Available online at www.sciencedirect.com
ScienceDirect
Int. J. Production Economics 112 (2008) 816–828
international journal of production economics
www.elsevier.com/locate/ijpe

The investment model in preventive maintenance in multi-level production systems

Hsu-Hua Lee

Graduate Institute of Management Sciences Tamkang University 151 Ying-Chuan Road, Tamsui, Taipei County, Taiwan 25137, Republic of China

Received 24 October 2005; accepted 4 July 2007
Available online 28 July 2007

Abstract

In this research, cost/benefit models for investments made in quality improvements are developed to measure the impact of quality programs and to predict the return of an investment in these programs in a multi-level assembly system. Using these models, the decision makers can decide whether and how much to invest in quality improvement projects. The relationship between the investment and return on investment can be developed based on the tangible variables. The investment model in preventive maintenance is developed in a multi-level assembly system. The investment in preventive maintenance is to reduce the variance and the deviation of the mean from the target value of the quality characteristic, and hence to reduce the proportion of defectives and also to increase reliability. The proportion of defectives can be linked to manufacturing cost, inventory cost, and profit loss. The reliability is linked to warranty cost. The total costs in this investment model include manufacturing cost, setup cost, holding cost, profit loss, and warranty cost.

Keywords: Batch quantity; Investment in quality improvement; Variance; Mean; Defective proportion; Reliability; Multi-level system

1. Introduction

The development of cost/benefit models for investments in quality improvement is crucial because it can help the manufacturers in evaluating the effectiveness of the amount of investment they spend and selecting optimal investment opportunities. The investment in quality improvement should not be based on faith, and should be analyzed by the ‘‘quantified’’ measures of quality. Therefore, what is needed is a way of measuring the impact of quality programs and a mechanism for predicting the return of an investment in these programs.

Investment models are used to evaluate the effect of investment in prevention and appraisal activities on the resulting internal and external failure costs and to predict the return of the investment (Gupta and Campbell, 1995). Goyal and Gunasekaran (1990), Chen (1996), Lee et al. (1997), and Deleveaux (1997) developed investment models in quality improvement. Porteus (1986a, b), Trevino et al. (1993), and Leschke and Weiss (1997) presented the investment models in setup reduction. Hwang et al. (1993), Hong et al. (1993), Hong and Hayya (1995), Gunasekaran (1995), and Hong (1997) developed the investment models in quality and setup improvement. Ben-Daya

E-mail address: HXL120@gmail.com

0925-5273/$ - see front matter
doi:10.1016/j.ijpe.2007.07.004

Nomenclature

The two-tuple notation (i_h,j) indicates the jth stage of component/subassembly i_h at level h. The following notations are used at stage (i_h,j) to develop the investment model.

$X(i_h,j)$	quality characteristic
$\mu(i_h,j)$	mean of quality characteristic
$\sigma^2(i_h,j)$	variance of quality characteristic
$x_0(i_h,j)$	target value of quality characteristic
$C(i_h,j)$	manufacturing cost per unit
$S(i_h,j)$	setup cost per batch
$V(i_h,j)$	sales value
$p(i_h,j)$	proportion of defectives
$d(i_h,j)$	demand rate
$\mathrm{PO}(i_h,j)$	production rate
$F(i_h,j)$	production quantity required for one unit of final product
$\omega(i_h,j)$	replenishment time
$y(i_h,j)$	maximum inventory level
$\mathrm{PL}(i_h,j)$	profit loss per batch
$\mathrm{LSL}(i_h,j)$	lower specification limit
$\mathrm{USL}(i_h,j)$	upper specification limit
$\mathrm{Cpm}(i_h,j)$	process capability index
$R(t/X(i_h,j))$	reliability function
$\xi(i_h,j)$	shape parameter of characteristic
$B(i_h,j)$	new scale parameter of characteristic

Other notations will be introduced in the text.

(1999) considered the effect of imperfect quality on lot-sizing decisions and inspection errors. Salameh and Jaber (2000) studied the model for economic production quantity of the items with imperfect quality. Sheu and Chen (2004) developed a lot-sizing model for the determination of the level of preventive maintenance for an imperfect process control. Zequeira et al. (2004) presented a model to determine the optimal length of production periods between maintenance actions and the optimal buffer inventory during preventive maintenance. Hoque and Goyal (2005) presented a cost model of setup, transportation, and inventory with respect to setup and transportation times in multi-stage production systems. Lee (2005a–c) developed the model to increase the service level and reduce the defectives in imperfect production systems with imperfect products' quality and imperfect supplied quantity. Lee (2006) presented the investment model with respect to repetitive inspections and measurement equipment in imperfect production systems. Papachristos and Konstantaras (2006) considered the timing of withdrawing the imperfect quality items from stock in economic ordering quantity models. Eroglu and Ozdemir (2007) studied an economic order quantity model by considering defective items and shortages back-ordered. Sana et al. (2007) derived a flexible inventory model of imperfect quality items with a reduced selling price. Lee et al. (1997) studied the problem of selecting the optimum production batch size in multi-stage manufacturing facilities with scrap and determining the optimal amount of investment. They analyzed the effect of investment for quality improvement on proportion of rejection, and the effect of proportion of rejection on processing cost, setup cost, holding cost, and profit loss. The purpose of the investment was to reduce the variance of the quality characteristic and hence the proportion of defectives. Taguchi's loss function was simplified so that it included the variance and the deviation of the mean from the target value as well as the proportion of defectives in the expression. The model assumed known demand, which must be satisfied completely, scrap at each stage, and profit loss due to scrap. Using this model, the optimal values of the production quantity and the proportion of defective products for minimizing the total cost were obtained. The optimal investment was then obtained using the relationship between the investment and the proportion of defectives. Deleveaux (1997) extended the investment model in a multi-stage system into a multi-component multi-stage system with imperfect processes to develop the lot-size models as a function of quality level, and the cycle time model as a function of lot size. A conditional bi-variate Weibull distribution for reliability was developed that incorporated the impact of variance and mean setting on time to failure.

In this research, the investment model will be extended to a multi-level multi-stage system. The investment in preventive maintenance is to reduce the variance and the deviation of the mean from the target value of the quality characteristic, and hence to reduce the proportion of defectives and also to increase reliability. The proportion of defectives can be linked to manufacturing cost, inventory cost, and profit loss, which is the loss in profit due to the defective units. The reliability is linked to the warranty cost. The total costs in this investment model include manufacturing cost, setup cost, holding cost, profit loss, and warranty cost.

