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Decision Support

Incremental analysis for MCDM with an application to group TOPSIS

Hsu-Shih Shih *

Graduate Institute of Management Sciences, Tamkang University, Tamsui, Taipei 25137, Taiwan, ROC

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Abstract

The study aims to exploit incremental analysis or marginal analysis to overcome the drawbacks of ratio scales utilized in various multi-criteria or multi-attribute decision making (MCDM/MADM) techniques. In the proposed 11-step procedure, multiple criteria of alternatives are first reorganized as two categories – benefits and costs – and decision information will be manipulated separately. The performances of alternatives are then evaluated on their incremental benefit–cost ratio, and the rank can be obtained by applying the group TOPSIS (technique for order preference by similarity to ideal solution) model (Shih et al., 2007). Two representations of cost, i.e., a cost index and utility index, are proposed in the model to better-fit real-world situations. In addition, some considerations on costs and input–output relations are also discussed in order to understand the essentials of incremental analysis. In the final part, a case of robot selection demonstrates the suggested model to be both robust and efficient in a group decision-making environment.

Keywords: Incremental analysis; Multi-criteria analysis; Benefit–cost ratio; TOPSIS; Engineering economy; Ratio scale; Preference aggregation

1. Introduction

Incremental or marginal analysis is a crucial tool for evaluating alternatives in an engineering economy. It can be defined as the examination of the differences between two alternatives from the aspect of benefits and costs. By emphasizing alternatives, in an ascending order of costs, decision makers (DMs) decide whether or not differential costs are justified by differential benefits (Newnan et al., 2002). It is also valuable for checking the ratio of differential benefits to differential costs so that the best alternative can finally be detected.

Multi-criteria or multi-attribute decision making (MCDM/MADM) techniques traditionally rank and select alternatives by their composite values or scores in a ratio scale. However, there are two major drawbacks for this evaluation. The first concerns the ranking of alternatives, in which they are ranked from the highest to the lowest value. For example, alternative A_2 with the highest value is chosen from the set of alternatives $\{A_1$

* Tel.: +886 2 8631 3221; fax: +886 2 8631 3214.
E-mail address: hshih@mail.tku.edu.tw

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(with the performance value of 0.85), A_2 (with 0.88), and A_3 (with 0.86)}. For the example set of alternatives, DMs can assume that alternative A_2 is better than alternative A_3, which is better than alternative A_1. However, the ranking may be biased due to relative values or the ratio scale used in the evaluation. Another drawback concerns the choice of alternatives, usually depending on the operation of normalized composite values, between zero and one. Although DMs can say that $A_2 \succ A_3$ (i.e., A_2 is superior to A_3) with a narrow gap of 0.02 between the performed measures, the comparison might have little meaning. Therefore, based on the ratio scale, MCDM approaches are not good for the purpose of ranking and selection and cannot reflect the true dominance of alternatives. For this reason we think the incremental analysis to be a necessary tool to solve the problems and help MCDM ranking and selecting alternatives in a more robust fashion.

There exists a post-selection operation called sensitivity analysis, which attempts to guarantee the evaluation results to be robust. The analysis investigates the change in the optimal solution resulting from a perturbation in a variety of parameters or trade-off rates, even on the weights of criteria or the uncertainty on performance measures (Steuer, 1986; Taha, 2003). Nevertheless, the aim of the analysis provides additional information about the range of parameters of alternatives so that DMs can be cautious in making decisions. It has no approach to finding a best alternative or to give the relative dominance among alternatives. Accordingly, we think about incremental analysis as an effective tool for helping MCDM to hurdle the shortcomings and at least to indicate the raking as robust.

Despite many MCDM techniques, e.g., the analytic hierarchy process (AHP) (Saaty, 1980), can be enhanced with incremental analysis by a benefit–cost ratio, TOPSIS (technique for order preference by similarity to ideal solution) (Hwang and Yoon, 1981) is chosen as the target for the analysis because of its stability and ease of use with cardinal information. The proposed procedure through a group TOPSIS (Shih et al., 2007) is demonstrated with incremental analysis. In addition, other techniques with cardinal information, e.g., simple additive weighting method, can be also extended with similar steps. The suggested concept of incremental analysis is applicable without loss of generality.

The paper is organized as follows: Section 2 offers a literature review about the benefit–cost ratio for AHP as the basis of our development. Some developments of TOPSIS in a group-decision environment are also described to better understand the insides of TOPSIS. Section 3 focuses on the proposed incremental analysis for the group TOPSIS model in a step-by-step fashion. Following that, considerations about cost and input–output are illustrated. In the last section, conclusions and remarks are drawn for future studies.

2. Literature review

The use of MCDM techniques, where multiple criteria/attributes are considered in a simple score model (see MacCrimmon, 1968), can be dated back to four decades. Since then, the theory and applications have been developed significantly. Despite the fact that MCDM has been notably utilized for an alternative evaluation in the past, its use does not result in controlling the budget effectively. Thus, the engineering economy is still recognized as an effective tool for capital investments. Hence, this study attempts to bridge the gap between the two disciplines through incremental analysis, so that MCDM techniques can deal with investments and improve their robustness toward ranking.

In the following, the first part deals with the topic of discrimination of alternatives. Following that, the utilization of benefit–cost ratios in AHP is discussed. The third part discusses the contents of TOPSIS in a group-decision making environment as the basis for our study.

2.1. Discrimination of alternatives

Decision makers always like to know which option is best among several alternatives. For the category of cardinal information on the criteria/attribute of MCDM methods, alternatives are ranked by their cardinal values of performance (Hwang and Yoon, 1981). The previous example has revealed that $A_2 \succ A_3$ with a gap of 0.02 between the performed measures. Since the gap is rather narrow, the differences may not mean much. It is better to conclude that alternatives A_2 and A_3 have little difference. However, DMs would not be able to recognize the situation and may be unwilling to adopt the fact in a complex decision-making environment. Thus, the decision could be inappropriate.

Tzeng et al. (2005) suggest an acceptable advantage as a standard for discriminating any two alternatives by the value of $1/(m-1)$ with m alternatives. It is implicitly assumed that the performance of m alternatives would be uniformly distributed with the range 0–1 for the normalized values. However, the performance measures with the lower values are usually unacceptable so that the assumption would be too loose to make a judgment. In this case, it is necessary to develop a new index for the discrimination of alternatives based on statistical information of the specified alternatives' performance. The work of Tzeng et al. (2005) is just a starting point for further development. From this viewpoint, the representation of order information for decision making, e.g., elimination by aspect (see Hwang and Yoon, 1981), can alleviate the difficulty in making a better choice.

2.2. Benefit–cost ratios for AHP

Benefit–cost ratios in an engineering economy attempt to quantify all positive and negative aspects as monetary values and therefore provide a basis for evaluation of projects. Even though the evaluation has been criticized for its strong assumptions (see French, 1988), it has still been considered a major tool in practice.

