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Optimal pricing, shipment and payment policy for an integrated supplier–buyer inventory model with two-part trade credit

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Abstract

In this article, we develop an integrated supplier–buyer inventory model with the assumption that the market demand is sensitive to the retail price and the supplier adopts a trade credit policy. The trade credit policy discussed in this paper is a "two-part" strategy: cash discount and delayed payment. That is, if the buyer pays within M_1, the buyer receives a cash discount; otherwise, the full purchasing price must be paid before M_2, where $M_2 > M_1 \geqslant 0$. The objective of this research is to determine the optimal pricing, ordering, shipping, and payment policy to maximize the joint expected total profit per unit time. An iterative algorithm is established to obtain the optimal strategy. Furthermore, numerical examples and sensitivity analysis are presented to illustrate the results of the proposed model and to draw managerial insights.
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1. Introduction

The traditional economic order quantity (EOQ) model assumes that the buyer must pay off as soon as the items are received. In fact, allowing customers to delay payment for goods already delivered is a very common business practice. Suppliers often offer trade credit as a marketing strategy to increase sales and reduce on-hand stock level. Once a trade credit has been offered, the amount of time that the buyer's capital tied up in stock is reduced, and that leads to a reduction in the buyer's holding cost of finance. In addition, during the time of the credit period, buyers may earn interest on the money. In fact, buyers, especially small businesses which tend to have a limited number of financing opportunities, rely on trade credit as a source of short-term funds. Goyal (1985) was the first to establish an economic order quantity model with a constant demand rate under the condition of permissible delay in payments. After that, numerous studies dealing with

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the trade credit problem have been presented. For example, Aggarwal and Jaggi (1995), Kim et al. (1995), Jamal et al. (1997), Shinn (1997), Chu et al. (1998), Chen and Chuang (1999), Chang and Dye (2001), Teng (2002), Chung and Huang (2003), Shinn and Hwang (2003), Chung and Liao (2004, 2006), Chung et al. (2005), Teng et al. (2005), Ouyang et al. (2005), and so on.

The above papers assume that the supplier offers the buyer a "one-part" trade credit, i.e., the supplier offers a specified period without interest charge to the buyer that is to be paid off within a permissible delay period. As a result, with no incentive for making early payments, and earning interest through the accumulated revenue received during the credit period, the buyer postpones payment up to the last moment of the permissible period allowed by the supplier. Therefore, from the supplier's perspective, offering trade credit leads to delayed cash inflow and increases the risk of cash flow shortage and bad debt. To accelerate cash inflow and reduce the risk of a cash crisis and bad debt, the supplier may provide a cash discount to encourage the buyer to pay for goods quickly. In other words, the supplier offers a "two-part" trade credit to the buyer to balance the trade-off between delayed payment and cash discount. For example, under a contract, the supplier agrees to a 2% discount deducted from the buyer's purchasing price if payment is made within 10 days. Otherwise, full payment is required within 30 days after the delivery. This credit term in financial management is denoted as "2/10 net 30". If the supplier only offers the buyer a 30 days delay payment, i.e., "one-part" trade credit, then this credit term is denoted as "net 30" (Brigham, 1995, p. 741). There are more papers related to this trade credit policy such as Lieber and Orgler (1975), Hill and Riener (1979), Kim and Chung (1990), Arcelus and Srinivasan (1993). Recently, Ouyang et al. (2002), Chang (2002) and Huang and Chung (2003) developed inventory models in which the supplier provides a permissible delay and a cash discount for early payment.

However, the previous inventory models on trade credit focused only on the supplier's or buyer's performance. They ignored the fact that each parties' local objectives may often conflict. Lee et al. (1997) pointed out that without coordinated inventory management throughout the supply chain results in excessive inventory investment, revenue reduction and delays in response to customer requirements. Therefore, determining the optimal policies based on the maximum/minimum integrated total profit/cost is more reasonable than considering the buyer's or the supplier's individual profit/cost. Goyal (1976) first developed a single supplier–single customer integrated inventory model. Subsequently, Banerjee (1986) extended Goyal's (1976) model and assumed that the supplier followed a lot-for-lot shipment policy with respect to a buyer. Later, Goyal (1988) illustrated that the inventory cost can be reduced if the supplier's economic production quantity is an integer multiple of the buyer's purchase quantity. Lu (1995) then generalized Goyal's (1988) model by relaxing the assumption that the supplier can supply to the buyer only after completing the entire lot size. Many researchers (e.g. Bhatnagar et al., 1993; Goyal, 1995; Viswanathan, 1998; Hill, 1997, 1999; Kim and Ha, 2003; Kelle et al., 2003; Li and Liu, 2006) continued to propose more batching and shipping policies for an integrated inventory model. These studies on integrated inventory problems did not take the effect of trade credit on the optimal policy between the supplier and buyer into account. Abad and Jaggi (2003) first offered a supplier–buyer integrated model following a lot-for-lot shipment policy under a permissible delay in payment. In Abad and Jaggi's model (2003), the supplier offered a "one-part" trade credit to the buyer.

In light of the lack of research dealing with the operational impact of a "two-part" trade credit policy in the integrated inventory model, we develop an integrated inventory model with a retail price sensitive demand. We assume that the supplier offers the buyer a cash discount if payment is made before a specified period, and if the buyer does not pay within the specified period, the full purchasing price must be paid before the delay payment due date. The goal of this research is to determine the optimal payment policy, retail price, lot size, and number of shipments from supplier to buyer in one production run in order to maximize the joint expected total profit. An algorithm is designed to determine the optimal policy. Numerical examples are presented to illustrate the theoretical results. Furthermore, the sensitivity of the optimal solutions with respect to some parameters is also examined.