2. Model development

The operation process chart in Fig. 1 is used to develop the investment model. Let N' denote the total number of levels for a multi-level assembly system. Subassemblies including components that are assembled at level h are manufactured in level $h+1$, $h=1,\ldots,N'-1$. The level of the main assembly is 1. The number of subassemblies at level h is $N(h)$. The component/subassembly at level h, denoted by i_h, requires $n(i_h)$ stages. The two-tuple notation (i_h, j) indicates the jth stage of component/subassembly i_h at level h, $j=1,\ldots,n(i_h)$. Let $m(i_h, k_{h-1})$ be the number of components/subassemblies i_h required per subassembly k_{h-1}.

2.1. Assumption

The following assumptions are made while developing this model:

(1) Demand for the products is constant and uniform.
(2) Setup costs are constant and price per unit of product is constant.
(3) Inventory holding cost is based on average inventory.
(4) All defective items are scrapped.
(5) The demand is satisfied completely and no shortages are allowed.
(6) The loss in profit is due to the defective units.
(7) Demand of the item occurs continuously. The production rate exceeds the demand rate.
(8) The quality characteristic of the product is normally distributed.

2.2. Investment in preventive maintenance

Let the quality characteristic be $X(i_h,j)$ with a mean $\mu(i_h,j)$ and variance $\sigma^2(i_h,j)$. The target value for characteristic $X(i_h,j)$ is denoted by $x_0(i_h,j)$. The investment in preventive maintenance or new technology/equipment for the multi-level assembly system can be written as (Deleveaux, 1997)

$$\mathrm{TC}_{\mathrm{PM}} = \sum_{h=1}^{N'} \sum_{i=1}^{N(h)} \sum_{j=1}^{n(i_h)} f(\sigma^2(i_h,j), (x_0(i_h,j) - \mu(i_h,j))^2), \tag{2.1}$$

in which the investment in preventive maintenance or new technology/equipment is expressed as a function

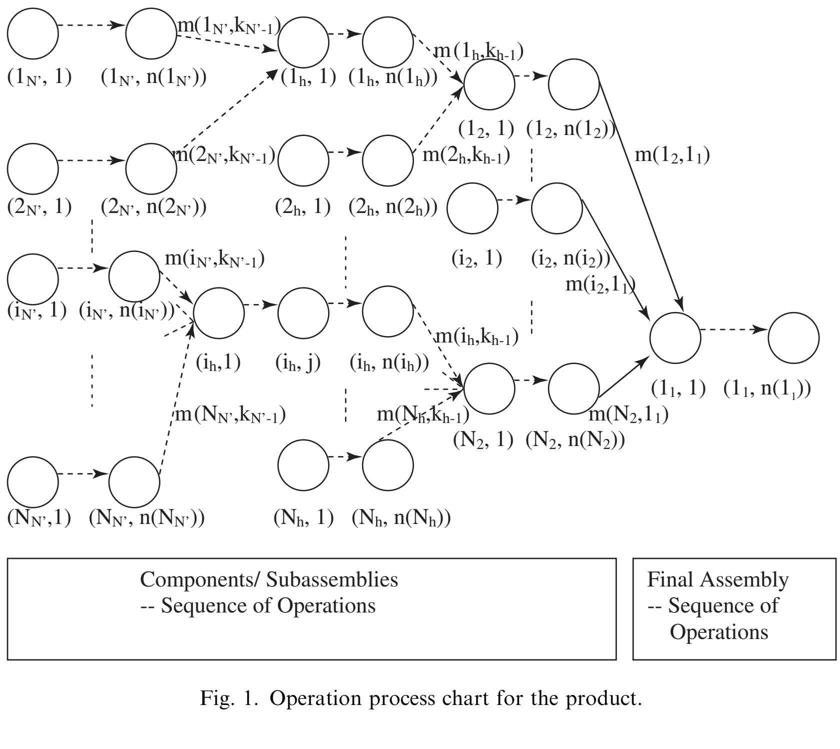

Fig. 1. Operation process chart for the product.

of the deviation of process mean $\mu(i_h,j)$ from the target value $x_0(i_h,j)$, and the variance for the quality characteristic $\sigma^2(i_h,j)$.

2.3. Manufacturing cost

Let $m(i_h, k_{h-1})$ denote the number of components/subassemblies i_h required per subassembly k_{h-1}. It can be deduced that i_h is used as a subassembly of one and only one subassembly in level $h-1$, i.e., k_{h-1}. k_{h-1} is also a subassembly of one and only one subassembly in level $h-2$, and so on. Let Q indicate the batch quantity and let the annual demand for the final products be D, which will be satisfied completely as per the assumptions. Thus, at each stage of component/subassembly at each level, the number of units required to assemble Q units of final product will be manufactured. Let the manufacturing cost per unit and the proportion of defectives be $C(i_h,j)$ and $p(i_h,j)$, respectively.

The total production quantity, $M(i_h)$, of component/subassembly i_h at level h is illustrated in Appendix A, and can be written as

$$M(i_h) = \prod_{r=2}^{h}\left\{\frac{m(i_r,k_{r-1})Q}{\prod_{s=1}^{n(i_{r-1})}[1-p(i_{r-1},s)]}\right\}\sum_{j=1}^{n(i_h)}\frac{1}{\prod_{g=j}^{n(i_h)}[1-p(i_h,g)]}. \tag{2.2}$$

Thus, from (2.2), the total manufacturing cost per year is the total cost per batch times the number of batches, D/Q, of the multi-level assembly system, and is written as

$$\mathrm{TC}_M = \frac{D}{Q}\sum_{h=1}^{N'}\sum_{i=1}^{N(h)}\sum_{j=1}^{n(h)}\prod_{r=2}^{h}\left\{\frac{C(i_h,j)m(i_r,k_{r-1})Q}{\prod_{s=1}^{n(i_{r-1})}[1-p(i_{r-1},s)]}\right\} \times \sum_{j=1}^{n(i_h)}\frac{1}{\prod_{g=j}^{n(i_h)}[1-p(i_h,g)]}. \tag{2.3}$$

2.4. Holding cost

This cost includes all the expenses incurred because of carrying inventory. The demand rate at the final stage, $n(1_1)$, for the final product is denoted by d. Thus, the demand rate at stage j for final product, $d(1_1,j)$, is

$$d(1_1,j) = \frac{d}{\prod_{s=j+1}^{n(1_1)}[1-p(1_1,s)]}, \quad \text{for} \quad j = 1,2,\ldots,n(1_1). \tag{2.4}$$

