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AN EOQ MODEL WITH CONTROLLABLE SELLING RATE

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According to the marketing principle, a decision maker may control demand rate through selling price and the unit facility cost of promoting transaction. In fact, the upper bound of willing-to-pay price and the transaction cost probably depend upon the subjective judgment of individual consumer in purchasing merchandise. This study therefore attempts to construct a bivariate distribution function to simultaneously incorporate the willing-to-pay price and the transaction cost into the classical economic order quantity (EOQ) model. Through the manipulation of the constructed bivariate distribution function, the demand function faced by the supplier can be expressed as a concrete form. The proposed mathematical model mainly concerns how to determine the initial inventory level for each business cycle, so that the profit per unit time is maximized by means of the selling price and the unit-transaction cost to control the selling rate. Furthermore, the sensitivity analysis of optimal solution is performed and the implication of this extended inventory model is also discussed.

Keywords: Transaction cost; demand function; inventory model; economic order quantity model.

1. Introduction

The EOQ (economic order quantity) model is one of the earliest developed, and most widespread, quantitative analysis models in inventory management. The EOQ model can be generally categorized into two types: the controllable (Type 1) or uncontrollable demand rate (Type 2) (i.e. the demand quantity per unit time).

Type 1 EOQ model is based on a constant demand rate that the inventory decision makers need to determine the ordering frequency and quantity in a business cycle, so as to minimize the inventory costs per unit time. Many inventory

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researchers therefore supplement this type model with some assumptions in order to enhance the practical applications. For example, Hwang (1999) considered the deteriorating product. Ouyang *et al.* (1999) deliberated the lead-time cost between the period of order receiving and order delivering. Chen and Chuang (1999) proposed an inventory model for focusing on the permissible delay in payment. Considine and Heo (2000) developed an inventory model for considering the cost of allowable shortage. Horowitz (2000) discussed the inflation uncertainty in his proposed EOQ model. Recently, Sun and Queyranne (2002) asserted the net present value in production and inventory to analyze the EOQ models. Furthermore, the extensions of previous studies so-called the integrated inventory models are considered (e.g. Chung and Tsai, 2001; Dye and Ouyang, 2005; Dye *et al.*, 2007; Teng *et al.*, 2005).

Contrarily, Type 2 EOQ model is often assumed that the demand rate is controllable through some decision variables. (cf. Chen and Lai, 1992; Chen, 1998; Chen and Lin, 2002; Chen and Chu, 2003; Chen and Chu, 2001; Ho *et al.*, 2007). The framework proposed in this study, is part of Type 2 model distinguishing from others through simultaneously considering the unit-selling price p_s and the transaction costs of suppliers e_s as the decision variables which would influence the demand rate r.

The term "transaction cost" originates from terminology in accounting that refers to the cost of providing or concluding some goods or service through the market in the transaction process (Liang and Huang, 1998). In recent years, the concept of transaction cost has been widely applied to marketing research, and regarded as the main determinants affecting consumer behaviors. Anderson (1985), John and Weitz (1989), Stump and Heide (1996), Girlich (2003), and Chen *et al.* (2006) discussed the transaction costs in finance and inventory management. Other empirical research found that transaction costs would affect the consumer choice behavior, such as the store choice (Crafton, 1979; Kim and Park, 1997; Bell *et al.*, 1998), the offline versus online shopping choice (Greenfield Online, 2000), and the website choice (Forrester Research, 1998). Tyagi (2004) further examined the effects of reducing consumer transaction costs by market-level technological advances, especially for internet shopping. Till date, however, the transaction cost of supplier is still not mentioned even though it is an important consideration for transaction costs.

According to the economic theory, the economic gains accrue to consumers and suppliers when they engage in transaction. The gains of consumers are the difference between the price they are willing to pay (or reservation price) and the actual price. That is called the consumer surplus (i.e. $p - (p_s + x)$, where p is the highest price that the consumers are willing to pay, p_s is unit-selling price and x is the transaction cost of consumer). The gains of suppliers are the difference between the price they actually receive and the price they are willing to supply. That is called the supplier surplus (i.e. $p_s - (c + e_s)$, where c is unit cost and e_s is the transaction cost of supplier).

In this study, the unit-selling price p_s and the transaction cost of supplier e_s are regarded as the decision variables affecting the demand rate r, and further used to construct a novel EOQ model, in which the consumer's average transaction cost μ_x depends upon the transaction cost e_s invested by the supplier. In order to facilitate product delivery to the consumer, the supplier usually attempts to improve the delivery efficiency by investing in facility or providing extra services, such as parking lots near the retailing shops, home delivery, extensively retailing shops, or some manners that can absorb the consumer's transaction cost (e.g. the time cost of acquiring some good). It represents the part of consumer's transaction cost that may be transferred to the supplier. Therefore, it is reasonable when the transaction cost e_s burdened by the supplier is greater; the average transaction cost $\mu_x(e_s)$ that the consumer needs to pay would be lower.

From the supplier's perspective, the supply function for a certain product can be formulated according to the bivariate distribution function of consumer's willing-to-pay price, p, and the unit-transaction cost, x. Through statistical sampling technique, the realistic data of these combinations of consumers' distribution variables (p, x) can be obtained, and then a more concrete product supply function can be further constructed to serve to analyze the EOQ problem.