There are only a few studies dealing with the benefit–cost ratio for MCDM, and mostly for AHP. Hence, the review can solely be made for AHP. Saaty (1980) is the pioneer for using benefit–cost ratios for AHP where resource-allocation problems are split into cost and benefit hierarchies, and the ratio of composite priorities from the two hierarchies is combined for decision-making. The choice of alternatives is made according to the highest benefit–cost ratio in a descending order with a river-crossing example. Saaty (1990) realizes the importance of marginal benefit to cost ratios or incremental analysis. He suggests that the succeeding alternative (with the next higher cost) be dropped from consideration if the difference in a numerator (i.e., benefit) is negative. However, a comprehensive study regarding the ratio has not been made.

From the standpoint of an engineering economy, Bernhard and Canada (1990) argue that the use of benefit–cost ratios by Saaty (1980) will not necessarily yield an optimal choice, and incremental analysis is therefore required. In addition, to include DM's relative preferences for the increments of benefits vs. increments of costs, an optimal decision can be made depending upon the level of the cutoff ratio. The cutoff ratio is defined as the total amount of costs, in dollars, divided by total amount of benefits, in dollars, of all projects. They also indicate that the ratio is not necessarily equal to one. For the river-crossing example, the ratio is 0.670. However, there are four optimal alternatives corresponding to the ranges of cutoff ratios: 0–0.763, 0.763–1.064, 1.064–1.340, and >1.340, respectively. Since the correct choice is sensitive to the level of the ratio, the DM's relative willingness to incur levels of the ratio should be carefully considered during for the evaluation. How to decide the values of cutoff ratios is not mentioned. It is noted that Wedley et al. (2001) also spotlight the fallacious results by utilizing the ratio of benefit priorities to cost priorities of Saaty's in AHP (1990), and they recommend magnitude adjustments to the ratio for correction. Nevertheless, their work is not related to incremental analysis.

Yang et al. (2004) make some modifications on the work of Bernhard and Canada (1990). Due to the transitivity characteristic of a theorem, they point out that the previous work cannot select a best policy for the river-crossing example. After illustrating some examples, they claim that the incremental benefit/cost analysis is not suitable for the AHP situation. Under such observations, they present that the benefits, costs, opportunities and risks method of Saaty and Cho (2001) will be good for selecting the best project. However, the selection of a reasonable cutoff ratio has not yet been investigated.

Before discussing the cutoff ratio, we would like to point out some confusion in the works of Bernhard and Canada (1990) and Yang et al. (2004). Both try to simply use monetary values to illustrate a cost–benefit analysis with intangible factors. In fact, if the monetary values can be obtained, then we can do the analysis by any technique of engineering economics, eliminating AHP or other MCDM techniques. It appears that they make a simple problem more complex. Nevertheless, we will rank and select the alternatives through MCDM if intangible factors exist. Due to the scaling effect, their results cannot be appropriately compared with each other.

On the other hand, the threshold value for judgment has drawn much attention in the past. Traditionally, the incremental benefit–cost ratio is greater than one in the engineering economy (Blank and Tarquin, 1989). This concept implies that the benefit acquired should be in excess of the cost exhausted. Bernhard and Canada

(1990) define a cutoff ratio so that the value would vary based on the project. The ratio can be roughly considered as an average benefit–cost ratio of all projects. Miller (2001) introduces Bayesian statistics to provide an analytical solution for combining prior information with an engineer's estimates (sample information). In that way he claims that the ratio should exceed one. Because there is not much information about an effective ratio for judgment from the literature, we will use one, just like the logic of engineering economics, as the threshold for judging conservatively in the study.

2.3. TOPSIS in a group-decision environment

TOPSIS is an MCDM technique with cardinal information, a ratio scale, on the criteria/attributes given as AHP (Hwang and Yoon, 1981). It derives from the concept of displaced ideal point from which the compromised solution would have the shortest distance. The ranking of alternatives depends upon the shortest distance from the (positive) ideal solution (PIS) and the farthest from the negative-ideal solution (NIS). TOPSIS simultaneously takes into account the distances to both PIS and NIS, and a preference order is ranked by the composite distances. According to the simulation comparison of Zanakis et al. (1998), TOPSIS has the fewest rank reversals among the eight methods of MCDM. Thus, TOPSIS is chosen as the target technique of development. Interested readers can check the contents of Shih et al. (2007) for more details of TOPSIS.

Because certain groups are constantly making complex decisions within organizations, group decision-making is now considered. To extend TOPSIS to a group-decision environment, a couple of papers have involved the preference aggregation among the group. These papers can be classified as external or internal aggregation as shown in Table 1. The former utilizes some operations to manipulate the alternative ratings and weight ratings, or uses a social welfare function to obtain a final ranking from individual DMs of the group (i.e., outside the TOPSIS procedure). The latter tries to aggregate the preference of individuals within the TOPSIS procedure (i.e., an integrated procedure). Moreover, in the external aggregation class, we can further distinguish the methods into pre-operation (e.g., mean operators for weight rating) and post-operations (e.g., Borda's function for ranking of alternatives), which rely on whether the aggregation is done before or after the TOPSIS procedure. A heterogeneous structure for different groups (Pochampally and Gupta, 2004) is added to the classification of Shih et al. (2007).

There is only cardinal information in the external aggregation class. It appears that external aggregation intends to provide more information to support a complex decision, and the internal aggregation focuses on an integrated decision-making procedure. From a practical viewpoint, MCDM techniques are known for its relief of applications, and the integrated procedure keeps this strength and develops multiple sources of knowledge and experience. As shown in Table 1, our previous work (Shih et al., 2007) for the internal aggre-

Table 1
The preference aggregation for TOPSIS in the group-decision environment

Aggregation method	Target	Note
I. External aggregation		
I.1 Pre-operation (cardinal information)		
1. Weighted sum	Alternative rating on subjective criteria (1–9 scale)	
2. Arithmetical mean	Alternative rating on criteria (fuzzy) weight rating of criteria (fuzzy)	
3. Arithmetical mean	Alternative rating on subjective criteria (fuzzy) weight rating of criteria (fuzzy)	
I.2 Post-operation (ordinal information)		
4. Borda's count	TOPSIS ranking (homogeneous structure)	Shih et al. (2004)
5. Borda's count	TOPSIS ranking (heterogeneous structure)	Pochampally and Gupta (2004)
II. Internal aggregation (cardinal information)		
1. Arithmetic mean	Separation measure	Shih et al. (2007)
2. Geometric mean	Separation measure	Shih et al. (2007)

Note: (1) The table is an extension of Table 3 in Shih et al. (2007).

gation truly manipulates the core part, distance measures, of TOPSIS for group decision, and it deserves the name of "group TOPSIS". Thus, this study deals with the group TOPSIS (Shih et al., 2007) as the object for exploring incremental analysis.