The demand rate at stage j for component/subassembly i_h at level h is

$$d(i_h,j) = \frac{d}{\prod_{g=j+1}^{n(i_h)}[1-p(i_h,g)]}\prod_{r=2}^{h}\left\{\frac{m(i_r,k_{r-1})}{\prod_{s=1}^{n(i_{r-1})}[1-p(i_{r-1},s)]}\right\}, \quad \text{for} \quad j = 1,2,\ldots,n(i_h). \tag{2.5}$$

The production quantity required for one unit of final product is

$$F(i_h,j) = \frac{1}{\prod_{g=j+1}^{n(i_h)}[1-p(i_h,g)]}\prod_{r=2}^{h}\left\{\frac{m(i_r,k_{r-1})}{\prod_{s=1}^{n(i_{r-1})}[1-p(i_{r-1},s)]}\right\}. \tag{2.6}$$

The number of conforming component/subassembly i_h required for one unit of final product is

$$L(i_h) = \prod_{r=2}^{h} m(i_r,k_{r-1}). \tag{2.7}$$

The production rate is assumed to be $\mathrm{PO}(i_h,j)$. Let the replenishment time and the maximum inventory level be $\omega(i_h,j)$ and $y(i_h,j)$, respectively. The annual holding cost per unit of average inventory is H. The inventory graph for each batch for all stages for component/subassembly i_h at level h is shown in Fig. 2. Then the total units produced at stage j for component/subassembly i_h at level h is

$$\frac{Q}{\prod_{g=j}^{n(i_h)}[1-p(i_h,g)]}\prod_{r=2}^{h}\left\{\frac{m(i_r,k_{r-1})}{\prod_{s=1}^{n(i_{r-1})}[1-p(i_{r-1},s)]}\right\} = \mathrm{PO}(i_h,j)\omega(i_h,j)\prod_{r=2}^{h} m(i_r,k_{r-1}), \quad \text{for} \quad j = 1,2,\ldots,n(i_h). \tag{2.8}$$

Rearranging Eq. (2.8), we obtain

$$\omega(i_h,j) = \frac{Q/\mathrm{PO}(i_h,j)}{\prod_{g=j}^{n(i_h)}[1-p(i_h,g)]}\prod_{r=2}^{h}\left\{\frac{1}{\prod_{s=1}^{n(i_{r-1})}[1-p(i_{r-1},s)]}\right\}, \quad \text{for} \quad j = 1,2,\ldots,n(i_h). \tag{2.9}$$

The maximum inventory level of good items, $y(i_h,j)$, is

$$y(i_h,j) = \left\{\prod_{r=2}^{h} m(i_r,k_{r-1})\mathrm{PO}(i_h,j)[1-p(i_h,j)] - d(i_h,j)\right\}\omega(i_h,j), \quad \text{for} \quad j = 1,2,\ldots,n(i_h). \tag{2.10}$$

Let H be the holding cost per unit. Then from Eq. (2.10), the holding cost/year at all stages in multi-level assembly system including component,

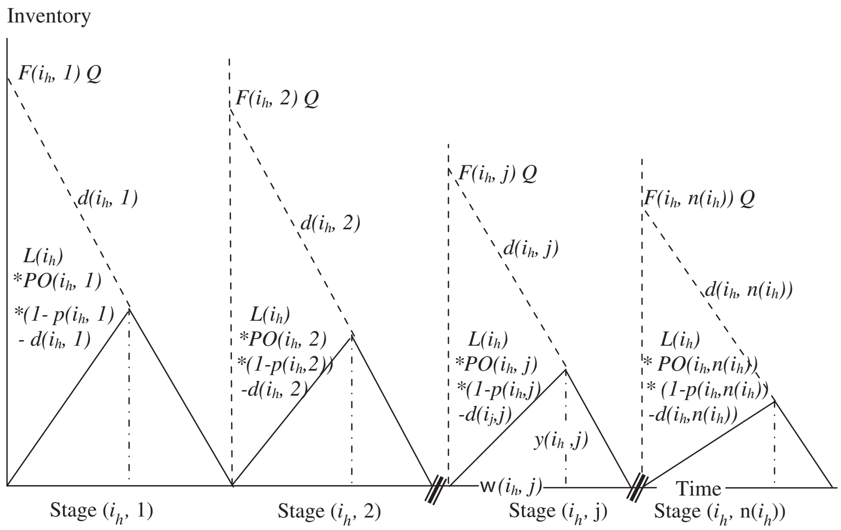

Fig. 2. Inventory graph.

subassembly, and final assembly system is

$$\begin{aligned}\mathrm{THC} &= \frac{H}{2}\sum_{h=1}^{N'}\sum_{i=1}^{N(h)}\sum_{j=1}^{n(i_h)}\left\{\prod_{r=2}^{h} m(i_r,k_{r-1})\mathrm{PO}(i_h,j)[1-p(i_h,j)] - d(i_h,j)\right\}x(i_h,j)\\ &= \frac{H}{2}\sum_{h=1}^{N'}\sum_{i=1}^{N(h)}\sum_{j=1}^{n(i_h)}\left\{\prod_{r=2}^{h} m(i_r,k_{r-1})\mathrm{PO}(i_h,j)[1-p(i_h,j)]\right.\\ &\quad\left. - \frac{d}{\prod_{g=j+1}^{n(i_h)}[1-p(i_h,g)]}\prod_{r=2}^{h}\left\{\frac{m(i_r,k_{r-1})}{\prod_{s=1}^{n(i_{r-1})}[1-p(i_{r-1},s)]}\right\}\right\}\\ &\quad\times\frac{Q/\mathrm{PO}(i_h,j)}{\prod_{g=j}^{n(i_h)}[1-p(i_h,g)]}\prod_{r=2}^{h}\left\{\frac{1}{\prod_{s=1}^{n(i_{r-1})}[1-p(i_{r-1},s)]}\right\}.\end{aligned} \tag{2.11}$$

Let $$\pi(i_h,j) = \frac{1}{\prod_{g=j}^{n(i_h)}[1-p(i_h,g)]}\prod_{r=2}^{h}\left\{\frac{1}{\prod_{s=1}^{n(i_{r-1})}[1-p(i_{r-1},s)]}\right\}. \tag{2.12}$$

Then from Eq. (2.12), Eq. (2.11) can be simplified as

$$\begin{aligned}\mathrm{THC} &= \frac{HQ}{2}\sum_{h=1}^{N'}\sum_{i=1}^{N(h)}\sum_{j=1}^{n(i_h)}\left\{\prod_{r=2}^{h} m(i_r,k_{r-1})\right\}[1-p(i_h,j)]\\ &\quad\times\left\{1-\frac{d}{\mathrm{PO}(i_h,g)}\pi(i_h,j)\right\}\pi(i_h,j).\end{aligned} \tag{2.13}$$