Through the mathematical deduction for the transaction costs of consumer and supplier, this study would therefore reveal a novel extension for the classical inventory management and further refine the classical EOQ model for practical application.

2. Notations and Assumptions

2.1. *Notations*

x: The transaction cost paid by a consumer to acquire a unit of product. It may consist of the delivery cost, which is transferred from the supplier to the consumer, the time cost resulted from a potentially deferred delivery, and the lead-time cost after completing the transaction. Since different consumers would pay different transaction cost, x can be regarded as an independent variable, and its mean value and standard deviation of x are notated as μ_x and σ_x, respectively.

p: The upper bound of willing-to-pay price per unit product for consumers. This price includes the selling price p_s and the transaction cost x, when $p \geq x$. Since different consumers have different preference, p is an independent variable, and its mean value and standard deviation of p are notated as μ_p and σ_p, respectively.

$f(y, z)$: The bivariate continuous distribution function of (y, z), where $y = \frac{p-\mu_p}{\sigma_p}$ and $z = \frac{x-\mu_x}{\sigma_x}$. If $y \leq \frac{-\mu_p}{\sigma_p}$ or $z \leq \frac{-\mu_x}{\sigma_x}$, then $f(y, z) = 0$.

p_s: The unit price that the consumer pay to the supplier.

e_s: The unit-transaction cost of the supplier.

A: The setup cost.

c: The unit-purchasing cost of the supplier.

h: The inventory cost per unit product for a unit time.
Q_s: The initial inventory level for an inventory cycle.
N: The potential demand per unit time without considering the price.

2.2. *Assumptions*

In the real life, if the suppliers are willing to invest more (i.e. higher e_s) in their facilities, the expected transaction cost of the consumers, μ_x (where μ_x is a strictly decreasing function of e_s), would be relatively reduced so that the consumers would be easier to engage in transaction for acquiring products. Therefore, this study assumes

$$\mu'_x(e_s) < 0, \quad \mu''_x(e_s) > 0, \qquad \forall\ e_s \tag{2.1}$$

Meanwhile, this study assumes that the suppliers would adopt a cycle purchase-and-sales policy, so the necessary condition for the consumer to purchase a product should satisfy

$$p \geq p_s + x. \tag{2.2}$$

If the inequality (2.2) is satisfied, the potential demand can be transferred into the real demand. The demand function of a product is a relational expression incorporating with the unit price of product p_s and amount of product q, but such the relational expression do not provide an explicit instruction on the relationship between p_s and the number of consumers n who buys the products. However, if each consumer only buys one product, i.e. $q = n$, the above-mentioned statements for the demand function will be equivalent in meaning. In other words, when a consumer pays kp_s and additionally spends transaction cost e_k to buys k unit of products, that can be regarded as k the same consumers who pays the same price p_s and the transaction cost $\frac{e_k}{k}$. For analytical convenience, this study assumes that each consumer buys only one product.

For all above notations, p_s, e_s, and Q_s represent the decision variables for a supplier and the others are the given environment parameters with respect to a supplier.

3. The Selling Rate r and the Consumer Surplus CS

From (2.2), the necessary condition for a consumer to make a purchase decision is

$$\sigma_p y + \mu_p = p \geq p_s + x = p_s + \sigma_x z + \mu_x, \tag{3.1}$$

where $\mu_x = \mu_x(e_s)$ (cf. Fig. 1). Thus the consumer surplus can be expressed as $(\sigma_p y + \mu_p) - (p_s + \sigma_x z + \mu_x)$. When (p_s, e_s) is determined by a supplier such that the point (y, z) corresponding to a consumer falls into the area of R in Fig. 1, a potential consumer would become a real consumer and then purchases the product. Consequently, the selling rate of the product r is the demand volume N times the

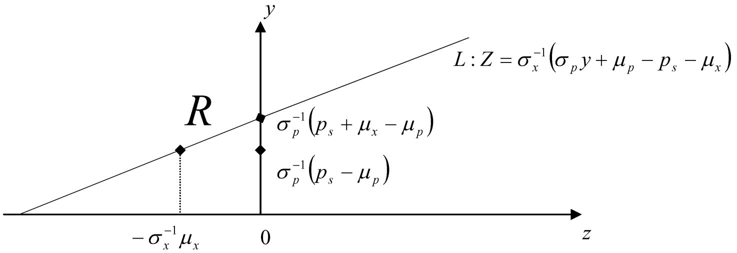

Fig. 1. The relationship between selling rate $r = N \cdot \iint_R f$ and zero consumer surplus line L.

integral of the bivariate distribution function f in area R, that is

$$\begin{aligned} r(p_s, e_s) &= N \cdot \iint_R f(y,z)dzdy \\ &= N \cdot \int_{\sigma_p^{-1}(p_s-\mu_p)}^{\infty} \left[\int_{-\sigma_x^{-1}\mu_x}^{\sigma_x^{-1}(\sigma_p \cdot y + \mu_p - p_s - \mu_x)} f(y,z)dz \right] dy, \end{aligned} \tag{3.2}$$

where $\mu_x = \mu_x(e_s)$.

By Eq. (3.1), the relationship between selling rate $r = N \cdot \iint_R f$ and zero consumer surplus line L can be depicted as Fig. 1, where the line L is the lower boundary of the area R.