3. Incremental analysis for group TOPSIS

Inspired by the incremental analysis for AHP, we propose an incremental analysis procedure for group TOPSIS. The initial part of the analysis classifies multiple criteria into benefit and cost categories. The procedure for the benefit category is similar to the original TOPSIS (Hwang and Yoon, 1981). The procedure for the cost category is complicated involving cost and utility. Later, incremental analysis will be tackled based on the judgment between differential cost and differential benefit indices. A best alternative can then be identified through the comparisons of alternatives with a judging ratio of one, and the second best one can be discovered for the rest of the alternatives consecutively, and so on. The procedure is organized as 11 steps, which are partitioned into five parts:

- Collecting decision information (Step 1).
- Processing benefit criteria (Steps 2–7).
- Identifying cost criteria (Steps 8 and 9).
- Executing incremental analysis (Step 10).
- Ranking alternatives (Step 11).

The following illustrates the details of the designated 11 steps.

Step 1. Construct a decision matrix $\boldsymbol{D}^k$, $k = 1, \ldots, K$, by each decision maker. The structure of the matrix can be depicted as

$$\boldsymbol{D}^k = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccccc} X_1 & X_2 & \cdots & X_j & \cdots & X_n \end{array} \\ \begin{bmatrix} x_{11}^k & x_{12}^k & \cdots & x_{1j}^k & \cdots & x_{1n}^k \\ x_{21}^k & x_{22}^k & \cdots & x_{2j}^k & \cdots & x_{2n}^k \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{i1}^k & x_{i2}^k & \cdots & x_{ij}^k & \cdots & x_{in}^k \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{m1}^k & x_{m2}^k & \cdots & x_{mj}^k & \cdots & x_{mn}^k \end{bmatrix} \end{array}, \qquad (1)$$

where A_i denotes the ith alternative, $i = 1, \ldots, m$; X_j represents the jth criterion or attribute, $j = 1, \ldots, n$; with quantitative and qualitative data. The element of $\boldsymbol{D}^k$ is x_{ij}^k, which indicates the performance rating of alternative A_i with respect to criterion X_j by DM k, $k = 1, \ldots, K$. Please note that there should be K decision matrices for the K members of the group. Observe that DMs can also set the outcomes of qualitative or intangible criterion for each alternative as discrete values, e.g., 1–10 or other linguistics values, so that the quantitative values will be placed in the above decision matrix (Hwang and Yoon, 1981).

For the convenience of incremental analysis, we must separate all the elements of $\boldsymbol{D}^k$ into two categories of benefit or cost criteria. It is further assumed that there are p criteria for benefits and q criteria for costs where $p + q = n$.

Step 2. Construct the normalized decision matrix $\boldsymbol{R}_{\mathrm{b}}^k$, $k = 1, \ldots, K$, by each DM for p benefit criteria.

For DM k, $k = 1, \ldots, K$, the normalized value r_{ij}^k in the decision matrix $\boldsymbol{R}_{\mathrm{b}}^k$, where the subscript b stands for benefits, can be any transformation of the column of the benefit criteria of $\boldsymbol{D}^k$ with the value keeping $0 \leqslant r_{ij}^k \leqslant 1$.

For vector normalization, the normalized value for criterion j can be

$$r_{ij}^k = \frac{x_{ij}^k}{\sqrt{\sum_{j=1}^n (x_{ij}^k)^2}}, \quad \text{where } i = 1, \ldots, m; \ j = 1, \ldots, p. \tag{2}$$

For a linear normalization, the normalized value for criterion j can be $r_{ij}^k = \frac{x_{ij}^k}{x_j^{k*}}$ or $r_{ij}^k = \frac{x_{ij}^{k-}}{x_j^k}$, where $x_j^{k*} = \max_i\{x_{ij}^k\}$, $x_j^{k-} = \min_i\{x_{ij}^k\}$ for $i = 1, \ldots, m$, $j = 1, \ldots, p$.

Step 3. Assign a weight vector $\boldsymbol{W}_\mathrm{b}^k$, $k = 1, \ldots, K$, for the benefit criteria of each DM.

Each DM will first elicit weights for criteria as w_j^k, where $j = 1, \ldots, p$, and $\sum_{j=1}^p w_j^k = 1$; and for each DM k, $k = 1, \ldots, K$. The weight vector of criteria $\boldsymbol{W}^{k'}$, by DM k, is $\boldsymbol{W}^{k'} = \{w_1^k, w_2^k, \ldots, w_p^k\}$ for p benefit criteria, respectively.

The weight of criteria can be obtained from various techniques, e.g., analytic hierarchy process or entropy method (refer to Hwang and Yoon, 1981).

Step 4. Construct the modified normalized weighted decision matrix $\boldsymbol{V}_\mathrm{b}^k$, $k = 1, \ldots, K$, by each DM for benefit criteria.

For DM k, $k = 1, \ldots, K$, the modified normalized weighted decision matrix is

$$\boldsymbol{V}_\mathrm{b}^k = \begin{bmatrix} v_{\mathrm{b}11}^k & v_{\mathrm{b}12}^k & \cdots & v_{\mathrm{b}1p}^k \\ v_{\mathrm{b}21}^k & v_{\mathrm{b}22}^k & \cdots & v_{\mathrm{b}2p}^k \\ \vdots & \vdots & \cdots & \vdots \\ v_{\mathrm{b}m1}^k & v_{\mathrm{b}m2}^k & \cdots & v_{\mathrm{b}mp}^k \end{bmatrix} = \begin{bmatrix} w_{\mathrm{b}1}^k r_{\mathrm{b}11}^k & w_{\mathrm{b}2}^k r_{\mathrm{b}12}^k & \cdots & w_{\mathrm{b}p}^k r_{\mathrm{b}1p}^k \\ w_{\mathrm{b}1}^k r_{\mathrm{b}21}^k & w_{\mathrm{b}2}^k r_{\mathrm{b}22}^k & \cdots & w_{\mathrm{b}p}^k r_{\mathrm{b}2p}^k \\ \vdots & \vdots & \cdots & \vdots \\ w_{\mathrm{b}1}^k r_{\mathrm{b}m1}^k & w_{\mathrm{b}2}^k r_{\mathrm{b}m2}^k & \cdots & w_{\mathrm{b}p}^k r_{\mathrm{b}mp}^k \end{bmatrix}. \tag{3}$$

It is observed that $0 \leqslant v_{\mathrm{b}ij}^k \leqslant 1$, where $i = 1, \ldots, m$; $j = 1, \ldots, p$.

Step 5. Determine the ideal and negative-ideal solutions $\boldsymbol{V}_\mathrm{b}^{k+}$ (PIS) and $\boldsymbol{V}_\mathrm{b}^{k-}$ (NIS), respectively, for the benefit criteria, for each DM.