2.5. Setup cost

Let the setup cost per batch be $S(i_h,j)$, where $h = 1,\ldots,N'$, $i = 1,\ldots,N(h)$, and $j = 1,2,\ldots,n(i_h)$. The total setup cost/year at stage j for component/subassembly i_h at level h is $S(i_h,j)\ D/Q$. Then the total setup cost/year at all stages in multi-level assembly system including component, subassembly, and final assembly systems will be the sum of total setup cost/year at all stages, and can be written as

$$\mathrm{TSC} = \frac{D}{Q}\sum_{h=1}^{N'}\sum_{i=1}^{N(h)}\sum_{j=1}^{n(i_h)} S(i_h,j). \tag{2.14}$$

2.6. Profit loss

The profit loss per batch is the loss in profit due to the defective units. It is

$$\mathrm{PL}(i_h,j) = [V(i_h,j) - C(i_h,j)]\left\{\frac{1}{\prod_{g=j}^{n(i_h)}[1-p(i_h,g)]}\right.$$

$$-\frac{1}{\prod_{g=j+1}^{n(i_h)}[1-p(i_h,g)]}\Bigg\} \times \prod_{r=2}^{h}\left\{\frac{m(i_r,k_{r-1})Q}{\prod_{s=1}^{n(i_{r-1})}[1-p(i_{r-1},s)]}\right\}, \quad (2.15)$$

where $V(i_h,j)$ and $C(i_h,j)$ denote the sales value added to the component/subassembly and the manufacturing cost, respectively, and the quantity within parenthesis is the number of defective items manufactured at stage j for component/subassembly i_h at level h.

Thus, the total profit loss/year at all stages in a multi-level assembly system including component, subassembly, and final assembly system is the total profit loss per batches at all stage times the number of batches, D/Q, and can be written as

$$\mathrm{TPL} = \frac{D}{Q}\sum_{h=1}^{N'}\sum_{i=1}^{N(h)}\sum_{j=1}^{n(i_h)}\mathrm{PL}(i_h,j). \quad (2.16)$$

2.7. Warranty cost

The quality characteristic of component/subassembly is normally distributed with a mean, $\mu(i_h,j)$, and standard deviation, $\sigma(i_h,j)$. The target value is $x_0(i_h,j)$. Let the lower and upper specification limits of $X(i_h,j)$ be $\mathrm{LSL}(i_h,j)$ and $\mathrm{USL}(i_h,j)$, respectively. The purpose of the investment is to reduce the variance of the quality characteristic and deviation of the mean from the target value, which in turn reduces the proportion of defectives. Then, the proportion of defectives is denoted by $p(i_h,j)$, and can be written as

$$p(i_h,j) = \int_{-\infty}^{\mathrm{LSL}(i_h,j)} f(x(i_h,j))\,\mathrm{d}(x(i_h,j)) + \int_{\mathrm{USL}(i_h,j)}^{\infty} f(x(i_h,j))\,\mathrm{d}(x(i_h,j)). \quad (2.17)$$

Process capability index attempts to analyze the magnitude of the process variance as it relates to specification. For normal-the-best type quality characteristic, the process capability index $\mathrm{Cpm}(i_h,j)$ for the population of the characteristic $X(i_h,j)$ is defined as (Chan et al., 1988)

$$\mathrm{Cpm}(i_h,j) = \frac{\mathrm{USL}(i_h,j) - \mathrm{LSL}(i_h,j)}{6\sqrt{\sigma(i_h,j)^2 + (\mu(i_h,j) - X_0(i_h,j)}}. \quad (2.18)$$

The reliability functions for the component/subassembly i_h at stage j of level h is (Deleveaux, 1997)

$$R(t/X(i_h,j)) = \mathrm{e}^{-t^{\xi(i_h,j)}\mathrm{e}^{-B(i_h,j)\mathrm{Cpm}(i_h,j)^2}}, \quad (2.19)$$

where $\xi(i_h,j)$ and $B(i_h,j)$ are shape parameter and new scale parameter for characteristic $X(i_h,j)$. There exists only one quality characteristic $X(i_h,j)$ at stage j of component/subassembly i_h at level h, and all quality characteristics of component/subassembly i_h at each stage of level h, $X(i_h,j)$'s, are assumed to be independent of each other. Then the reliability of component/subassembly i_h at level h can be determined by the minimum reliability corresponding to all quality characteristics at all stages of component/subassembly i_h at level h. Thus, from Eq. (2.19), the reliability for quality characteristic value $X(i_h)$ of component/subassembly i_h at level h is

$$R(t/X(i_h)) = \mathrm{Min}\{R[(t/X(i_h,j)), j = 1,2,\ldots,n(i_h)]\}. \quad (2.20)$$

In the assembly system, the product is assumed to be composed of independent components/subassemblies. Then the system reliability, R_{sys}, can be determined by the configuration of components/subassemblies in the product (O'Connor, 1991).

The expected number of failures of final product during the warranty period [0, W] can be obtained from the system reliability as (Kececioglu, 1991; Djamaludin et al., 1994)

$$\int_0^W \varphi_{\mathrm{sys}}(t)\,\mathrm{d}t = -\ln(R_{\mathrm{sys}})|_0^W, \quad (2.21)$$

where $\phi_{\mathrm{sys}}(t)$ is the system failure rate for a final product.

The warranty cost is

$$\mathrm{TWC} = C_{\mathrm{R}}(D)\int_0^W \varphi_{\mathrm{sys}}(t)\,\mathrm{d}t, \quad (2.22)$$

where C_{R} is the repair cost per unit of final product and D is the annual demand.