Thus, the consumer surplus is

$$\begin{aligned} CS(p_s, e_s) = N \cdot & \int_{\sigma_p^{-1}(p_s-\mu_p)}^{\infty} \\ & \times \left[\int_{-\sigma_x^{-1}\mu_x}^{\sigma_x^{-1}(\sigma_p \cdot y + \mu_p - p_s - \mu_x)} (\sigma_p y + \mu_p - p_s - \mu_x - \sigma_x z) f(y,z)dz \right] dy. \end{aligned} \tag{3.3}$$

Afterward, taking the partial derivatives from Eq. (3.2) with respect to p_s and e_s to yield

$$\frac{1}{N} \cdot \frac{\partial r}{\partial p_s} = -\sigma_x^{-1} \int_{\sigma_p^{-1}(p_s-\mu_p)}^{\infty} f(y, \sigma_x^{-1}(\sigma_p y + \mu_p - p_s - \mu_x))dy \tag{3.4}$$

and

$$\frac{1}{N} \cdot \frac{\partial r}{\partial e_s} = -\left[\sigma_x^{-1} \int_{\sigma_p^{-1}(p_s-\mu_p)}^{\infty} f(y, \sigma_x^{-1}(\sigma_p y + \mu_p - p_s - \mu_x))dy \right] \mu_x'(e_x). \tag{3.5}$$

Combining (3.4) and (3.5) to become

$$\frac{\partial r}{\partial e_s} = \mu_x'(e_s) \frac{\partial r}{\partial p_s}. \tag{3.6}$$

4. Static State Model Analysis

Since the inventory cost per unit time for the product

$$= \frac{\text{the cycle inventory cost}}{\text{the length of a cycle}} = \frac{\mathrm{A} + h\left(\frac{Q_s}{2}\right) \cdot \frac{Q_s}{r}}{\frac{Q_s}{r}},$$

the maximal profit per unit time Π^* is

$$\Pi^* = \operatorname*{Max}_{p_s, e_s, Q_s} \Pi, \quad \text{where}$$

$$\Pi(p_s, e_s, Q_s) = r(p_s, e_s)[p_s - e_s - c] - \left[\frac{Ar(p_s, e_s)}{Q_s} + h\frac{Q_s}{2}\right]. \tag{4.1}$$

Equation (4.1) is a general form of the classical EOQ model. It means that if p_s and e_s are determined, $r(p_s, e_s)$ can be obtained by Eq. (3.2), and the classical EOQ model can also be rearranged as

$$\operatorname*{Min}_{Q_s} \left[\frac{Ar(p_s, e_s)}{Q_s} + h\frac{Q_s}{2}\right]. \tag{4.2}$$

Considering all probable combination of (p_s, e_s) which satisfies $r(p_s, e_s) = \bar{r}$, Eq. (4.2) is equivalent to solve the following problem:

$$\operatorname*{Max}_{\bar{r}} g(\bar{r}), \tag{4.3}$$

where

$$g(\bar{r}) = \operatorname*{Max}_{\substack{p_s,\, e_s,\, Q_s \\ r(p_s,\, e_s) = \bar{r}}} \left[\bar{r} \cdot (p_s - e_s - c) - \left(\frac{A\bar{r}}{Q_s} + h\frac{Q_s}{2}\right)\right]. \tag{4.4}$$

In fact, if (p_s^*, e_s^*, Q_s^*) is the optimal solution of Eq. (4.1), then r^* would be the optimal solution of Eq. (4.3), where

$$r^* = r(p_s^*, e_s^*) \tag{4.5}$$

As $\bar{r}$ is given, using the relation of $r(p_s, e_s) = \bar{r}$, e_s can be considered as a function of p_s. That is $r(p_s, e_s(p_s)) = \bar{r}$.

According to $r(p_s, e_s(p_s)) = \bar{r}$ and Eq. (3.6),

$$\frac{de_s(p_s)}{dp_s} = -\left.\frac{\frac{\partial r(p_s, e_s)}{\partial p_s}}{\frac{\partial r(p_s, e_s)}{\partial e_s}}\right|_{e_s = e_s(p_s)} = -\frac{1}{\mu_x'(e_s)} \tag{4.6}$$

Lemma 4.1. (The optimal solution of Eq. (4.4)) *Given* $\bar{r}$, *if* $(\bar{p}_s, \bar{e}_s, \bar{Q}_s)$ *is the optimal solution of Eq.* (4.4), *then* $\mu_x'(\bar{e}_s) = -1, \bar{Q}_s = \sqrt{\frac{2A\bar{r}}{h}}$, *and* $r(\bar{p}_s, \bar{e}_s) = \bar{r}$.