2. Assumptions and notation

The following assumptions and notation were made in developing the proposed model:

1. There is a single supplier and a single buyer for a single product in this model.
2. Shortages are not allowed.

3. The carrying cost rates excluding interest charges for the supplier is r_V and for the buyer is r_B.
4. To speed up cash inflow and reduce the risk of cash flow shortage, the supplier offers a discount, β $(0 < \beta < 1)$, off the purchasing price, if the buyer's payment is made within time M_1. Otherwise, the full cost of the purchase is due within time M_2, where $M_2 > M_1 \geqslant 0$.
5. The supplier's unit production cost is \$$c$ and unit sale price is \$$v$. The buyer's unit retail price is \$$p$. The relationship between these values is $p > v > (1-\beta)v > c$.
6. In offering trade credit to the buyer, the supplier agrees to give up an immediate cash inflow until a later date. With the time gap between delivery and payment of the product, the supplier endures a capital opportunity cost at rate I_{Vp}.
7. During period $[M_1, M_2]$, a cash flexibility rate, f_{Vc}, is used to quantize the advantage of early cash income for the supplier.
8. During the credit period (i.e., M_1 or M_2), the buyer sells the items and uses the sales revenue to earn interest at a rate of I_{Be}. At the end of this period, the buyer pays off all purchasing cost to the supplier and incurs a capital opportunity cost at a rate of I_{Bp} for the items in stock.
9. The market demand rate for the item is a downward sloping function of the retail price and is given by $D(p) = ap^{-\delta}$, where $a > 0$ is a scaling factor, and $\delta > 1$ is a price-elasticity coefficient. For notational simplicity, $D(p)$ and D will be used interchangeably in this paper.
10. The capacity utilization ρ is defined as the ratio of the market demand rate, D, to the production rate, R, i.e., $\rho = D/R$, where $\rho < 1$ and is fixed.
11. The buyer's inventory cycle length is T, order quantity is $Q(= DT)$ per order and ordering cost per order is S_B.
12. During the production period, the supplier manufactures in batches of size nQ (where n is an integer) and incurs a batch setup cost S_V. Once the first Q units are produced, the supplier delivers them to the buyer and then makes continuous delivery on average every T units of time until the supplier's inventory level falls to zero.

3. Model formulation

3.1. Supplier's expected total profit per unit time

Throughout each production run, the supplier manufactures in batches of size nQ and incurs a batch setup cost S_V. The expected cycle length for the supplier is $nQ/D(= nT)$. Therefore, the supplier's setup cost per unit time is $S_V/(nT)$. The inventory carrying cost includes the storage and handling expenses, insurance, taxes and obsolescence costs as well as the time value of capital tied up in inventories. With the unit production cost c, the carrying cost rate excluding interest charges r_V and the capital opportunity cost per dollar per unit time I_{Vp}, using the same approach as in Joglekar (1988), the supplier's carrying cost per unit time can be obtained and is given by $(cr_V + cI_{Vp})\frac{ap^{-\delta}T}{2}[(n-1)(1-\rho)+\rho]$.

For each unit of product, the supplier charges $(1-k_j\beta)v$ if the buyer pays the payment at time M_j, where $j = 1, 2$; $k_1 = 1$ and $k_2 = 0$. With a finance rate I_{Vp}, the opportunity cost per unit time for offering trade credit is $(1-k_j\beta)vI_{Vp}ap^{-\delta}M_j$. However, if the buyer pays at time M_1, during $M_2 - M_1$ the supplier can use the revenue $(1-\beta)v$ to avoiding a cash flow crisis or to generate profits. With a cash flexibility rate f_{Vc}, the advantage gain per unit time from early payment is $(1-\beta)vf_{Vc}ap^{-\delta}(M_2 - M_1)$.

The supplier's expected total profit per unit time is the sales revenue minus the total relevant cost (which consists of the production cost, set-up cost, carrying cost and opportunity cost for offering trade credit) and plus the advantage from early payment, which can be expressed as

$$\begin{aligned}\mathrm{TVP}_j(n) = {} & (1-k_j\beta)vap^{-\delta} - cap^{-\delta} - \frac{S_V}{nT} - \frac{ap^{-\delta}Tc(r_V + I_{Vp})}{2}[(n-1)(1-\rho)+\rho] \\ & - (1-k_j\beta)vI_{Vp}ap^{-\delta}M_j + k_j(1-\beta)vf_{Vc}ap^{-\delta}(M_2 - M_1), \quad j = 1, 2;\ k_1 = 1 \text{ and } k_2 = 0. \end{aligned} \tag{1}$$

3.2. Buyer's expected total profit per unit time

The buyer incurs an ordering cost S_B for each order of quantity Q, so the ordering cost per unit time is S_B/T. With the carrying cost rate excluding interest charges r_B, the unit purchasing cost $(1-k_j\beta)v$ and the average inventory over the cycle $Q/2$, the buyer's carrying cost per unit time, without the time value of the capital tied up in inventories, is $(1-k_j\beta)vr_BQ/2 = (1-k_j\beta)vr_Bap^{-\delta}T/2$.

As the payment is done before or after the total depletion of inventory, we have the following two possible cases: (i) $T < M_j$ and (ii) $T \geqslant M_j,\ j = 1,2$. These two cases are depicted in Fig. 1.

Case 1. $T < M_j,\ j = 1,2.$

In this case, the buyer's payment time ends after the inventory is depleted completely. So the buyer pays no opportunity cost for the items kept in stock. At the same time, the buyer uses the sales revenue to earn the interest at a rate of I_{Be}, hence, the interest earned per unit time is $\frac{1}{T}\left[pI_{Be}\int_0^T Dt\,\mathrm{d}t + pI_{Be}DT(M_j - T)\right] = ap^{-\delta+1}I_{Be}(M_j - \frac{T}{2})$.