The total cost including the investment in preventive maintenance, manufacturing cost, holding cost, setup cost, profit loss, and warranty cost as per Eqs. (2.1), (2.3), (2.13), (2.14), (2.16), and (2.22), respectively is

$$\mathrm{TC} = \sum_{h=1}^{N'}\sum_{i=1}^{N(h)}\sum_{j=1}^{n(i_h)} f(\sigma^2(i_h,j),(x_0(i_h,j)-\mu(i_h,j))^2) + \frac{D}{Q}\sum_{h=1}^{N'}\sum_{i=1}^{N(h)}\sum_{j=1}^{n(h)}\prod_{r=2}^{h}\left\{\frac{C(i_h,j)m(i_r,k_{r-1})Q}{\prod_{s=1}^{n(i_{r-1})}[1-p(i_{r-1},s)]}\right\}\sum_{j=1}^{n(i_h)}\frac{1}{\prod_{g=j}^{n(i_h)}[1-p(i_h,g)]} + \frac{HQ}{2}\sum_{h=1}^{N'}\sum_{i=1}^{N(h)}\sum_{j=1}^{n(i_h)}\left\{\prod_{r=2}^{h} m(i_r,k_{r-1})\right\}[1-p(i_h,j)] \times\left\{1-\frac{d}{\mathrm{PO}(i_h,g)}\pi(i_h,j)\right\}\pi(i_h,j) + \frac{D}{Q}\sum_{h=1}^{N'}\sum_{i=1}^{N(h)}\sum_{j=1}^{n(i_h)} S(i_h,j) + \frac{D}{Q}\sum_{h=1}^{N'}\sum_{i=1}^{N(h)}\sum_{j=1}^{n(i_h)}\mathrm{PL}(i_h,j) + C_{\mathrm{R}}(D)\int_0^W \varphi_{\mathrm{sys}}(t)\,\mathrm{d}t. \tag{2.23}$$

3. Numerical example

A numerical example is presented to illustrate the investment model in preventive maintenance in multi-level production systems. The optimum values of the proportion of defectives $p(i_h,j)$, the batch size Q, the standard deviation of quality characteristics $\sigma(i_h,j)$, the mean of quality characteristics $\mu(i_h,j)$, and the investments in preventive maintenance, and the predicted outputs of such investments, are obtained. The example for the multi-level production systems is given in Fig. 3.

This is a multi-level system with three levels marked in Fig. 3. Level 1 is the final assembly line with two stages marked as $(1_1,1)$ and $(1_1,2)$. Level 2 has one subassembly labeled as Subassembly 1_2, and two components, Components 2_2 and 3_2. Subassembly 1_2 has two stages, $(1_2,1)$ and $(1_2,2)$. Component 2_2 has one stage, $(2_2,1)$ whereas Component 3_2 has two stages, $(3_2,1)$ and $(3_2,2)$. One unit each of Subassembly 1_2 and Component 2_2 is required per assembly, indicated by $m(1_2,1_1)$ and $m(2_2,1_1)$, respectively. One assembly requires two units of Component 3_2, indicated by $m(3_2,1_1)$. Level

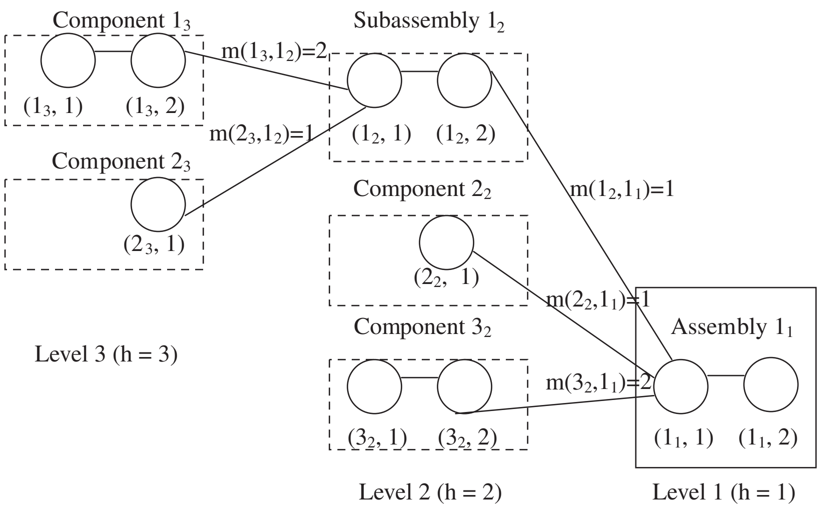

Fig. 3. Example for multi-level production systems.

3 has two components, Components 1_3 and 2_3. Component 1_3 has two stages, $(1_3, 1)$ and $(1_3, 2)$, whereas Component 2_3 has one stage, $(2_3, 1)$. One unit of Subassembly 1_2 requires two units of Component 1_3 and one unit of Component 2_3, indicated by $m(1_3, 1_2)$ and $m(2_3, 1_2)$, respectively.

The values assumed for the input parameters are given in the following tables.

Tables 1–5 relate to the characteristics in the multi-level production systems. These values in Table 5 are obtained using the lower and upper specification limits in Tables 1 and 2, respectively, and the means and standard deviations in Tables 3 and 4, respectively.

Table 1
Lower specification limits for characteristics, LSL(i_h, j)

		$j=1$	$j=2$
$h=1$	$i=1$	6	4
$h=2$	$i=1$	6	7
	$i=2$	4	
	$i=3$	3	4
$h=3$	$i=1$	5	5
	$i=2$	2	

Table 2
Upper specification limits, for characteristics, USL(i_h, j)

		$j=1$	$j=2$
$h=1$	$i=1$	10	12
$h=2$	$i=1$	12	15
	$i=2$	6	
	$i=3$	7	8
$h=3$	$i=1$	9	7
	$i=2$	6	

Table 3
Original means for characteristics, $\mu(i_h, j)$

		$j=1$	$j=2$
$h=1$	$i=1$	7.5	6.5
$h=2$	$i=1$	7.5	13
	$i=2$	5.1	
	$i=3$	4.7	5
$h=3$	$i=1$	5.8	6.21
	$i=2$	4	

Table 4
Original standard deviations for characteristics, $\sigma(i_h, j)$

		$j=1$	$j=2$
$h=1$	$i=1$	0.94	1.43
$h=2$	$i=1$	1.02	1.42
	$i=2$	0.50	
	$i=3$	0.93	0.61
$h=3$	$i=1$	0.57	0.49
	$i=2$	0.92	

Table 5
Original proportions of defectives, $p(i_h, j)$

		$j=1$	$j=2$
$h=1$	$i=I=1$	0.06	0.04
$h=2$	$i=1$	0.07	0.08
	$i=2$	0.05	
	$i=3$	0.04	0.05
$h=3$	$i=1$	0.08	0.06
	$i=2$	0.03	

Table 6
Shape parameters for characteristic, $\varepsilon(i_h, j)$

		$j=1$	$j=2$
$h=1$	$i=1$	1.4	1.93
$h=2$	$i=1$	1.1	1.5
	$i=2$	1.2	
	$i=3$	1.3	1.1
$h=3$	$i=1$	1.15	1.6
	$i=2$	1.3	

For example,

$$p(1_1, 1) = P\left[Z < \frac{6-7.5}{0.94}\right] + P\left[Z > \frac{10-7.5}{0.94}\right] = 0.06.$$

Tables 6 and 7 will be used to compute the reliability for characteristics.

Some other input parameters are given from Tables 8–13.