Proof. Let $F(p_s, Q_s)$ be the objective function of (4.4), i.e.

$$F(p_s, Q_s) = \left[\bar{r} \cdot (p_s - e_s(p_s) - c) - \left(\frac{A\bar{r}}{Q_s} + h\frac{Q_s}{2}\right)\right],$$

and then the necessary conditions of the optimal solution of Eq. (4.4) are

$$\begin{cases} 0 = \dfrac{\partial F(p_s, e_s)}{\partial p_s} = \bar{r}\left(1 - \dfrac{de_s}{dp_s}\right) & (4.7) \\ 0 = \dfrac{\partial F(p_s, e_s)}{\partial Q_s} = \dfrac{A\bar{r}}{Q_s^2} - \dfrac{h}{2}. & (4.8) \end{cases}$$

Since

$$\begin{bmatrix} \frac{\partial^2 F(p_s Q_s)}{\partial p_s^2} & \frac{\partial^2 F(p_s Q_s)}{\partial p_s \partial Q_s} \\ \frac{\partial^2 F(p_s Q_s)}{\partial Q_s \partial p_s} & \frac{\partial^2 F(p_s Q_s)}{\partial Q_s^2} \end{bmatrix} = \begin{bmatrix} -\bar{r}\frac{d^2 e_s}{d_s^2} & 0 \\ 0 & \frac{-2A\bar{r}}{Q_s^3} \end{bmatrix}$$

$$= \begin{bmatrix} -\bar{r}\frac{\mu_x''(e_s)\frac{de_s}{dp_s}}{[\mu_x'(e_s)]^2} & 0 \\ 0 & \frac{-2A\bar{r}}{Q_s^3} \end{bmatrix}$$

$$= \begin{bmatrix} \bar{r}\frac{\mu_x''(e_s)}{[\mu_x'(e_s)]^3} & 0 \\ 0 & \frac{-2A\bar{r}}{Q_s^3} \end{bmatrix} \quad \text{(by Eq. (4.6)), and}$$

$$\bar{r}\frac{\mu_x''(e_s)}{[\mu_x'(e_s)]^3} < 0 \quad \text{(cf. (2.1))},$$

the above matrix is positive determinant. Therefore, Eqs. (4.7) and (4.8) are the necessary and sufficient conditions of the optimal solution of Eq. (4.4).

From Eqs. (3.6), (4.6), and (4.7), $\bar{e}_s$ is shown to satisfy the following equation:

$$\mu_x'(\bar{e}_s) = -1, \quad \forall\, \bar{r}. \tag{4.9}$$

Thus, from Eq. (4.3),

$$\bar{Q}_s = \sqrt{\frac{2A\bar{r}}{h}}, \quad \forall\, \bar{r}. \tag{4.10}$$

□

In Eq. (4.9), $\bar{e}_s$ only depends on the function μ_x and is independent of the value $\bar{r}$. By (4.5) and (4.9), it yields $e_s^* = \bar{e}_s$. Thus,

$$\mu_x'(e_s^*) = -1 \quad \text{i.e. } e_s^* = \mu_x'^{-1}(-1) \tag{4.11}$$

Substituting Eqs. (4.10) and (4.11) into Eq. (4.4) to yield

$$g(\bar{r}) = \bar{r}\left[(p_s(\bar{r}) - e_s^* - c) - \sqrt{\frac{2Ah}{\bar{r}}}\right] = \bar{r}(p_s(\bar{r}) - e_s^* - c) - \sqrt{2Ah\bar{r}}, \tag{4.12}$$

where $p_s(\bar{r})$ is the inverse function of $\bar{r} = r(p_s, e_s^*)$ in Eq. (3.2).

Theorem 4.1. *The optimal demand function faced by the supplier is*

$$\begin{aligned}\bar{r} &= r(p_s, e_s^*)\\ &= N \cdot \int_{\sigma_p^{-1}(p_s-\mu_p)}^{\infty} \left[\int_{-\sigma_x^{-1}\mu^*}^{\sigma_x^{-1}(\sigma_p\cdot y+\mu_p-p_s-\mu^*)} f(y,z)dz\right] dy, \quad \text{where } \mu^* = \mu_x(e_s^*),\end{aligned} \tag{4.13}$$

and its inverse function $p_s = p_s(\bar{r})$ *is a decreasing function of* $\bar{r}$. *In fact,*

$$\begin{aligned}p_s'(\bar{r}) &= -\frac{\sigma_x}{N\int_{\sigma_p^{-1}(p_s-\mu_p)}^{\infty} f(y, \sigma_x^{-1}(\sigma_p y + \mu_p - p_s - \mu^*))dy}\\ &= -\frac{\textit{standard deviation of transaction cost}}{\textit{product quantity of zero consumer surplus}} < 0\end{aligned} \tag{4.14}$$

Proof. Equation (4.13) is followed immediately from (3.2) and (4.11). Differentiating Eq. (4.13) with respect to $\bar{r}$ yield

$$\begin{aligned}1 &= N\left[\int_{\sigma_p^{-1}(p_s-\mu_p)}^{\infty} f(y, \sigma_x^{-1}(\sigma_p y + \mu_p - p_s - \mu^*))dy\right] \cdot (-\sigma_x^{-1}) \cdot p_s'(\bar{r})\\ &= (-\sigma_x^{-1})N\left[\int_L f(y,z)\right] \cdot p_s'(\bar{r}),\end{aligned}$$

where $\int_L f(y,z)$ is the integral of f along with the consumer surplus line L (cf. Fig. 1). □

According to Eq. (4.12), the following Theorems can be established:

Theorem 4.2. (The optimal solution of Eq. (4.4)) *Let* r^* *be the optimal solution of Eq.* (4.4), *then* r^* *satisfies*

$$0 = g'(r^*) = (p_s(r^*) - e_s^* - c) - \sqrt{\frac{Ah}{2r^*}} + r^* \cdot p_s'(r^*) \tag{4.15}$$

and $g'(\bar{r})$ *is a decreasing function in a neighborhood of* r^* *shown in Fig.* 2.

