.2} = 0.208754$	208.7543	$Z_{2.2}(T_{2.2}) = 885.1045$
150	$T_{3.2} = 0.256175$	256.1749	$Z_{3.2}(T_{3.2}) = 1100.1911$
200	$T_{3.2} = 0.296096$	296.0960	$Z_{3.2}(T_{3.2}) = 1281.2616$
250	$T_{3.2} = 0.331240$	331.2402	$Z_{3.2}(T_{3.2}) = 1440.6658$
400	$T_{3.3} = 0.352231$	352.2309	$Z_{3.3}(T_{3.3}) = 1868.2402$
500	$T_{3.3} = 0.395758$	395.7584	$Z_{3.3}(T_{3.3}) = 2093.3185$
600	$T_{3.3} = 0.434952$	434.9516	$Z_{3.3}(T_{3.3}) = 2303.2284$

Table 2
Optimal solutions of Example 2

Ordering cost S	Replenishment cycle T^*	Order quantity Q^*	Total relevant cost per year $Z(T^*)$
50	0.139189	139.1888	608.0361
60	0.150430	150.4305	677.0922
100	0.188819	188.8189	912.9070
150	0.227885	227.8852	1152.8854
200	0.261172	261.1718	1357.3605
250	0.290671	290.6713	1538.5715

6. Conclusions and future research

In this paper, we introduced a new idea to the area of trade credits. Namely, the supplier charges the retailer progressive interest rates if the retailer prolongs its unpaid balance. By offering progressive interest rates to the retailers a supplier, can secure competitive market advantage over the competitors and possibly improve market share or/and profit. In the paper, we established the necessary and sufficient conditions for the unique optimal replenishment interval, and obtained the explicit closed-form optimal solution. Furthermore, we constructed two theoretical results, which provide us a simple way to obtain the optimal replenishment interval by examining the explicit conditions. Finally, we provided two numerical examples to show that the retailer will order more quantity and pay less total relevant cost per year if the supplier offers a short-term teaser interest rate.

The model proposed in this paper can be extended in several ways. For instance, we may extend the model to allow for a constant deterioration rate or a two-parameter Weibull distribution. Also, we could consider the demand as a function of price, quality as well as time varying. Furthermore, we could generalize the model to allow for shortages, quantity discounts, discount and inflation rates, and others. Finally, the supplier may extend two progressive interest charges to n progressive interest charges. However, in this paper, there are 6 (i.e., $1 + 2 + 3$ sub-cases) sub-cases when the supplier provides two progressive interest charges. If the supplier provides n progressive interest charges, then the problem has $(n + 1)(n + 2)/2$ sub-cases (i.e., $1 + 2 + \cdots + (n + 1)$ sub-cases), and becomes very complicated and tedious. The authors believe that this paper will work as a catalyst in the generation of numerous research papers in years to come.

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