Case 2. $T \geqslant M_j,\ j = 1,2.$

This situation indicates that the buyer's payment time ends on or before the inventory is depleted completely. Since the buyer does not pay the supplier until the end of the credit period, the buyer can use the sales revenue during the credit period to earn interest at a rate of I_{Be}. Hence, the interest earned per unit time is $\frac{pI_{Be}}{T}\int_0^{M_j} Dt\,\mathrm{d}t = \frac{ap^{-\delta+1}I_{Be}M_j^2}{2T}$. Moreover, after the due date M_j with some inventory on hand the buyer endures a capital opportunity cost at a rate of I_{Bp}, the capital opportunity cost per unit time is

$$\frac{(1-k_j\beta)vI_{Bp}}{T}\int_{M_j}^{T} D(t-M_j)\mathrm{d}t = \frac{(1-k_j\beta)vI_{Bp}D(T-M_j)^2}{2T} = \frac{(1-k_j\beta)vI_{Bp}ap^{-\delta}(T-M_j)^2}{2T}.$$

The buyer is charged $(1-k_j\beta)vD = (1-k_j\beta)vap^{-\delta}$ by the supplier, and receives $pD = ap^{-\delta+1}$ from customers. Therefore, the expected total profit per unit time for the buyer is the total sales revenue minus the total relevant costs (which consists of the purchasing cost, ordering cost, carrying cost excluding interest charges, interest earned and capital opportunity cost), that is

$$\mathrm{TBP}_j(p,T) = \begin{cases} \mathrm{TBP}_{j1}(p,T) & \text{if } T < M_j, \\ \mathrm{TBP}_{j2}(p,T) & \text{if } T \geqslant M_j, \end{cases} \quad j = 1,2, \tag{2}$$

where

$$\mathrm{TBP}_{j1}(p,T) = ap^{-\delta+1} - (1-k_j\beta)vap^{-\delta} - \frac{S_B}{T} - \frac{(1-k_j\beta)vr_Bap^{-\delta}T}{2} + ap^{-\delta+1}I_{Be}\left(M_j - \frac{T}{2}\right), \tag{3}$$

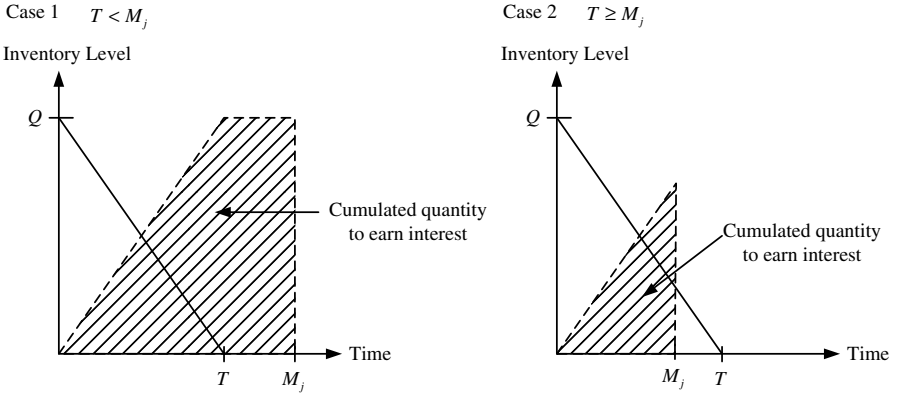

Fig. 1. Inventory and capital model for the buyer under trade credit.

and

$$\mathrm{TBP}_{j2}(p,T) = ap^{-\delta+1} - (1-k_j\beta)vap^{-\delta} - \frac{S_B}{T} - \frac{(1-k_j\beta)vr_Bap^{-\delta}T}{2} + \frac{ap^{-\delta+1}I_{Be}M_j^2}{2T} - \frac{(1-k_j\beta)vI_{Bp}ap^{-\delta}(T-M_j)^2}{2T}. \tag{4}$$

3.3. The joint expected total profit per unit time

Once the supplier and buyer have established a long-term strategic partnership and are contracted to commit to the relationship, they will determine the best joint policy in which to cooperate. Under this circumstance, the joint expected total profit per unit time for the supplier and buyer is

$$\Pi_j(n,p,T) = \begin{cases} \Pi_{j1}(n,p,T) & \text{if } T < M_j, \\ \Pi_{j2}(n,p,T) & \text{if } T \geqslant M_j, \end{cases} \quad j = 1,2, \tag{5}$$

where

$$\begin{aligned}\Pi_{j1}(n,p,T) &= \mathrm{TVP}_j(n) + \mathrm{TBP}_{j1}(p,T) \\ &= ap^{-\delta}\left\{p - c + [pI_{Be} - (1-k_j\beta)vI_{Vp}]M_j + k_j(1-\beta)vf_{Vc}(M_2 - M_1) \right. \\ &\quad \left. - \frac{T}{2}\{(1-k_j\beta)vr_B + pI_{Be} + c(r_V + I_{Vp})[(n-1)(1-\rho)+\rho]\}\right\} - \frac{1}{T}\left(\frac{S_V}{n} + S_B\right),\end{aligned} \tag{6}$$

and

$$\begin{aligned}\Pi_{j2}(n,p,T) &= \mathrm{TVP}_j(n) + \mathrm{TBP}_{j2}(p,T) \\ &= ap^{-\delta}\left\{p - c + (1-k_j\beta)v(I_{Bp} - I_{Vp})M_j + k_j(1-\beta)vf_{Vc}(M_2 - M_1) \right. \\ &\quad \left. - \frac{T}{2}\{(1-k_j\beta)v(r_B + I_{Bp}) + c(r_V + I_{Vp})[(n-1)(1-\rho)+\rho]\} + \frac{[pI_{Be} - (1-k_j\beta)vI_{Bp}]M_j^2}{2T}\right\} \\ &\quad - \frac{1}{T}\left(\frac{S_V}{n} + S_B\right).\end{aligned} \tag{7}$$

The problem now is to determine the optimal values of n, p and T such that $\Pi_j(n,p,T), j = 1,2$ in (5) is maximized.