Assuming that there are 250 days/year, the demand rate at the final stage for the final product, d, is 40 units/day. The original batch quantity is 269 units. The original relevant costs in Eq. (2.23) are given in Table 14.

Table 7
Scale parameters for characteristic, $B(i_h,j)$

		$j=1$	$j=2$
$h=1$	$i=1$	2.7	1.94
$h=2$	$i=1$	3.38	3.5
	$i=2$	3.42	
	$i=3$	3.2	3.90
$h=3$	$i=1$	5.22	3.0
	$i=2$	3.32	

Table 8
Production rate $PO(i_h,j)$ (unit/day)

		$j=1$	$j=2$
$h=1$	$i=1$	60	70
$h=2$	$i=1$	50	60
	$i=2$	60	
	$i=3$	70	80
$h=3$	$i=1$	60	70
	$i=2$	60	

Table 9
Values per unit, $V(i_h,j)$ ($/unit)

		$j=1$	$j=2$
$h=1$	$i=1$	11	9
$h=2$	$i=1$	17	14
	$i=2$	12	
	$i=3$	16	15
$h=3$	$i=1$	10	16
	$i=2$	8	

Table 10
Setup costs per batch, $S(i_h,j)$ ($/batch)

		$j=1$	$j=2$
$h=1$	$i=1$	20	15
$h=2$	$i=1$	15	10
	$i=2$	10	
	$i=3$	18	24
$h=3$	$i=1$	25	20
	$i=2$	12	

Table 11
Manufacturing costs per unit, $C(i_h,j)$ ($/unit)

		$j=1$	$j=2$
$h=1$	$i=1$	6	4
$h=2$	$i=1$	7	8
	$i=2$	5	
	$i=3$	4	5
$h=3$	$i=1$	8	6
	$i=2$	3	

Table 12
Given constant values for investment in preventive maintenance, $b(i_h,j)$ ($)

		$j=1$	$j=2$
$h=1$	$i=1$	10,000	12,000
$h=2$	$i=1$	25,000	30,000
	$i=2$	20,000	
	$i=3$	25,000	20,000
$h=3$	$i=1$	30,000	10,000
	$i=2$	20,000	

Table 13
Other input parameters for products

Annual demand (D)	10,000 units
Warranty period (W)	2 years
Repair cost per unit (C_R)	$200
Holding cost per unit (H)	$20
Value per unit of final product (V)	$128

Table 14
Original relevant costs ($)

Total cost	2,781,678.12
Manufacturing cost	1,012,384.22
Investment in preventive maintenance	0
Profit loss	293,988.84
Inventory cost	6282.53
Setup cost	6282.53
Warranty cost	1462,740

The sequential quadratic programming (SQP) method (Gill et al., 1981; Grace and Branch, 1996) can be used to solve this problem.

Table 15
Proportions of defectives after improvement, $p(i_h,j)$

		$j=1$	$j=2$
$h=1$	$i=1$	0.0045	0.0051
$h=2$	$i=1$	0.0196	0.0219
	$i=2$	0.05	
	$i=3$	0.04	0.0279
$h=3$	$i=1$	0.0582	0.0082
	$i=2$	0.03	

Table 16
Means of characteristics, $\mu(i_h,j)$ after improvement

		$j=1$	$j=2$
$h=1$	$i=1$	8	7
$h=2$	$i=1$	8	10.5
	$i=2$	5.1	
	$i=3$	4.7	5
$h=3$	$i=1$	6	6
	$i=2$	4	

Table 17
Standard deviations of characteristics, $\sigma(i_h,j)$ after improvement

		$j=1$	$j=2$
$h=1$	$i=1$	0.70	1.21
$h=2$	$i=1$	0.97	1.67
	$i=2$	0.50	
	$i=3$	0.93	0.52
$h=3$	$i=1$	0.64	0.38
	$i=2$	0.92	

Solution:

Step 1: The first derivative of the objective function (2.23) with respect to Q (batch quantity) is set equal to zero to obtain the expression of batch quantity, and then replace the batch quantity, Q, in the objective function (2.23).

Step 2: Because of the complexity of the problem, this step will be illustrated in a numerical example by using Matlab 5.3. The objective function is optimized and the optimal values of $p(i_h,j)$ are obtained. Then optimal batch quantity Q is computed. The batch quantity (Q) is 223 units after the investment in preventative maintenance. The optimal values of $p(i_h,j)$ are given in Table 15.

Step 3: The values of $\mu(i_h,j)$ and $\sigma(i_h,j)$ for quality characteristics after improvement are computed using $p(i_h,j)$ in Table 15 as per Eq. (2.17), and are given in Tables 16 and 17, respectively.

Step 4: The relevant costs are then computed.

In practice, the investment in preventive maintenance can be assumed to be a function of defective proportions, $p(i_h,j)$, in Table 15. The values of $b(i_h,j)$ are obtained from Table 12. The expression for the total investment in preventive maintenance can be obtained as (Porteus, 1986a, b)

$$\begin{aligned}
\mathrm{TC}_{\mathrm{PM}} &= \sum_{h=1}^{N'}\sum_{i=1}^{N(h)}\sum_{j=1}^{n(i_h)}\{a(i_h,j) - b(i_h,j)\ln[p(i_h,j)]\} \\
&= -28,134 - 10,000 \times \ln(p(1_1,1)) \\
&\quad - 38,626 - 12,000 \times \ln(p(1_1,2)) \\
&\quad - 66,481 - 25,000 \times \ln(p(1_2,1)) \\
&\quad - 75,771 - 30,000 \times \ln(p(1_2,2)) \\
&\quad - 59,914 - 20,000 \times \ln(p(2_2,1)) \\
&\quad - 80471 - 25,000 \times \ln(p(3_2,1)) \\
&\quad - 59,914 - 20,000 \times \ln(p(3_2,2)) \\
&\quad - 75,771 - 30,000 \times \ln(p(1_3,1)) \\
&\quad - 28,134 - 10,000 \times \ln(p(1_3,2)) \\
&\quad - 70,131 - 20,000 \times \ln(p(2_3,1)) \\
&= \$162,387.93,
\end{aligned}$$

where the constant values of $a(i_h,j)$ and $b(i_h,j)$ are obtained using $a(i_h,j) = b(i_h,j)\ln(p(i_h,j))$, corresponding to the original proportion of defective $p(i_h,j)$ in Table 5 (for example, as $b(1_1,1) = 10,000$, $a(1_1,1) = b(1_1,1)\ln(p(1_1,1)) = 10,000 \times \ln(0.06) = -28,134$ corresponding to $p(1_1,1) = 0.06$).