For the kth DM, $k = 1, \ldots, K$, his or her PIS and NIS are

$$\boldsymbol{V}_\mathrm{b}^{k+} = \{v_{\mathrm{b}1}^{k+}, \ldots, v_{\mathrm{b}p}^{k+}\} = \{(\max_i v_{\mathrm{b}ij}^k | j \in J) | i = 1, \ldots, m\}, \tag{4}$$

$$\boldsymbol{V}_\mathrm{b}^{k-} = \{v_{\mathrm{b}1}^{k-}, \ldots, v_{\mathrm{b}p}^{k-}\} = \{(\min_i v_{\mathrm{b}ij}^k | j \in J) | i = 1, \ldots, m\}, \tag{5}$$

where J is associated with the benefit criteria for $j = 1, \ldots, p$.

Step 6. Calculate the separation measures from the ideal and the negative-ideal solutions, $\overline{S_{\mathrm{b}i}^+}$ and $\overline{S_{\mathrm{b}i}^-}$, respectively, for the benefit criteria of the group.

There are two sub-steps to be considered here. The first one concerns the distance measure for individuals; the second one aggregates individual measures into a group measure.

Step 6a. Calculate the separation measures from PIS and NIS, for the benefit criteria individually.

For DM k, $k = 1, \ldots, K$, his or her separation measures from PIS and NIS are computed through Minkowski's L_p metric (Steuer, 1986). The individual separation measures of each alternative A_i, $i = 1, \ldots, m$, from the PIS and NIS are

$$S_{\mathrm{b}i}^{k+} = \left\{ \sum_{j=1}^n \left| v_{\mathrm{b}ij}^k - v_{\mathrm{b}j}^{k+} \right|^p \right\}^{1/p} \quad \text{for alternative } A_i, \ i = 1, \ldots, m, \tag{6}$$

and

$$S_{\mathrm{b}i}^{k-} = \left\{ \sum_{j=1}^n \left| v_{\mathrm{b}ij}^k - v_{\mathrm{b}j}^{k-} \right|^p \right\}^{1/p} \quad \text{for alternative } A_i, \ i = 1, \ldots, m, \tag{7}$$

where $p \geqslant 1$ and integer.

For the most common one, $p = 2$, the metric is a Euclidean distance. Eqs. (6) and (7) are

$$S_{\mathrm{b}i}^{k+} = \sqrt{\sum_{j=1}^{n} \left| v_{\mathrm{b}ij}^{k} - v_{\mathrm{b}j}^{k+} \right|^2} \quad \text{for alternative } A_i,\ i = 1,\ldots,m, \tag{8}$$

and

$$S_{\mathrm{b}i}^{k-} = \sqrt{\sum_{j=1}^{n} | v_{\mathrm{b}ij}^{k} - v_{\mathrm{b}j}^{k-} |^2} \quad \text{for alternative } A_i,\ i = 1,\ldots,m. \tag{9}$$

Note that Parkan and Wu (1997) propose four variants of TOPSIS with distance functions $p = 1$ and $p = 2$. The ranking results of the variants do not show the difference of the two examples. Olson (2004) makes a comparison of weights in TOPSIS models, and the average absolute rank differences appear less for $p = 1$ cases.

Step 6b. Calculate the measures of PIS and NIS for benefit criteria of the group.

The group separation measure of each alternative is combined through an operation $\otimes$ of all DMs. Thus, the two group measures to PIS and NIS are

$$\overline{S_{\mathrm{b}i}^{+}} = S_{\mathrm{b}i}^{1+} \otimes \cdots \otimes S_{\mathrm{b}i}^{K+} \quad \text{for alternative } A_i,\ i = 1,\ldots,m, \tag{10}$$

and

$$\overline{S_{\mathrm{b}i}^{-}} = S_{\mathrm{b}i}^{1-} \otimes \cdots \otimes S_{\mathrm{b}i}^{K-} \quad \text{for alternative } A_i,\ i = 1,\ldots,m. \tag{11}$$

The operation can have many choices, geometric mean, arithmetic mean, or their modifications. Upon taking the geometric mean of all individual measures, the group measures with K DMs, Eqs. (10) and (11), from PIS and NIS are

$$\overline{S_{\mathrm{b}i}^{+}} = \left(\prod_{k=1}^{K} S_{\mathrm{b}i}^{k+} \right)^{\frac{1}{K}} \quad \text{for alternative } A_i,\ i = 1,\ldots,m, \tag{12}$$

and

$$\overline{S_{\mathrm{b}i}^{-}} = \left(\prod_{k=1}^{K} S_{\mathrm{b}i}^{k-} \right)^{\frac{1}{K}} \quad \text{for alternative } A_i,\ i = 1,\ldots,m. \tag{13}$$

Step 7. Calculate the relative closeness $\overline{C_{\mathrm{b}i}^{*}}$ to the ideal solution, for the benefit criteria, of the group.

Calculate the relative closeness to the ideal solution and rank the alternatives in descending order. The relative closeness of the ith alternative A_i, $i = 1,\ldots,m$, with respect to PIS and NIS can be expressed as

$$\overline{C_{\mathrm{b}i}^{*}} = \frac{\overline{S_{\mathrm{b}i}^{-}}}{\overline{S_{\mathrm{b}i}^{+}} + \overline{S_{\mathrm{b}i}^{-}}}, \quad i = 1,\ldots,m, \tag{14}$$

where $0 \leqslant \overline{C_{\mathrm{b}i}^{*}} \leqslant 1$. The larger the value is, the better the performance of the alternative will be.

Step 8. Construct the cost index $D_{\mathrm{c}i}^{k}$ or utility $U_{\mathrm{c}i}^{k}$, $k = 1,\ldots,K$, and $i = 1,\ldots,m$, by each DM for cost criteria.

For cost criteria, DMs will first check and decide if they desire to use true cost or monetary values for evaluation in the case where monetary information is available. If it is not, then the cost criteria will be processed based on similar operations as described in Step 8a. Otherwise, the cost will be processed as explained in Step 8b.

Step 8a. Process each DM cost information individually.

Following the similar operations interpreted in the previous steps for benefit criteria, each DM will perform the following actions:

(i) Construct the normalized decision matrix $\boldsymbol{R}_{\mathrm{c}}^{k}$, $k=1,\ldots,K$, by each DM for cost criteria. The value of the elements of $\boldsymbol{R}_{\mathrm{c}}^{k}$, where subscript c stands for costs, will keep $0\leqslant r_{ij}^{k}\leqslant 1$, $i=1,\ldots,m$; $j=1,\ldots,q$.