Theorem 4.3. (The optimal solution of Eq. (4.1)) *Let* $(\bar{p}_s^*, \bar{e}_s^*, \bar{Q}_s^*)$ *be the optimal solution of Eq.* (4.1) *and* r^* *be determined by* (4.15), *then* $e_s^* = \mu_x'^{-1}(-1), Q_s^* = \sqrt{\frac{2Ar^*}{h}}$ *and* $r(p_s^*, \mu_x'^{-1}(-1)) = r^*$.

Proof. To link up (4.1), Lemma 4.1, and Theorem 4.2, the desired results can be obtained. □

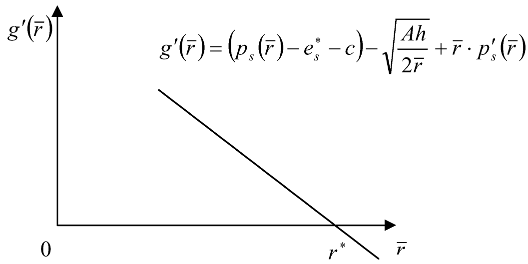

Fig. 2. The determination of the optimum solution r^*.

5. Sensitivity Analysis (Comparative Static State)

5.1. *The effects of changing A, h, or c*

From Eqs. (4.12) and (4.14), the function $p_s(\bar{r})$ will not be affected by changing parameters A, h or c. On the other hand, from Eq. (4.15), if A, h, or c increases, the function $g'(\bar{r})$ will be downward, and the original optimal selling rate r_0^* will decrease to r_n^* (see Fig. 3), so that the optimal selling price p_s^* will increase, and lead the optimal consumer surplus to be decreased (cf. Eq. (3.3) with $e_s = e^*$). Meanwhile, from Eq. (4.10), the optimal initial inventory level Q_s^* would be decreased.

It indicates that if the sales managers intend to transform the partial consumer surplus into profit by discrimination pricing from the different type consumers, the possibility of success would be decreased.

5.2. *The effects of increasing μ_p (the mean value of price ceiling which consumers are willing to pay)*

When other parameters except μ_p are fixed and $\bar{r}$ is determined, the partial differential of Eq. (4.13) with respect to μ_p would be

$$0 = \frac{\partial \bar{r}}{N \partial \mu_p} = \sigma_x^{-1}\left(1 - \frac{\partial p_s}{\partial \mu_p}\right)\int_{\sigma_p^{-1}(p_s-\mu_p)}^{\infty} f(y, \sigma_x^{-1}(\sigma_p y + \mu_p - p_s - \mu^*))dy,$$

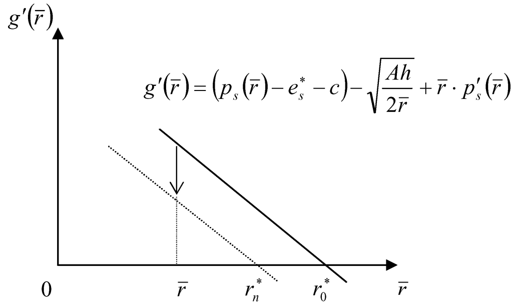

Fig. 3. The effects of changing A, h, or c.

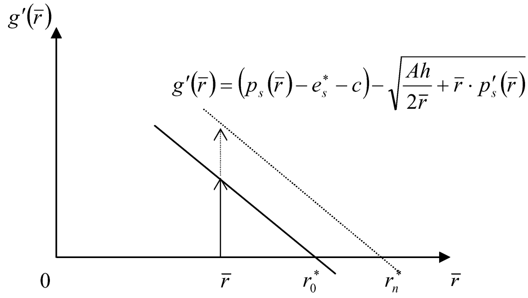

Fig. 4. The effects of increasing μ_p.

and hence $\frac{\partial p_s}{\partial \mu_p} = 1$.

$$\text{Equation (4.12) represents that if } \mu_p \text{ changes, the change of } p_s \text{ should be equal to the change of } \mu_p \text{ in order to remain } \bar{r} \text{ unchangeably. Furthermore, by Eq. (4.14), since } p_s - \mu_p \text{ is unchanged, } p_s'(\bar{r}) \text{ would also keep unchanged.} \tag{5.1}$$

From Eq. (5.1) and Fig. 2, it shows that: for a given $\bar{r}$, if μ_p and $p_s(\bar{r})$ increase, and $p_s'(\bar{r})$ remains unchanged, function $g'(\bar{r})$ will move upwards and the optimal selling rate r_0^* will shift to r_n^*. That is shown in Fig. 4, and thus the optimal initial inventory level Q_s^* will increase (cf. Eq. (4.10)).

Using the inequality $r_n^* > r_0^*$ and Eq. (4.13), the condition $\frac{\partial p_s^*}{\partial \mu_p} < 1$ will hold.

Such the result indicates that the price decision makers need to grasp the following implication:

Other things being equal, when the mean value of price ceiling μ_p which consumers are willing to pay is rising, the selling rate would rise, but the increasing margin of optimal price Δp_s^* would not excess the increasing margin of the average price ceiling $\Delta \mu_p$.