4. Solution procedure

To examine the effect of n on the joint expected total profit per unit time, taking the second order partial derivative of Eq. (5) with respect to n, we obtain $\frac{\partial^2\Pi_j(n,p,T)}{\partial n^2} = -\frac{2S_V}{n^3T} < 0$, for $j = 1,2$. It implies that for fixed p and T, $\Pi_j(n,p,T)$ is a concave function in n. This ensures that the search for the optimal shipment number, n^*, is reduced to find a local optimal solution.

4.1. Determination of the optimal replenishment cycle length T for any given n and p

For given n and p, taking the second order partial derivative of $\Pi_{j1}(n,p,T)$ in (6) with respect to T, we obtain $\frac{\partial^2\Pi_{j1}(n,p,T)}{\partial T^2} = -\frac{2}{T^3}\left(\frac{S_V}{n} + S_B\right) < 0$. Hence, for fixed n and p, $\Pi_{j1}(n,p,T)$ is a concave function in T. Consequently, there exists a unique value of T, denoted as $T_{j1}(n,p)$, which maximizes $\Pi_{j1}(n,p,T)$. $T_{j1}(n,p)$ can be obtained by solving the equation $\partial\Pi_{j1}(n,p,T)/\partial T = 0$, and is given by

$$T_{j1}(n,p) = \sqrt{\frac{2\left(\frac{S_V}{n} + S_B\right)}{ap^{-\delta}\{(1-k_j\beta)vr_B + pI_{Be} + c(r_V + I_{Vp})[(n-1)(1-\rho)+\rho]\}}}. \tag{8}$$

To ensure $T_{j1}(n,p) < M_j$, we substitute (8) into inequality $T_{j1}(n,p) < M_j$, and obtain

$$\begin{aligned}&\text{if and only if } T_{j1}(n,p) < M_j \text{ then}\\ &\frac{S_V}{n} + S_B < \frac{ap^{-\delta}M_j^2}{2}\left\{(1-k_j\beta)vr_B + pI_{Be} + c(r_V + I_{Vp})[(n-1)(1-\rho)+\rho]\right\}.\end{aligned} \tag{9}$$

Substituting (8) into (6), we can get the joint expected total profit function for Case 1

$$\begin{aligned}\Pi_{j1}(n,p) &\equiv \Pi_{j1}(n,p,T_{j1}(n,p))\\ &= ap^{-\delta}\left\{p - c + \left[pI_{Be} - (1-k_j\beta)vI_{Vp}\right]M_j + k_j(1-\beta)vf_{Vc}(M_2 - M_1)\right\}\\ &\quad - \sqrt{2ap^{-\delta}\left(\frac{S_V}{n} + S_B\right)\left\{(1-k_j\beta)vr_B + pI_{Be} + c(r_V + I_{Vp})[(n-1)(1-\rho)+\rho]\right\}}.\end{aligned} \tag{10}$$

Furthermore, from (9), we have

$$\begin{aligned}&\text{if and only if } T_{j2}(n,p) \geqslant M_j \text{ then}\\ &\frac{S_V}{n} + S_B \geqslant \frac{ap^{-\delta}M_j^2}{2}\left\{(1-k_j\beta)vr_B + pI_{Be} + c(r_V + I_{Vp})[(n-1)(1-\rho)+\rho]\right\}.\end{aligned} \tag{11}$$

Note that when

$$\frac{S_V}{n} + S_B \geqslant \frac{ap^{-\delta}M_j^2}{2}\left\{(1-k_j\beta)vr_B + pI_{Be} + c(r_V + I_{Vp})[(n-1)(1-\rho)+\rho]\right\},$$

it can be shown that

$$\begin{aligned}&ap^{-\delta}M_j^2[(1-k_j\beta)vI_{Bp} - pI_{Be}] + 2\left(\frac{S_V}{n} + S_B\right)\\ &\quad \geqslant ap^{-\delta}M_j^2[(1-k_j\beta)vI_{Bp} - pI_{Be}] + ap^{-\delta}M_j^2\left\{(1-k_j\beta)vr_B + pI_{Be} + c(r_V + I_{Vp})[(n-1)(1-\rho)+\rho]\right\}\\ &\quad = ap^{-\delta}M_j^2\left\{(1-k_j\beta)v(r_B + I_{Bp}) + c(r_V + I_{Vp})[(n-1)(1-\rho)+\rho]\right\} > 0,\end{aligned} \tag{12}$$

which implies the second order partial derivative of $\Pi_{j2}(n,p,T)$ in (7) is

$$\frac{\partial^2\Pi_{j2}(n,p,T)}{\partial T^2} = -\frac{1}{T^3}\left\{ap^{-\delta}M_j^2[(1-k_j\beta)vI_{Bp} - pI_{Be}] + 2\left(\frac{S_V}{n} + S_B\right)\right\} < 0.$$