(ii) Assign a weight vector $\boldsymbol{W}_{\mathrm{c}}^{k}$ for cost criteria by each DM.
For the weights of q criteria of costs, $\sum_{j=1}^{q}w_{\mathrm{c}j}^{k}=1$, $k=1,\ldots,K$. If there is only one criterion to be evaluated, then this action can be eliminated.

(iii) Establish the modified normalized weighted decision matrix $\boldsymbol{V}_{\mathrm{c}}^{k}$, $k=1,\ldots,K$, for each DM for cost criteria.
For DM k, $k=1,\ldots,K$, the modified normalized weighted decision matrix is

$$\boldsymbol{V}_{\mathrm{c}}^{k}=\begin{bmatrix} v_{\mathrm{c}11}^{k} & v_{\mathrm{c}12}^{k} & \cdots & v_{\mathrm{c}1q}^{k}\\ v_{\mathrm{c}21}^{k} & v_{\mathrm{c}22}^{k} & \cdots & v_{\mathrm{c}2q}^{k}\\ \vdots & \vdots & \cdots & \vdots\\ v_{\mathrm{c}m1}^{k} & v_{\mathrm{c}m2}^{k} & \cdots & v_{\mathrm{c}mq}^{k}\end{bmatrix}=\begin{bmatrix} w_{\mathrm{c}1}^{k}r_{\mathrm{c}11}^{k} & w_{\mathrm{c}2}^{k}r_{\mathrm{c}12}^{k} & \cdots & w_{\mathrm{c}q}^{k}r_{\mathrm{c}1q}^{k}\\ w_{\mathrm{c}1}^{k}r_{\mathrm{c}21}^{k} & w_{\mathrm{c}2}^{k}r_{\mathrm{c}22}^{k} & \cdots & w_{\mathrm{c}q}^{k}r_{\mathrm{c}2q}^{k}\\ \vdots & \vdots & \cdots & \vdots\\ w_{\mathrm{c}1}^{k}r_{\mathrm{c}m1}^{k} & w_{\mathrm{c}2}^{k}r_{\mathrm{c}m2}^{k} & \cdots & w_{\mathrm{c}q}^{k}r_{\mathrm{c}mq}^{k}\end{bmatrix}, \tag{15}$$

where there are q criteria for evaluating cost here, and $p+q=n$.
It is also observed that $0\leqslant v_{\mathrm{c}ij}^{k}\leqslant 1$, where $i=1,\ldots,m$; $j=1,\ldots,q$. In addition, if there is one only cost criterion, then the elements of $\boldsymbol{V}_{\mathrm{c}}^{k}$ are the same as the corresponding elements of $\boldsymbol{R}_{\mathrm{c}}^{k}$ for each DM with one column only.

(iv) Combine the modified performance of multiple cost criteria to be one index $D_{\mathrm{c}i}^{k}$ for each alternative by DM k, $k=1,\ldots,K$.
For alternative i, the cost index is

$$D_{\mathrm{c}i}^{k}=\sum_{j=1}^{q}v_{\mathrm{c}ij}^{k}\quad\text{for DM } k,\ k=1,\ldots,K.$$

Note that this action can be omitted if there is only one cost criterion.

It is common that the cost index is taken from an objective value, and the cost indices would be $D_{\mathrm{c}i}^{1}=D_{\mathrm{c}i}^{2}=\cdots=D_{\mathrm{c}i}^{k}$, for decision maker k, $k=1,\ldots,K$. Hence, we can eliminate their superscript so that the cost index is represented as $D_{\mathrm{c}i}$ for alternative i.

If DM prefers to use true cost data for the evaluation, then the cost will be processed as a true value or utility as described in Step 8b.

Step 8b. Each DM assesses his/her utility for cost criteria individually.

To incorporate attitudes toward risk, each DM assesses a utility function for the cost spent. There might be more than one cost item to be evaluated, and then these items will be combined. Two actions will be done for utility representation as follows.

(i) Assess utility by each DM.
Because utility is relative, DMs' utilities for consequences can be obtained through indifference comparison (Keeney and Raiffa, 1976). For simplicity, an exponential utility function for the total cost $R_{\mathrm{c}i}^{k}=\sum_{j=1}^{q}r_{\mathrm{c}ij}^{k}$ on j criteria, for alternative A_i, $i=1,\ldots,m$, by DM k, $k=1,\ldots,K$, can be expressed as

$$U_{\mathrm{c}i}^{k}=U(R_{\mathrm{c}i}^{k})=R\left(1-\mathrm{e}^{-\frac{R_{\mathrm{c}i}^{k}}{R}}\right), \tag{16}$$

where R is the risk tolerance of DMs.
Eq. (16) is frequently designed to fit a risk-averse individual in decision making. A great aversion to risk corresponds to a small value of R, whereas a small aversion to risk corresponds to a large value of R (refer to Hillier and Hillier, 2003).

(ii) Calculate utility index for each DM.
To make incremental analysis, we need to require the utility information to be <1. Thus, we define utility index instead of utilities, for the above action, being divided by the maximum value of their cost column-wise. The expression becomes

$$U_{ci}^{k'} = U_{ci}^{k} / \{(\max_i R_{ci}^{k} | j \in J') | i = 1, \ldots, m\}, \tag{17}$$

where J' is associated with the cost criteria for $j = 1, \ldots, q$, and $0 \leqslant U_{ci}^{k'} \leqslant 1$.
Note that it is possible to use monetary units directly for evaluation. However, we would not get into the contents of cost–benefit analysis in an engineering economy.

Step 9. Construct a group cost index $\overline{D_{ci}}$ or utility index $\overline{U'_{ci}}$, for alternative A_i, $i = 1, \ldots, m$, for cost criteria.

Based on the individual cost values from Step 8, DMs aggregate these cost values into a group value through cost indices or utilities for all alternatives. The aggregation process for cost index is illustrated in Step 9a, and the process for utility index is depicted in Step 9b. Moreover, the aggregation operation can be geometric mean, arithmetic mean, or their modification as stated in Step 6b.

Step 9a. Calculate the group cost indices $\overline{D_{ci}}$, for alternative A_i, $i = 1, \ldots, m$, of the decision group.

Table 2a
Decision matrix of Example 1 – objective performance capability of the robots

Robot	Velocity (m/s)	Load capacity (kg)	Cost ($)	Repeatability (mm)
1.	1.8	90	9500	0.45
2.	1.4	80	5500	0.30
3.	0.8	70	4500	0.20
4.	0.8	60	4000	0.15

Note: (1) The above data are originally taken from Goh et al. (1996). (2) The cost of robot 3 is modified for the purpose of incremental analysis.