5.3. *The effects of increasing σ_x (the difference level of transaction costs burdened by consumers)*

When other parameters except σ_x are fixed, for a given $\bar{r}$, the partial derivative of Eq. (4.12) with respect to σ_x is

$$
\begin{aligned}
0 &= \frac{1}{N} \cdot \frac{\partial \bar{r}}{\partial \sigma_x} \\
&= \int_{\sigma_p^{-1}(p_s - \mu_p)}^{\infty} \Bigg[f(y, \sigma_x^{-1}(\sigma_p y + \mu_p - p_s - \mu^*)) \\
&\quad \cdot \left(-\sigma_x^{-1} \frac{\partial p_s}{\partial \sigma_x} - (\sigma_p y + \mu_p - p_s - \mu^*)\sigma_x^{-2} \right) \Bigg] dy.
\end{aligned}
$$

Consequently,

$$\frac{\partial p_s(\bar{r})}{\partial \sigma_x} = \frac{-\int_L (\sigma_p y + \mu_p - p_s - \mu^*) f}{\sigma_x \int_L f} \tag{5.2}$$

Next, differentiating Eq. (4.14) with respect to σ_x to yield

$$\frac{1}{p_s'(\bar{r})} \cdot \frac{\partial p_s'(\bar{r})}{\partial \sigma_x} = -\frac{1}{\sigma_x} + \frac{\frac{\partial}{\partial \sigma_x} \int_L f}{\int_L f}.$$

Since $p_s'(\bar{r}) < 0$ (see Eq. (4.14)), then

$$\frac{\partial p_s'(\bar{r})}{\partial \sigma_x} < 0 \quad \text{if and only if } \frac{\int_L f}{\sigma_x} < \frac{\partial}{\partial \sigma_x} \int_L f. \tag{5.3}$$

Therefore, if $\sigma_x \to 0$, any value of y will lead to $[\sigma_x^{-1}(\sigma_p y + \mu_p - p_s - \mu^*)] \to \infty$, and hence $f(y, \sigma_x^{-1}(\sigma_p y + \mu_p - p_s - \mu^*)) \to 0$ and $\int_L f \to 0$.

Applying the above property to the mean value theorem, it can be shown that:

if $\displaystyle\int_L f$ is a convex function of σ_x (i.e. $\displaystyle\frac{\partial^2}{\partial \sigma_x^2} \int_L f > 0$), then the inequality Eq. (5.3) is true, and hence $\displaystyle\frac{\partial p_s'(\bar{r})}{\partial \sigma_x} < 0$. On the other hand, if $\displaystyle\int_L f$ is a concave function of σ_x $\left(\text{i.e.} \displaystyle\frac{\partial^2}{\partial \sigma_x^2} \int_L f < 0\right)$, then $\displaystyle\frac{\partial p_s'(\bar{r})}{\partial \sigma_x} > 0$. (5.4)

Afterward, this study further assumes that the supplier will adopt the optimal solution of the transaction cost $\mu^*, \mu^* = \mu_x(e_s^*)$. According to Eqs. (5.3) and (5.4), and the same argument about Fig. 4, the following two cases can be inferred:

Case (1): If the difference between the average transaction cost of the zero consumer surplus group and that of the whole consumer group, $\int_L (p - p_s - \mu^*) f$, is negative, and the product quantity of the zero consumer surplus group $\int_L f$ is a concave function of σ_x, then $\frac{\partial r^*}{\partial \sigma_x} > 0$.

Case (2): If the difference between the average transaction cost of the zero consumer surplus group and that of the whole consumer group, $\int_L (p - p_s - \mu^*) f$, is positive, and the product quantity of the zero consumer surplus group $\int_L f$ is a convex function of σ_x, then $\frac{\partial r^*}{\partial \sigma_x} < 0$.

The time that a consumer spends on acquiring merchandise is one of the important factors of the transaction cost. Such the time cost would vary in accordance with the difference among consumers. In other words, when the geographical distance is closer or the communications and transportation is more convenient, or the retail sales store is near the consumers' life circle, the difference of time cost for transaction, σ_x, would be smaller.

Based on the above inference, when the difference of transaction cost σ_x is changing, the sales manager can therefore know how to adjust the optimal solutions of p^*, O^*, r^* according to the corresponding plus/minus sign of $\int_L (p - p_s - \mu^*) f$ and $\frac{\partial^2}{\partial \sigma_x} \int_L f$.

5.4. *The effects of increasing σ_p (the difference level of price ceiling that consumer is willing to pay)*

When other parameters except σ_x are fixed, for a given $\bar{r}$, the partial derivative of Eq. (4.12) with respect to σ_p is

$$0 = \frac{1}{N} \cdot \frac{\partial \bar{r}}{\partial \sigma_p} = \frac{1}{\sigma_x} \int_{\sigma_x^{-1}(p_s-\mu_p)}^{\infty} \left(y - \frac{\partial p_s}{\partial \sigma_p} \right) f(y, \sigma_x^{-1}(\sigma_p y + p_s - \mu_p - \mu^*))dy,$$

and then

$$\frac{\partial p_s(\bar{r})}{\partial \sigma_p} = \frac{\int_{\sigma_x^{-1}(p_s-\mu_p)}^{\infty} y f(y, \sigma_x^{-1}(\sigma_p y + p_s - \mu_p - \mu^*))dy}{\int_{\sigma_p^{-1}(p_s-\mu_p)}^{\infty} f(y, \sigma_x^{-1}(\sigma_p y + p_s - \mu_p - \mu^*))dy}$$

$$= \frac{\int_l (p-\mu_p) f}{\sigma_p \int_L f} \quad \left(\text{because } y = \frac{p-\mu_p}{\sigma_p} \right) \tag{5.5}$$