Therefore, for fixed n and p, $\Pi_{j2}(n,p,T)$ is a concave function in T. By solving the equation $\partial\Pi_{j2}(n,p,T)/\partial T = 0$, we obtain the value of T (denoted by $T_{j2}(n,p)$) which maximizing $\Pi_{j2}(n,p,T)$ is given by

$$T_{j2}(n,p) = \sqrt{\frac{2\left(\frac{S_V}{n} + S_B\right) + ap^{-\delta}M_j^2[(1-k_j\beta)vI_{Bp} - pI_{Be}]}{ap^{-\delta}\left\{(1-k_j\beta)v(r_B + I_{Bp}) + c(r_V + I_{Vp})[(n-1)(1-\rho)+\rho]\right\}}}. \tag{13}$$

Substituting (13) into (7), we can get the joint expected total profit function for Case 2

$$\begin{aligned}\Pi_{j2}(n,p) &\equiv \Pi_{j2}(n,p,T_{j2}(n,p))\\ &= ap^{-\delta}\left\{p - c + (1-k_j\beta)v(I_{Bp} - I_{Vp})M_j + k_j(1-\beta)vf_{Vc}(M_2 - M_1)\right\}\\ &\quad - \sqrt{ap^{-\delta}\left\{(1-k_j\beta)v(r_B + I_{Bp}) + c(r_V + I_{Vp})[(n-1)(1-\rho)+\rho]\right\}}\\ &\quad \times \sqrt{2\left(\frac{S_V}{n} + S_B\right) + ap^{-\delta}M_j^2[(1-k_j\beta)vI_{Bp} - pI_{Be}]}.\end{aligned} \tag{14}$$

For convenience, we let

$$\Delta_j = \frac{ap^{-\delta}M_j^2}{2}\left\{(1-k_j\beta)vr_B + pI_{Be} + c(r_V + I_{Vp})[(n-1)(1-\rho)+\rho]\right\}, \quad j = 1,2. \tag{15}$$

Because $M_2 > M_1 \geqslant 0$, $k_1 = 1$ and $k_2 = 0$, we have $\Delta_2 > \Delta_1$. Furthermore, let $T^{(n)}$ denote the optimal replenishment cycle length for any given n and p, thus we obtain the following result.

Theorem 1. *For any given n and p,*

(a) *when* $\frac{S_V}{n} + S_B < \Delta_1$, *if* $\max\{\Pi_{11}(n,p), \Pi_{21}(n,p)\} = \Pi_{11}(n,p)$, *then the optimal payment time is* M_1 *and* $T^{(n)} = T_{11}(n,p)$; *otherwise, the optimal payment time is* M_2 *and* $T^{(n)} = T_{21}(n,p)$;
(b) *when* $\Delta_1 \leqslant \frac{S_V}{n} + S_B < \Delta_2$, *if* $\max\{\Pi_{21}(n,p), \Pi_{12}(n,p)\} = \Pi_{21}(n,p)$, *then the optimal payment time is* M_2 *and* $T^{(n)} = T_{21}(n,p)$; *otherwise, the optimal payment time is* M_1 *and* $T^{(n)} = T_{12}(n,p)$;
(c) *when* $\Delta_2 \leqslant \frac{S_V}{n} + S_B$, *if* $\max\{\Pi_{12}(n,p), \Pi_{22}(n,p)\} = \Pi_{12}(n,p)$, *then the optimal payment time is* M_1 *and* $T^{(n)} = T_{12}(n,p)$; *otherwise, the optimal payment time is* M_2 *and* $T^{(n)} = T_{22}(n,p)$.

Proof 1. It immediately follows from (9), (11) and (15). □

4.2. Determination of the buyer's optimal retail price for any given n

To find the optimal solution of p, we use the similar solution processes asserted by Teng et al. (2005). Motivated by (9) and (11), we let

$$f_j(p) = \frac{ap^{-\delta}M_j^2}{2}\{(1-k_j)vr_B + pI_{Be} + c(r_V + I_{Vp})[(n-1)(1-\rho)+\rho]\}, \quad j = 1,2. \tag{16}$$

Because of $\frac{\mathrm{d}f_j(p)}{\mathrm{d}p} < 0$, $f_j(p)$ is a strictly decreasing function of p when n is fixed. Furthermore, because $\lim_{p\to 0} f_j(p) = \infty$ and $\lim_{p\to\infty} f_j(p) = 0$, therefore, for fixed n, we can find a unique value p_{j0} such that $f_j(p_{j0}) = \frac{S_V}{n} + S_B$, that is

$$\frac{S_V}{n} + S_B = \frac{ap_{j0}^{-\delta}M_j^2}{2}\{(1-k_j\beta)vr_B + p_{j0}I_{Be} + c(r_V + I_{Vp})[(n-1)(1-\rho)+\rho]\}. \tag{17}$$

Thus, (9) and (11) reduce to

$$\text{if and only if} \quad p < p_{j0}, \ \text{then } T_{j1}(n,p) < M_j, \tag{18}$$

and

$$\text{if and only if} \quad p \geqslant p_{j0}, \ \text{then } T_{j2}(n,p) \geqslant M_j, \tag{19}$$

respectively, where p_{j0} is the value that satisfies (17).

Consequently, from (5), (10), (14) and (17)–(19), our problem becomes to find the optimal value of p which maximize the following joint expected total profit function when n is fixed,

$$\Pi_j(n,p) = \begin{cases} \Pi_{j1}(n,p) & \text{if } p < p_{j0}, \\ \Pi_{j2}(n,p) & \text{if } p \geqslant p_{j0}, \end{cases} \quad j = 1,2. \tag{20}$$

For fixed n, the optimal value of p which maximizes $\Pi_{ji}(n,p)$, $j = 1,2$ and $i = 1,2$, can be determined by solving the first order necessary condition (i.e., $\partial\Pi_{ji}(n,p)/\partial p = 0$) and examining the second order sufficient condition for concavity (i.e., $\partial^2\Pi_{ji}(n,p)/\partial p^2 < 0$).