Before the reliability of the assembly can be computed from the reliabilities of the components and the subassemblies, the stage that determines the reliability of the component/system has to be determined. For the component/subassembly processed in multiple stages, the stage associated with

Table 18
Accepted conforming capabilities, $\mathrm{Cpm}(i_h,j)$ for characteristics

		$j=1$	$j=2$
$h=1$	$i=1$	0.9524	0.8494
$h=2$	$i=1$	0.7178	0.7465
	$i=2$	0.6667	
	$i=3$	0.7168	0.5915
$h=3$	$i=1$	0.5615	0.8772
	$i=2$	0.7246	

the quality characteristic yielding the minimum reliability will be selected. To accomplish this, the reliabilities of all stages in Fig. 3 can be computed next.

First, the capabilities $\mathrm{Cpm}(i_h,j)$ are computed using the values of $\mu(i_h,j)$ in Table 16 and $\sigma(i_h,j)$ in Table 17 as per Eq. (2.18) and are given in Table 18. The reliabilities are obtained using $\mathrm{Cpm}(i_h,j)$ in Table 18, $\varepsilon(i_h,j)$ in Table 6, and $b(i_h,j)$ in Table 7 with $t=2$ as per Eq. (2.19) and are given in Table 19.

Table 19
Reliabilities, $R(t/x(i_h, j))$ for characteristics

		$j=1$	$j=2$
$h=1$	$i=1$	0.7961	0.3856
$h=2$	$i=1$	0.6868	0.6941
	$i=2$	0.6051	
	$i=3$	0.6215	0.5783
$h=3$	$i=1$	0.6518	0.7398
	$i=2$	0.65	

Table 20
Stages for components/subassemblies

Component/subassembly (i_h)	Stage (i_h,j)
1_1	$(1_1, 2)$
1_2	$(1_2, 1)$
2_2	$(2_2, 1)$
3_2	$(3_2, 2)$
1_3	$(1_3, 1)$
2_3	$(2_3, 1)$

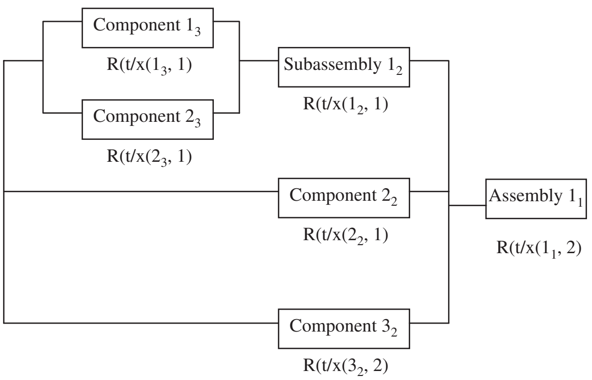

Fig. 4. Configuration of components/subassemblies in this example.

Table 21
Relevant costs after investments ($)

Total cost	2,525,854.71
Manufacturing cost	840,944.93
Investment in preventive maintenance	162,428.24
Profit loss	59,004.58
Inventory cost	7578.48
Setup cost	7578.48
Warranty cost	1,448,320

Based on the results in Table 19, the following stages are selected for determining the reliabilities of components/subassemblies as per Eq. (2.20), and can be shown in Table 20.

The following configuration of components/subassemblies in the product is assumed in Fig. 4.

Then the system reliability can be computed and the expected number of failures of products is computed as per Eq. (2.21). The reliabilities for conforming components/subassemblies are computed using capability $\mathrm{Cpm}(i_h,j)$ in Table 17. From the configuration in Fig. 4, the system reliability, R_{sys}, is computed as

$$\begin{aligned}R_{\mathrm{sys}} = \{1-\{[1-[(1-(1-R_1(t/x(1_3,1)))\\ \times(1-R_1(t/x(2_3,1))))\times R_1(t/x(1_2,1))]]\\ \times(1-R_1(t/x(2_2,1)))\times(1-R_1(t/x(3_2,2)))\}\}\\ \times R_1(t/x(1_1,2)).\end{aligned}$$

The expected number of failures for the products during warranty period [0, W] for $W=2$ as per Eq. (2.21) is

$$\int_0^W \varphi_{\mathrm{sys}}(t)\mathrm{d}t = -\ln(R_{\mathrm{sys}})|_0^W = -\ln(R_{\mathrm{sys}})|_0^2 = 0.72416.$$

The warranty cost as per (2.22) is

$$\begin{aligned}\mathrm{TWC} &= C_{\mathrm{R}}(D)\int_0^W \varphi_{\mathrm{sys}}(t)\,\mathrm{d}t\\ &= 200\times 10,000\times 0.72416 = \$1,448,320.\end{aligned}$$

The relevant costs in Eq. (2.23) are summarized in Table 21.

The original total cost before the investments in quality improvement is $2,781,678.12 from Table 14. The total cost after the investments is reduced to $2,525,854.71 from Table 21. The return on investments in improvement is $2,781,678.12–2,525,854.71 = $255,823.41. The investments in quality improvement projects are profitable.

4. Summary

The investment in preventive maintenance is related to $\sigma(i_h,j)$ (the standard deviation of quality characteristic) and $\mu(i_h,j)$, (the mean of quality characteristic), which will reduce the defective proportion $p(i_h,j)$ and then affect the batch quantity Q. Because the standard deviation of quality characteristic $\sigma(i_h,j)$, the mean of quality characteristic $\mu(i_h,j)$ and the batch quantity Q are all related to proportion of defectives $p(i_h,j)$, it is chosen as the decision variable for the investment preventive maintenance. Thus, the objective is to find the optimal values of $p(i_h,j)$, which minimize the total annual cost. Then the optimal values of standard deviation $\sigma(i_h,j)$, mean $\mu(i_h,j)$, batch quantity Q, and the optimal amount of investment in preventive maintenance will be obtained.

The SQP method (Gill et al., 1981; Grace and Branch, 1996) can be used to solve this problem. The final quality investment models can be used to predict the benefits of investment before it is made and justify investment in quality improvement projects, and thus can help the industries to make optimal selection of quality improvement projects for investment.

Acknowledgment

The researches NSC 90-2218-E-231-004 and NSC 95-2221-E-231-003 are sponsored by National Science Council.