Consequently, the necessary and sufficient condition for $\dfrac{\partial p_s(\bar{r})}{\partial \sigma_p} > 0$ is that: the mean value of $(p - \mu_p)$ on the group with zero consumer surplus $\displaystyle\int_L (p-\mu_p) f$ is positive. (5.6)

Also, by $f\left(y, \frac{-\mu_x}{\sigma_x}\right) = 0$ (cf. the definition of f), Eq. (4.14) can be rewritten as:

$$\frac{\partial p_s'(\bar{r})}{\partial \sigma_p} = \frac{\sigma_x}{N} \cdot \frac{\left[\frac{\partial}{\partial \sigma_p} \int_{\sigma_p^{-1}(p_s-\mu_p)}^{\infty} f(y, \sigma_x^{-1}(\sigma_p y + p_s - \mu_p - \mu^*))dy \right]}{\left[\int_{\sigma_p^{-1}(p_s-\mu_p)}^{\infty} f(y, \sigma_x^{-1}(\sigma_p y + p_s - \mu_p - \mu^*)) \right]^2}$$

$$= \frac{1}{N} \cdot \frac{\frac{\partial}{\partial \sigma_p} \int_L f}{\left[\int_L f \right]^2}. \tag{5.7}$$

Similarly, for $\mu^* = \mu_x(e_s^*)$, from Eqs. (5.6) and (5.7), and the same argument about Fig. 4, the following two cases can be inferred:

Case (3): If the difference between the average willing-to-pay price of $(p - \mu_p)$ of the zero consumer surplus group and that of the whole consumer group, $\int_L (p-\mu_p) f$, is positive, and the product changing rate of the zero consumer surplus group $\frac{\partial}{\partial \sigma_p} \int_L f$ is positive, then $\frac{\partial r^*}{\partial \sigma_p} > 0$ and $\frac{\partial Q^*}{\partial \sigma_p} > 0$.

Case (4): If the difference between the average willing-to-pay price of $(p - \mu_p)$ of the zero consumer surplus group and that of the whole consumer group, $\int_L (p-\mu_p) f$ is negative, and the changing rate of the product quantity of the group with zero consumer surplus $\frac{\partial}{\partial \sigma_p} \int_L f$ is negative, then $\frac{\partial r^*}{\partial \sigma_p} < 0$ and $\frac{\partial Q^*}{\partial \sigma_p} < 0$.

In practice, if the greater part of consumers have need for some function of a product, and have the adequate purchasing experience (e.g. the experience in purchasing the necessaries), the difference in valuation, σ_p, would be smaller. Contrarily, if the function of a product can only satisfy a few consumers (e.g. luxury goods), the difference in valuation would be greater due to the consumers cannot realize the supply function of a new product. However, if a product has been available in the market for a span time, due to the diffusion effect of price discovery, the consumers will re-evaluate the product that may lead to a diminishing difference in valuation. In other words, even though the consumers buy the same product, the difference in valuation for the consumers will decrease in accordance with the length of available time in market.

Based on this, when the difference of valuation is changing, the consumers can therefore know how to adjust the optimal solutions according to the corresponding plus/minus sign of $\int_L (p-\mu_p)f$ and $\frac{\partial}{\partial\sigma_p}\int_L f$ in the zero-consumption group.

5.5. *The effects of increasing the marginal transaction cost μ'_x*

When other conditions are unchanged, if function μ' increases from its original function μ'_o to a new function μ'_n, that is

$$\mu'_o(e) < \mu'_x(e), \quad \forall\ e. \tag{5.8}$$

By Eq. (2.1), Eq. (5.8), and Fig. 5, it can be conducted as

$$e_n^* < e_0^*, \tag{5.9}$$

and

$$\begin{aligned}\mu_n^* - \mu_o^* = \mu_n(e_n^*) - \mu_o(e_0^*) &= \left[\int_0^{e_n^*} \mu'_n(e)de + \mu_0\right] - \left[\int_0^{e_0^*} \mu'_o(e)de + \mu_0\right] \\ &= \int_0^{e_n^*} \mu'_n(e)de - \int_0^{e_0^*} \mu'_o(e)de.\end{aligned} \tag{5.10}$$

It means that the area of $ABCDe_0^*e_n^* > 0$ (see Fig. 5).

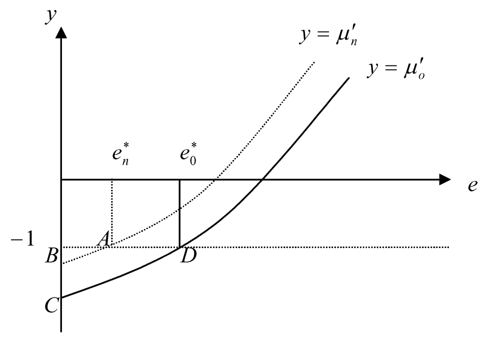

Fig. 5. The effects of increasing the marginal transaction cost μ'_x.