Summarizing the above arguments, we establish the following algorithm to obtain the optimal solution (n^*, p^*, T^*).

Algorithm 1

Step 0. Let $n = 0$ and set $\Pi(n, p^{(n)}, T^{(n)}) = 0$.
Step 1. Let $n = 1$.
Step 2. For $j = 1, 2$,
(i) Determine p_{j0} by solving Eq. (17).
(ii) If there exists a p_{j1} such that $p_{j1} < p_{j0}$, $\frac{\partial \Pi_{j1}(n,p)}{\partial p}\Big|_{p=p_{j1}} = 0$ and $\frac{\partial^2 \Pi_{j1}(n,p)}{\partial p^2}\Big|_{p=p_{j1}} < 0$, then we determine $T_{j1}(n, p_{j1})$ by (8) and $\Pi_{j1}(n, p_{j1}, T_{j1}(n, p_{j1}))$ by (10). Otherwise, we set $\Pi_{j1}(n, p_{j1}, T_{j1}(n, p_{j1})) = 0$.
(iii) If there exists a p_{j2} such that $p_{j2} \geqslant p_{j0}$, $\frac{\partial \Pi_{j2}(n,p)}{\partial p}\Big|_{p=p_{j2}} = 0$ and $\frac{\partial^2 \Pi_{j2}(n,p)}{\partial p^2}\Big|_{p=p_{j2}} < 0$, then we determine $T_{j2}(n, p_{j2})$ by (13) and determine $\Pi_{j2}(n, p_{j2}, T_{j2}(n, p_{j2}))$ by (14). Otherwise, we set $\Pi_{j2}(n, p_{j2}, T_{j2}(n, p_{j2})) = 0$.
Step 3. Find $\max_{j=1,2;i=1,2}\Pi_{ji}(n, p_{ji}, T_{ji}(n, p_{ji}))$. Set $\Pi(n, p^{(n)}, T^{(n)}) = \max_{j=1,2;i=1,2}\Pi_{ji}(n, p_{ji}, T_{ji}(n, p_{ji}))$, then $(p^{(n)}, T^{(n)})$ is the optimal solution for given n.
Step 4. If $\Pi(n, p^{(n)}, T^{(n)}) \geqslant \Pi(n-1, p^{(n-1)}, T^{(n-1)})$, then go to Step 5. Otherwise, go to Step 6.
Step 5. Set $n = n + 1$, go to Step 2.
Step *6*. Set $\Pi(n^*, p^*, T^*) = \Pi(n-1, p^{(n-1)}, T^{(n-1)})$, then (n^*, p^*, T^*) is the optimal solution.

Once the optimal solution (n^*, p^*, T^*) is obtained, the optimal order quantity per order for the buyer $Q^* = D(p^*)T^*$follows.

5. Numerical study

Example 1. In order to illustrate the solution procedure, we consider an integrated inventory system with the following numerical data: $a = 250\,000$, $\rho = 0.9$, $\delta = 1.25$, $c = \$2$/unit, $v = \$4.5$/unit, $S_V = \$1000$/setup, $S_B = \$300$/order, $r_V = 0.05$, $r_B = 0.08$, $I_{Vp} = 0.09$/\$/year, $I_{Bp} = 0.16$/\$/year, $I_{Be} = 0.18$/\$/year and $f_{Vc} = 0.17$/\$/year. In addition, a credit term "2/10 net 30" (i.e., $M_1 = 10$ days, $M_2 = 30$ days and $\beta = 2\%$) is offered by the supplier.

The three-dimensional graphs of joint expected total annual profit of the entire supply chain system are presented in Fig. 2. The graphs reveal that for any given n (e.g., $n = 8, 12, 50$) there exists a corresponding optimal solution $(p^{(n)}, T^{(n)})$ which maximizes the joint expected total annual profit. Furthermore, we run the numerical results with values of $n = 1, 2, \ldots, 100$. The numerical results indicate that there is a unique integer n which maximizes the value of $\Pi(n) \equiv \Pi(n, p^{(n)}, T^{(n)})$, as shown in Fig. 3. Consequently, the solution obtained through Algorithm 1 is the optimal solution.

Using the Algorithm 1, the maximum joint expected total annual profit of the entire supply chain system is $\Pi(n^*, p^*, T^*) = \$109\,063$ and the optimal policy is: The buyer makes the payment within 10 days and takes the advantage of 2% discount, the retail price $p^* = \$10.52$/unit, the replenishment cycle length $T^* = T_{12} = 0.188$ year=68.61 days and the ordering quantity $Q^* = 2481$ units/order. In such case, the optimal shipment number from supplier to buyer $n^* = 12$.

Example 2. To analyze the effects of credit terms on performance, using the same data in Example 1, we obtain the computational results for various values of M_1 and M_2 as shown in Table 1. In addition, to illustrate the relationship between credit terms and profit progress, we demonstrate profit gain (comparing with no trade credit) in percentage in the last three columns of Table 1.

Table 1 shows the profit gains in percentage are positive for the entire supply chain system, which means total profit for the supply chain as a whole under the two-part trade credit strategy is greater than the total profit before adopting this strategy. Therefore, the two-part trade credit is beneficial to the supply chain as a whole. However, the profit gains in percentage are not always positive for the supplier. Under credit terms "2/20, net 30" or as the supplier extends the due date to 90 days after delivery (i.e., $M_2 = 90$) the supplier's profit gains in percentage are negative.