Table 2b
Decision matrix of Example 1 – subjective evaluation of the robots by experts

Robot	Expert 1		Expert 2		Expert 3		Expert 4		Expert 5	
	VSQ	PF	VSQ	PF	VSQ	PF	VSQ	PF	VSQ	PF
1.	2	9	9	3	8	4	7	5	9	3
2.	3	8	8	4	7	5	6	6	8	4
3.	4	7	7	5	6	6	5	7	7	5
4.	9	2	3	8	4	7	5	6	3	8

Note: (1) The above data are originally taken from Goh et al. (1996). (2) VSQ – vendor's service quality; PF – programming flexibility.

Table 3
Normalized weights of benefit criteria by experts

Expert	Velocity (m/s)	Load capacity (kg)	Repeatability (mm)	Vendor's service quality	Programming flexibility	Summation
1.	0.0870	0.0870	0.3478	0.0870	0.3913	1.0000
2.	0.2368	0.2368	0.1316	0.2368	0.1579	1.0000
3.	0.2162	0.2162	0.1622	0.2162	0.1892	1.0000
4.	0.1944	0.1944	0.1944	0.1944	0.2222	1.0000
5.	0.2368	0.2368	0.13160	0.2368	0.1579	1.0000

Note: (1) The original weights of criteria are taken from Goh et al. (1996). (2) The weights of criteria are normalized by an operation of division with their total scores by each expert. In addition, the information of cost criteria is excluded here so that there are only five criteria to be calculated.

If geometric mean is considered, then the group cost index $\overline{D_{ci}}$ for alternative A_i is

$$\overline{D_{ci}} = \left(\prod_{k=1}^{K} D_{ci}^{k}\right)^{\frac{1}{K}}, \tag{18}$$

where $i = 1, \ldots, m$, $k = 1, \ldots, K$.

It is popular that the cost index be an objective value, and the group cost indices are $\overline{D_{ci}} = D_{ci}^{1} = \cdots = D_{ci}^{k}$, for all group members, and $0 \leqslant \overline{D_{ci}} \leqslant 1$.

Step 9b. Calculate group utility indices $\overline{U'_{ci}}$ for alternative A_i, $i = 1, \ldots, m$, for the group.

If geometric mean is considered, then the group utility indices $\overline{U'_{ci}}$, $0 \leqslant \overline{U'_{ci}} \leqslant 1$, of costs for alternative i can be expressed as

$$\overline{U'_{ci}} = \left(\prod_{k=1}^{K} U_{ci}^{k'}\right)^{\frac{1}{K}}, \quad \text{where } i = 1, \ldots, m, \ k = 1, \ldots, K. \tag{19}$$

Step 10. Process incremental analysis for all alternatives.

Rearrange all alternatives by their cost or utility indices in ascending order. The incremental analysis is then processed pair-wise based on the order of cost information in the following sub-steps.

Step 10a. Calculate the differences in benefits $\Delta\overline{C_{bi}^{*}}$, $i = 1, \ldots, m$, and differences in costs $\Delta\overline{D_{ci}}$ or in associated utilities $\Delta\overline{U_{ci}}$ of two alternatives with the smallest cost index and the next smallest one.

This will examine the differences between alternatives, with the smallest cost index and the next smallest one, from the aspect of benefits and costs. If the ratio of the differences of benefit and cost, $\Delta\overline{C_{bi}^{*}}/\Delta\overline{D_{ci}}$ or $\Delta\overline{C_{bi}^{*}}/\Delta\overline{U_{ci}}$, is greater than 1, then the latter one is kept; otherwise, the former one is reserved. By emphasizing alternatives, DMs can really decide whether or not differential costs are justified by differential benefits (Newnan et al., 2002). This means that the exchange is worthwhile.

Step 10b. Calculate the differences of benefits $\Delta\overline{C_{bi}^{*}}$ and differences of costs $\Delta\overline{D_{ci}}$ or $\Delta\overline{U_{ci}}$ of the alternative from Step 10a and the alternative with the next smallest cost index.

This will examine the differences between the alternative with the next smallest cost index and the alternative from Step 10a. If the ratio of the differences, $\Delta\overline{C_{bi}^{*}}/\Delta\overline{D_{ci}}$ or $\Delta\overline{C_{bi}^{*}}/\Delta\overline{U_{ci}}$, is >1, then the latter one is kept; otherwise, the former one is reserved. The alternative left is manipulated with the alternative with the next smallest cost index of the order until the alternative with the largest cost index is compared.

Step 10c. The best alternative is obtained.

The remaining alternative in Step 10b is the best alternative in the incremental analysis.

Step 11. Rank the alternatives.

After the best alternative is acquired, DMs continue to do the same routine of Step 10 iteratively excluding the alternatives obtained from the previous routines. In each analysis, DMs obtain the best one, and the ranking of all alternatives can be obtained through the process. For n alternatives, one should carry out $n - 1$ analyses totally to rank all alternatives.

As the procedure is hence outlined, we now illustrate the model through an example.

Example 1. A robot selection problem (Goh et al., 1996).

Four robots are evaluated by five experts or DMs in a group. There are four objective criteria (velocity, load capacity, cost, and repeatability) and two subjective criteria (vendor's service quality and programming flexibility) to be used for evaluation. The objective and subjective data are illustrated in Tables 2a and 2b (Step 1), respectively. To better highlight the proposed model, we change the cost data of Robot 3 to be $4500. Its normalized decision matrix for benefit criteria is then constructed (Step 2), and the normalized weights of criteria can be obtained by experts (Step 3) as shown in Table 3.

In Step 4, each expert constructs the modified normalized weighted decision matrix for benefit criteria, from Step 2, and the decision matrix by Expert 1 is illustrated in Table 4. Later, each expert calculates the separation measures to PIS and NIS for benefit criteria so that the relative closeness is secured. The decision information for Expert 1 is shown in Table 5. In addition, the cost index, normalized by Eq. (2), is listed here for reference. Going through Steps 5–9, the relative closeness of the group, by geometric mean and arithmetical mean, is depicted in Table 6. Their values of the benefit–cost ratio are also presented for reference. This will not be good for ranking (see Bernhard and Canada, 1990).