Appendix A

Let N' denote the total number of levels for a multi-level assembly system. Subassemblies including components that are assembled at level h are manufactured in level $h+1$, $h=1,\ldots,N'-1$. The level of the main assembly is 1. The number of subassemblies at level h is $N(h)$. The component/subassembly at level h denoted by i_h, requires $n(i_h)$ stages. The two-tuple notation (i_h,j) indicates the jth stage of component/subassembly i_h at level h, $j=1,\ldots,n(i_h)$. Let $m(i_h, k_{h-1})$ denote the number of components/subassemblies i_h required per subassembly k_{h-1}. At each stage of component/subassembly at each level, the number of units required to assemble Q units of final product will be manufactured. Let the proportion of defectives be $p(i_h,j)$. The total production quantity in Eq. (2.2), $M(i_h)$, of component/subassembly i_h at level h is illustrated as follows:

At the level of the main assembly, level 1, the total production quantity is

$$M(1_1)=\sum_{j=1}^{n(1_1)}\frac{Q}{\prod_{g=j}^{n(1_1)}[1-p(1_1,g)]}.$$

At level 2, the total production quantity is

$$M(i_2)=\frac{m(i_2,k_1)Q}{\prod_{s=1}^{n(i_1)}[1-p(i_1,s)]}\sum_{j=1}^{n(i_2)}\frac{1}{\prod_{g=j}^{n(i_2)}[1-p(i_2,g)]}.$$

Then, the total production quantity at level $h-1$ is

$$M(i_{h-1})=\prod_{r=2}^{h-1}\left\{\frac{m(i_r,k_{r-1})Q}{\prod_{s=1}^{n(i_{r-1})}[1-p(i_{r-1},s)]}\right\}\times\sum_{j=1}^{n(i_{h-1})}\frac{1}{\prod_{g=j}^{n(i_{h-1})}[1-p(i_{h-1},g)]}.$$

Hence, at level h, the total production quantity can be obtained as

$$M(i_h)=\prod_{r=2}^{h}\left\{\frac{m(i_r,k_{r-1})Q}{\prod_{s=1}^{n(i_{r-1})}[1-p(i_{r-1},s)]}\right\}\times\sum_{j=1}^{n(i_h)}\frac{1}{\prod_{g=j}^{n(i_h)}[1-p(i_h,g)]}.$$

References

Ben-daya, M., 1999. Multi-stage lot-sizing models with imperfect processes and inspection errors. Production Planning & Control 10, 118–126.

Chan, L.K., Cheng, S.W., Spiring, F.A., 1998. A new measure of process capability: Cpm. Journal of Quality Technology 20, 162–175.

Chen, W.H., 1996. The effects of SPC on the target of process quality improvement. Journal of Quality Technology 28, 224–232.

Deleveaux, V.J., 1997. Analytical models for the justification of investment in continuous quality improvement. Ph.D. Thesis, Penn State.

Djamaludin, I., Murthy, D.N.P., Wilso, R.J., 1994. Quality control through lot sizing for items sold with warranty. International Journal of Production Economics 33, 97–107.

Eroglu, A., Ozdemir, G., 2007. An economic order quantity model with defective items and shortages. International Journal of Production Economics 106, 544–549.

Gill, P.E., Murray, W., Wright, M.H., 1981. Practical Optimization. Academic Press, London.

Goyal, S.K., Gunasekaran, A., 1990. Effect of dynamic process quality control on the economics of production. International Journal of Operations & Production Management 10, 69–77.

Grace, A., Branch, M.A., 1996. Optimization Toolbox. The Mathworks, Inc., Natick, MA.

Gunasekaran, A., 1995. Optimal investment and lot-sizing policies for improved productivity and quality. International Journal of Production Research 33, 261–278.

Gupta, M., Campbell, V.S., 1995. The cost of quality. Production and Inventory Management Journal 36, 43–49.

Hong, J., Hayya, J.C., 1995. Joint investment in quality improvement and setup reduction. Computer Operations Research 22, 567–574.

Hong, J., Xu, S.H., Hayya, J.C., 1993. Process quality improvement and setup reduction in dynamic lot-sizing. International Journal of Production Research 31, 2693–2708.

Hong, J.D., 1997. Optimal production cycles, procurement schedules, and joint investment in an imperfect production system. European Journal of Operational Research 100, 413–428.

Hoque, M.A., Goyal, S.K., 2005. On lot streaming in multistage production systems. International Journal of Production Economics 95, 195–202.

Hwang, H., Kim, D.B., Kim, Y.D., 1993. Multiproduct economic lot size models with investment costs for setup reduction and quality improvement. International Journal of Production Research 31, 691–703.

Kececioglu, D., 1991. Reliability Engineering Handbook. Prentice-Hall, Inc., New Jersey.

Lee, H.-H., 2005a. Quality Management. Tsang-Hai, Inc., ROC.

Lee, H.-H., 2005b. Cost/benefit model for investments in inventory and preventive maintenance in imperfect production system. Computers and Industrial Engineering 48, 55–68.

Lee, H.-H., 2005c. Cost model with rolled throughput yield for six sigma management. Journal of Quality 33, 231–237.

Lee, H.-H., 2006. Investment model development for repetitive inspections and measurement equipment in imperfect production systems. International Journal of Advanced Manufacturing Technology 31, 278–282.

Lee, H.-H., Chandra, M.J., Deleveaux, V.J., 1997. Optimal batch size and investment in multistage production systems with scrap. Production Planning & Control 8, 586–596.

Leschke, J.P., Weiss, E.N., 1997. Multi-item setup-reduction investment—allocation problem with continuous investment-cost functions. Management Science 43, 890–894.

O'Connor, P.D.T., 1991. Practical Reliability Engineering. Wiley, New York.

Papachristos, S., Konstantaras, I., 2006. Economic ordering quantity models for items with imperfect quality. International Journal of Production Economics 100, 148–154.

Porteus, E.L., 1986a. Optimal lot sizing, process quality improvement and setup cost reduction. Operations Research 34, 137–144.

Porteus, E.L., 1986b. Investing in new parameter values in the discounted EOQ model. Naval Research Logistics Quarterly 33, 39–48.

Salameh, M.K., Jaber, M.Y., 2000. Economic production quantity model for items with imperfect quality. International Journal of Production Economics 64, 59–64.

Sana, S.S., Goyal, S.K., Chaudhuri, K., 2007. An imperfect production process in a volume flexible inventory model. International Journal of Production Economics 105, 548–559.

Sheu, S.H., Chen, J.A., 2004. Optimal lot-sizing problem with imperfect maintenance and imperfect production. International Journal of System Science 35, 69–77.

Trevino, J., Hurley, B.J., Friedrich, W., 1993. A mathematical model for the economic justification of setup time reduction. International Journal of Production Research 31, 191–202.

Zequeira, R., Prida, B., Valdes, J.E., 2004. Optimal buffer inventory and preventive maintenance for an imperfect production process. International Journal of Production Research 42, 959–974.