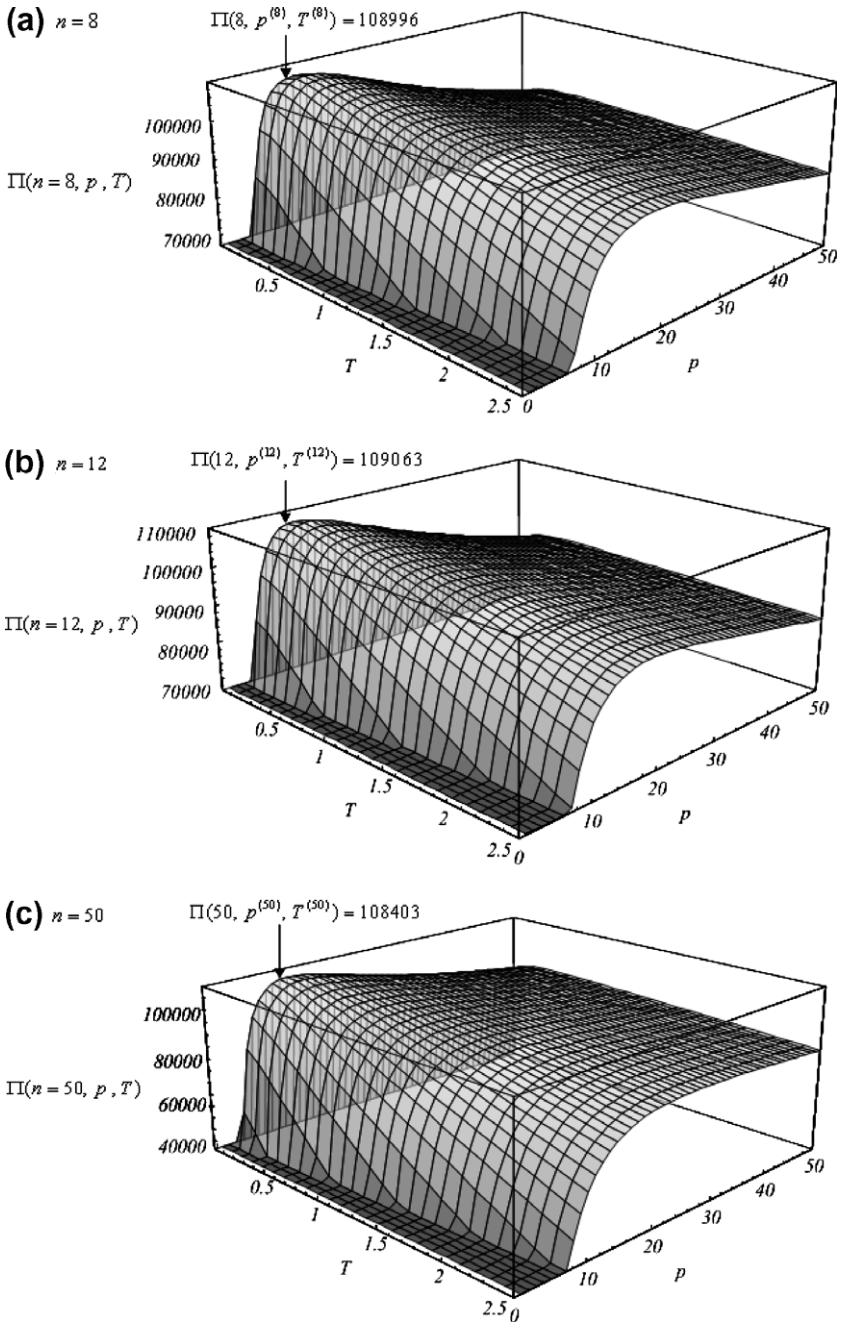

Fig. 2. The total expected profit per unit time for some given n (e.g., $n = 8, 12, 50$).

Table 1 also reveals that if the supplier sets the payment due date at 30 days then offering a 2% discount can drive the buyer to pay early. However, if the supplier considers extending the due date to 60 days or 90 days, the channel's profit will be maximized as the buyer pays at the end of the net period. This indicates that as the supplier sets the due date at 60 days or 90 days after delivery, then offering a 2% discount will not help to speed up his/her cash inflows. Therefore, in an integrated supply chain system, the supplier must consider the credit policy very carefully to achieve mutual benefit from a two-part trade credit strategy.

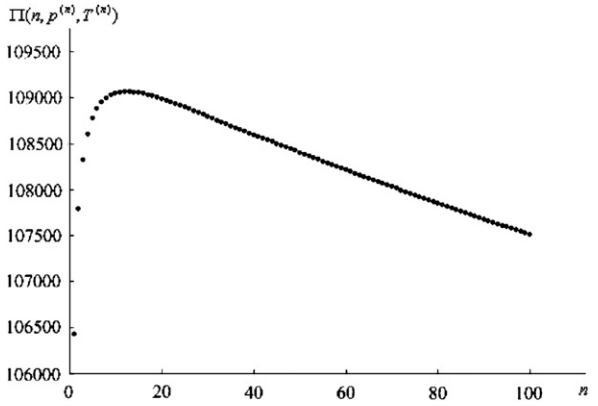

Fig. 3. The optimal total expected profit per unit time for $n = 1, 2, \ldots, 100$.