Based on the information obtained, DMs make incremental analyses. At Step 10, the robots are first ranked by cost index or utility index in ascending order. The DMs compare the robots with the two lowest indices to see if their differential costs are justified by differential benefits. The operation of aggregation using geometric mean is shown in Table 7, in which Robot 4 and Robot 3 are first justified, and their $\Delta\overline{C_{bi}^{*}}/\Delta\overline{D_{ci}}$ ratio of 3.097 is >1. Thus, Robot 3 is supported. Later, the $\Delta\overline{C_{bi}^{*}}/\Delta\overline{D_{ci}}$ ratios of Robot 3 and Robot 2 are justified, and Robot

Table 4
Modified weighted normalized matrix of objective and subjective data for Expert 1

Robot	Velocity (m/s)	Load capacity (kg)	Repeatability (mm)	Vendor's service quality	Programming flexibility
1.	0.0615	0.0516	0.2627	0.0166	0.2503
2.	0.0478	0.0459	0.1751	0.0249	0.2225
3.	0.0273	0.0401	0.1168	0.0332	0.1947
4.	0.0273	0.0344	0.0876	0.0746	0.3195

Table 5
Decision information of benefit and cost for Expert 1

Robot	Benefit criteria			Cost criteria
	Separation measures		Relative closeness	Cost index
	S_{bi}^{k+}	S_{bi}^{k-}	C_{bi}^{k*}	D_{ci}
1.	0.0580	0.2646	0.8201	0.7588
2.	0.1055	0.1901	0.6430	0.4393
3.	0.1656	0.1432	0.4637	0.3594
4.	0.2646	0.0580	0.1799	0.3195

Note: (1) Benefit criteria are processed following TOPSIS process and the values of cost criteria are normalized using Eq. (2b). (2) The above data are for Expert 1. Thus, the superscript here is $k = 1$, and the subscript $i = 1, \ldots, 4$ for robots. (3) Since cost criteria are objective for the example, cost index does not change for different experts.

Table 6
Decision information of benefit and cost for the group

Robot	Benefit criteria			Cost criteria	Benefit–cost ratio
	Separation measures		Relative closeness	Cost index	
	$\overline{S_{bi}^{+}}$	$\overline{S_{bi}^{-}}$	$\overline{C_{bi}^{*}}$	D_{ci}	
Geometric mean					
1.	0.0568	0.1663	0.7455	0.7588	0.983
2.	0.0782	0.1096	0.5835	0.4393	1.328
3.	0.1299	0.0707	0.3524	0.3594	0.981
4.	0.1666	0.0494	0.2288	0.3195	0.716
Arithmetical mean					
1.	0.1675	0.1406	0.4563	0.7588	0.601
2.	0.0642	0.1648	0.7196	0.4393	1.638
3.	0.0843	0.1300	0.6067	0.3594	1.688
4.	0.1366	0.0900	0.3972	0.3195	1.243

Note: (1) Benefit criteria are processed following TOPSIS process and the values of cost criteria are normalized using Eq. (2b). (2) The above data are aggregated by geometric and arithmetical means of experts. The subscripts $i = 1, \ldots, 4$ are for robots. (3) Since cost criteria are objective for the example, cost index does not change for the group.

Table 7
Incremental analysis by group cost for Example 1

Robot	Benefit criteria Relative closeness	Cost criteria Cost index	Order by cost	Incremental analysis Comparison of two robots		
	$\overline{C^*_{bi}}$	$\overline{D_{ci}}$		Robot 3–Robot 4	Robot 2–Robot 3	Robot 1–Robot 2
1.	0.7544	0.7588	4			0.507 < 1
2.	0.5835	0.4393	3		2.893 > 1	*
3.	0.3524	0.3594	2	3.097 > 1		
4.	0.2288	0.3195	1			

Note: (1) The basic data are from the information of geometric mean in Table 5. (3) The robot marked by "*" is the best one in this turn. (2) In the analysis, based on the order of cost, there will be a comparison of two alternatives each time.

2 is seen as a better one. The $\Delta\overline{C^*_{bi}}/\Delta\overline{D_{ci}}$ ratio between Robot 2 and Robot 1 is validated, and Robot 2 is still better. After the comparisons are made, DMs can conclude that Robot 2 is the best among the four.

For ranking the remaining robots, DMs follow the action in Step 11. Performing incremental analysis twice, the ranking order of the four robots with the same aggregation is Robot 2 $\succ$ Robot 3 $\succ$ Robot 1 $\succ$ Robot 4. Furthermore, the ranking with arithmetical mean is Robot 2 $\succ$ Robot 3 $\succ$ Robot 1 $\succ$ Robot 4. However, the result by the incremental benefit–cost ratio is Robot 2 $\succ$ Robot 1 $\succ$ Robot 3 $\succ$ Robot 4 and Robot 3 $\succ$ Robot 2 $\succ$ Robot 4 $\succ$ Robot 1 through geometric mean and arithmetical mean, respectively. The results from incremental analysis appear rather robust and reliable.

Table 8
Utility information assessed by experts

Robot	Cost ($)	Expert 1		Expert 2		Expert 3		Expert 4		Expert 5	
		Utility	Utility index	Utility	Utility index	Utility	Utility index	Utility	Utility index	Utility	Utility index
		$R = 1{,}000{,}000$		$R = 2{,}000{,}000$		$R = 3{,}000{,}000$		$R = 4{,}000{,}000$		$R = 5{,}000{,}000$	
		U^k_{ci}	$U^{k'}_{ci}$	U^k_{ci}	$U^{k'}_{ci}$	U^k_{ci}	$U^{k'}_{ci}$	U^k_{ci}	$U^{k'}_{ci}$	U^k_{ci}	$U^{k'}_{ci}$
1.	9500	9455	0.9953	9477	0.9976	9485	0.9984	9489	0.9988	9491	0.9991
2.	5500	5485	0.5774	5492	0.5782	5495	0.5784	5496	0.5785	5497	0.5786
3.	4500	4490	0.4726	4495	0.4732	4497	0.4733	4497	0.4734	4498	0.4735
4.	4000	3992	0.4202	3996	0.4206	3997	0.4208	3998	0.4208	3998	0.4209

Note: (1) For DM k, $k = 1, \ldots, 4$, his/her utility can be calculated by the function $U^k_{ci} = U(R^k_{ci}) = R\left(1 - e^{-\frac{R^k_{ci}}{R}}\right)$, where R^k_{ci} is the total cost as defined. (2) The utility index $U^{k'}_{ci} = U^k_{ci}/\{(\max_i R^k_{ci} | j \in J') | i = 1, \ldots, m \text{ for alternatives}\}$, where J' is associated with the cost criteria for $j = 1, \ldots, q$.

Table 9
Incremental analysis by group utility for Example 1

Robot	Benefit criteria Relative closeness	Cost criteria Utility index	Order by utility	Incremental analysis Comparison of two robots		
	$\overline{C^*_{bi}}$	$\overline{D_{ci}}$		Robot 3–Robot 4	Robot 2–Robot 3	Robot 1–Robot 2
1.	0.7544	0.9978	4			0.386 < 1
2.	0.5835	0.5782	3		2.220 > 1	*
3.	0.3524	0.4732	2	2.354 > 1		
4.	0.2288	0.4207	1			

Note: (1) The basic data are from the operation of geometric mean from the information in Table 7. (2) In the analysis, based on the order of utility, we make a comparison of two alternatives each time. (3) The robot marked by "*" is the best one in this turn.