For given a $\bar{r}$, the partial derivative of Eq. (4.12) with respect to μ^* is

$$0 = \frac{1}{N} \cdot \frac{\partial \bar{r}}{\partial \mu^*}$$

$$= \left[\int_{\sigma_p^{-1}(p_s-\mu_p)}^{\infty} -\sigma_x^{-1} f(y, \sigma_x^{-1}(\sigma_p y + \mu_p - p_s - \mu^*))dy\right] \left(\frac{\partial p_s(\bar{r})}{\partial \mu^*} + 1\right).$$

Hence,

$$\frac{\partial p_s(\bar{r})}{\partial \mu^*} = -1 \quad (\text{i.e. } \Delta p_s(\bar{r}) + \Delta\mu^* = 0). \tag{5.11}$$

It implies that

$$\frac{\partial(p_s(\bar{r}) - e_s^*)}{\partial \mu^*} = \left(-1 + \frac{1}{-\frac{\partial \mu^*}{\partial e_s^*}}\right) > 0 \quad \text{if and only if } \frac{\partial \mu^*}{\partial e_s^*} < -1. \tag{5.12}$$

From Eq. (5.11), if $\bar{r}$ is unchanged, p_s has to decrease to keep $p_s + \mu^*$ unchanged since μ^* increases. Therefore, the denominator on the right-hand side of Eq. (4.14) would be increased, and

$$\frac{\partial p'_s(\bar{r})}{\partial \mu^*} > 0. \tag{5.13}$$

According to Eqs. (5.12) and (5.13), the following results can be inferred: If the function of the marginal transaction cost μ'_x shift increasingly, then the decrement of the transaction cost e_s^* of supplier is smaller than that of the average transaction cost μ^* of consumer (i.e. if μ'_x increases, $\left|\frac{\partial \mu^*}{\partial e_s^*}\right| > 1$). Thus, from Eqs. (5.12) and (5.13), the function $g'(\bar{r})$ in Fig. 4 will move upwards so that the optimal selling rate r^* will be increased and the optimal selling price p_s^* is decreased as well as the increase of the optimal initial inventory level Q_s^*. (cf. Eq. (4.10)).

The effects of changing the mean value μ_p, the standard deviation σ_p, and other factors of the willing-to-pay price are summarized in Table 1.

Table 1. The effects of changing the parameters.

Parameters	Parameter change	p_s^*	e_s^*	Q_s^*	r^*	Annotations
A	$A\uparrow$	$\uparrow$	—	#	$\downarrow$	see Fig. 3
h	$h\uparrow$	$\uparrow$	—	$\downarrow$	$\downarrow$	
c	$c\uparrow$	$\uparrow$	—	$\downarrow$	$\downarrow$	
μ_p	$\mu_p\uparrow$	$\Delta p_s^* < \Delta\mu_p$	—	$\uparrow$	$\uparrow$	see Fig. 4
μ'_x	$\mu'_x\uparrow$ in the case: $(\lvert\Delta\mu^*\rvert > \lvert\Delta e^*\rvert)$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	see Fig. 5
σ_p	$\sigma_p\uparrow$ in Case (3)	#	—	$\uparrow$	$\uparrow$	see Sec. 5.4
	$\sigma_p\uparrow$ in Case (4)	$\uparrow$	—	$\downarrow$	$\downarrow$	
σ_x	$\sigma_x\uparrow$ in Case (1)	$\downarrow$	—	$\uparrow$	$\uparrow$	see Sec. 5.3
	$\sigma_x\uparrow$ in Case (2)	#	—	$\downarrow$	$\downarrow$	

Remarks: The sign "—" means steadfast; "↑" means increase; "↓" means decrease, and "#" means the effect of changing the parameter depends on other parameters.

6. Conclusions

Based on the bivariate distribution function of the willing-to-pay price p and unit-transaction cost x of acquiring a product for a consumer, the EOQ model with controllable selling rate is concretely constructed in this study. By the distribution function f, the demand function faced by the suppliers can be expressed in a concrete form. Especially, according to the following two contentions, this proposed model may be regarded as a generalized form of the classical EOQ model.

(1) The proposed EOQ model with controllable selling rate not only takes the quantity and the price of products into account, but also discusses the individually dependent transaction cost to deal with the selling rate. If the selling price p_s and the transaction cost of the supplier e_s are given, the present model will become the classical EOQ model. Thus, one may conclude that the proposed model is one extension of the classical EOQ model.
(2) In practical application, only conducting a sampling survey is needed to estimate the bivariate variables of the willing-to-pay price p and unit-transaction cost of consumers μ_x in the demand function. The optimal demand function faced by the supplier can then be expressed by Theorem 4.2. Using this optimal demand function, the optimal selling rate r^*, the optimal selling price p_s^*, and the initial inventory level Q^* could further be formulated in Theorem 4.3.

In conclusion, the main contribution of this paper is to incorporate a relevant realistic factor such as transaction cost into the classical EOQ model in order to improve the existing inventory control models and to make the inventory model closer to a real market situation. In practice, suppliers can apply this model to determine the optimal inventory level by controlling the optimal selling rate r^*. Therefore, such issue should be considered as an important direction for further research in inventory management.

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