Table 10
Comparison of ranking order from different aspects of Example 1

Ranking base	Aggregation type	Rank
Benefit–cost ratio		
	Geometric mean	Robot 2 ≻ Robot 1 ≻ Robot 3 ≻ Robot 4
	Arithmetical mean	Robot 3 ≻ Robot 2 ≻ Robot 4 ≻ Robot 1
Incremental analysis		
(i) Cost index	Geometric mean	Robot 2 ≻ Robot 3 ≻ Robot 1 ≻ Robot 4
	Arithmetical mean	Robot 2 ≻ Robot 3 ≻ Robot 4 ≻ Robot 1
(ii) Utility index	Geometric mean	Robot 2 ≻ Robot 3 ≻ Robot 4 ≻ Robot 1
	Arithmetical mean	Robot 2 ≻ Robot 3 ≻ Robot 4 ≻ Robot 1

Note: (1) There are four robots, Robot 1, Robot 2, Robot 3, and Robot 4, to be ranked (Goh et al., 1996).

In addition to the investigation by cost index, experts express their attitude to risk through the incremental analysis. Here, each expert can elicit utility, and his/her utility index is established. For simplicity, an exponential utility function is suggested as in Eq. (16). The value of R is assumed to be 1,000,000, 2,000,000, 3,000,000, 4,000,000, and 5,000,000, respectively, by five experts with a risk-averse attitude. The utility information is described in Table 8. The incremental analysis by the group is executed as illustrated in Table 9. It is shown that Robot 2 is also the best under the operation of geometric mean, and the ranking order is Robot 2 ≻ Robot 3 ≻ Robot 4 ≻ Robot 1. Moreover, the ranking order by arithmetical mean is also Robot 2 ≻ Robot 3 ≻ Robot 4 ≻ Robot 1. All the above ranking orders, from different aspects, are collected in Table 10.

4. Other considerations

The above research is executed based on the incremental benefit–cost ratio. We further examine cost itself and also conduct a more general analysis.

4.1. Cost considerations

Cost is the central part of incremental analysis, however, the word "cost" is used in various way by different people. It also has many meanings from different aspects and must be clarified carefully (Fisher, 1971). Thus, an inappropriate analysis resulting from unsuitable classification can be avoided.

In addition to the analysis with cost requiring monetary measures traditionally, the study broadly tries to explain cost from different viewpoints. By its nature, cost can be expressed by: characteristics, scales, representation, for group-decision environment, and examples. In Table 11 we first divide cost information into subjective and objective characteristics. Under subjective characteristics, cost should be on relative scales. The representations for cost by the group of DMs would not be the same. An example of subjective characteristics is using linguistic variables, e.g., cheap or expensive. On the other hand, relative and absolute scales could represent objective characteristics. For this relative scale, we choose fixed scores for all DMs in the group, and

Table 11
The classifications and their nature of cost for evaluation

Characteristics	Scale	Representation	Group decision environment	Examples
Subjective	Relative (implicit)	–	Different scores by DMs	Linguistic variables
Objective	Relative (implicit)	–	Fixed scores	Financial ratios
	Absolute (explicit)	Partial	Fixed scores	Purchase cost
		Full	Fixed scores	Total cost of ownership (life cycle cost)

Note: (1) We do not further classify partial or full representation for relative scales. (2) In a group decision environment, the performance of alternatives could be various scores for subjective evaluation by different decision makers. It would be fixed scores for objective evaluation, and the scores are irrelevant to decision makers.

financial ratios as the examples in the class. For an absolute scale, we see that the representation could be partial or full, and both are fixed scores for the group. The examples for partial or full representation are the purchase cost or total cost of ownership (or life cycle cost), respectively. Moreover, the absolute scale usually reflects the dollar expenditures, which has physical meaning for project evaluation.

Although we almost make an exhaustive description on the use of cost (criteria) in MCDM models, it does not include entire situation for evaluation. For investment problems, the DM(s) will be concerned with how much money is spent. In such a way, absolute scales with total cost of ownership seem preferable. The same situation has happened in the fields of engineering economy, accounting, finance, etc. The problem addressed here can direct some research in the applications of MCDM to the above fields.

4.2. Input–output considerations

In an engineering economy, incremental analysis is commonly applied to the methods of rate of return (ROR) and cost–benefit analysis (Newnan et al., 2002; Rose, 1988). This study concentrates on cost–benefit analysis only because of some similarities between MCDM and cost–benefit analysis. Since we have not found any clue between ROR and MCDM, the incremental analysis will not be easy to apply. However, checking the essences of ROR and cost–benefit analysis, we think it is possible to use incremental analysis on any MCDM problem with an input–output relation. If the multiple criteria can be recognized as two parts, input-related and output-related criteria, then incremental analysis is associable. It makes sense that input-related criteria involve the resources consumed and output-related criteria manage the performance demonstrated. For instance, alternative-fuel buses are referred by eleven criteria such as energy supply, energy efficiency, air pollution, noise pollution, industrial relations, employment cost, maintenance cost, capability vehicle, road facility, speed of traffic, and sense of comfort (Tzeng et al., 2005). Among these criteria, three (i.e., energy supply, employment cost, and maintenance cost) can be considered as input-related criteria, and the rest are categorized as output-related criteria. Consequently, the above proposed procedure can be used for ranking and selection of alternatives based on the ratio scale. The input–output relation can be deemed as a general cost–benefit analysis.

5. Conclusions and remarks

This study first examines the necessity of incremental analysis for MCDM. It then has applied incremental analysis for group TOPSIS and developed an 11-step procedure for the evaluation. An example of robot selection proves that the proposed procedure is robust and effective. Moreover, some comprehensive discussions extend the concepts' ease for applications to other MCDM techniques without loss of generality.

As a by-product, we have a better knowledge about the discrepancy between engineering economy and MCDM. If the monetary values can be obtained for benefits and costs, we can make an evaluation by any technique of engineering economics. Otherwise, we rank and select alternatives through MCDM techniques if there exist intangible information.

Microsoft Excel executes all calculation in this study. Since the software can handle a huge number of cells, larger size examples can also be computed efficiently. In addition, we have not demonstrated the effectiveness of the model. An application or simulation work for the recommended model would be valuable for future study.

This study does not infer that the alternatives are mutually exclusive or independent. It is common that alternatives are mutually exclusive in MCDM problems so that only one alternative, normally the best one, is to be selected from a set of options. Notwithstanding this, the situation of alternatives being independent is popular for resource allocation problems in which more than one alternative can be chosen. In the latter case, the choice, usually in the ranking order, is terminated until the resources are dispersed. Interested readers can refer to the work of Blank and Tarquin (1989).

The incremental benefit–cost ratio or cutoff ratio is the key for judgment in making an incremental analysis. However, we have little information about how to obtain the value so that an incremental benefit–cost ratio in excess of one is adopted as the standard of discrimination. Statistical analyses for the types of alternatives to be selected will be a direction for future study. Furthermore, research on the scaling effect for the ratio scale and other considerations is also left for future study.

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