Table 1
Performances of supply chain for various credit terms

M_1 (days)	M_2 (days)	Optimal time for payment	n^*	p^* ($)	T^* (days)	$D(p^*)$	Q^*	Profit ($)			Profit gain (%)[a]		
								Buyer	Supplier	Channel	Buyer	Supplier	Channel
0	0	–	12	10.78	$T = 69.77$	12793	2446	77499	30862	108361	–	–	–
0	30	30	14	10.62	$T_2 = 62.00$	13034	2214	77941	31048	108988	+0.57	+0.60	+0.58
10		10	12	10.52	$T_{12} = 68.61$	13201	2481	77987	31076	109063	+0.63	+0.69	+0.65
20		20	13	10.57	$T_{12} = 66.03$	13116	2373	78529	30471	109000	+1.33	−1.27	+0.59
0	60	60	17	10.51	$T_1 = 49.51$	13211	1792	79276	31088	110365	+2.29	+0.73	+1.85
10		60	17	10.51	$T_{21} = 49.51$	13211	1792	79276	31088	110365	+2.29	+0.73	+1.85
20		60	17	10.51	$T_{21} = 49.51$	13211	1792	79276	31088	110365	+2.29	+0.73	+1.85
0	90	90	17	10.52	$T_1 = 49.53$	13195	1790	81370	30609	111979	+5.00	−0.82	+3.34
10		90	17	10.52	$T_{21} = 49.53$	13195	1790	81370	30609	111979	+5.00	−0.82	+3.34
20		90	17	10.52	$T_{21} = 49.53$	13195	1790	81370	30609	111979	+5.00	−0.82	+3.34

[a] Profit gain = [(profit with trade credit – profit without trade credit)/profit without trade credit] × 100%.

Example 3. To explore the impact of trade credit when choosing between an independent or coordinated decision on the supply chain performance, using the same data as in Example 1, we list the optimal solutions of "cash on delivery" (i.e., $M_1 = M_2 = 0$ and $\beta = 0$) and "2/10 net 30" for each in Table 2.

The independent supply chain system here means that the supplier and the buyer act independently and each aim to maximize their own profit respectively. In this case the buyer makes his/her optimal pricing and ordering decisions in advance and then the supplier determines his/her own optimal production and delivery

Table 2
The optimal solutions of supply chain under different policies

Decision making	Credit term(s)	Time for payment	n^*	p^* ($)	T^* (days)	$D(p^*)$	Q^*	nQ^*	Profit ($)		
									Buyer	Supplier	Channel
Independent	Cash on delivery	0	11	23.42	$T = 123.48$	4854	1642	18062	90034	11430	101464
	Trade credit (2/10, net 30)	10	11	22.83	$T_{12} = 121.48$	5009	1667	18337	90610	11506	102116
Coordinated	Cash on delivery	0	12	10.78	$T = 69.77$	12793	2446	29352	77499	30862	108361
	Trade credit (2/10, net 30)	10	12	10.52	$T_{12} = 68.61$	13201	2481	29772	77987	31076	109063
Allocated									96774	12289	109063

policy. Using the same data in Example 1 and applying similar procedures (see Appendix for detailed computation and proof), we can obtain the optimal solutions for both the buyer and the supplier.

Table 2 illustrates that under both an independent and coordinated policy, offering trade credit to the buyer results in a lower retail price and thus a higher market demand and channel profit. However, when the supplier and the buyer act independently, regardless of whether or not the supplier offers trade credit to the buyer, the retail price set in order to maximize the buyer's profit is more than double that associated with a coordinated policy. The escalating price in turn reduces market demand causing the buyer's order quantity to drop for each subsequent order. With the reduction in order lot, the profit of the supplier as well as the entire supply chain shrinks significantly. Therefore, the adoption of lot size coordination policy can significantly improve the profit of the entire supply chain. Furthermore, from the supplier's perspective, a joint decision is much more advantageous than an independent decision although the reverse is true for the buyer. Therefore, in order to benefit both the buyer and supplier, we applied a simple and useful compensation method suggested by Goyal (1976) so that the long term partnership between the supplier and buyer can remain intact and the benefits can continue to accrue. We reallocated $\Pi(n^*, p^*, T^*)$ and obtained

$$\text{buyer's profit} = \Pi(n^*, p^*, T^*) \times \frac{\text{TBP}(p_B^*, T_B^*)}{\left[\text{TBP}(p_B^*, T_B^*) + \text{TVP}(n_V^*)\right]} = 109\,063 \times \frac{90\,610}{102\,116} = 96\,774,$$

and

$$\text{supplier's profit} = \Pi(n^*, p^*, T^*) \times \frac{\text{TVP}(n_V^*)}{\left[\text{TBP}(p_B^*, T_B^*) + \text{TVP}(n_V^*)\right]} = 109\,063 \times \frac{11\,506}{102\,116} = 12\,289.$$

The allocated results are also list at the bottom of Table 2.

To illustrate the benefit of a coordinated lot size trade credit policy more clearly, a summary of channel profit under four scenarios is listed in Table 3. This shows that the profit increase of a coordinated supply chain system is \$6897(=\$108 361 − 101 464) for the "cash on delivery" scenario and \$6947 (=\$109 063 − 102 116) for the "2/10 net 30" scenario, respectively. The percentage increase is 6.80% in both instances. Turning now to the benefit of a trade credit policy, the profit increase in the supply chain system resulting from a trade credit policy is \$652 (=\$102 116 − 101 464) for the "independent policy" scenario and \$702 (=\$109 063 − 108 361) for the "coordination policy" scenario, respectively. The percentage increase is about 0.65% in both instances. Also, the surplus capital generated for the supply chain by jointly optimizing the supplier–buyer lot sizing and trade credit policies is \$7599 (=\$109 063 − 101 464) and the percentage increase in total profit is 7.5%. In conclusion, it can be determined from this data that one can expect larger channel profit expansion from lot size coordination than from trade credit policy. In addition, the joint procedure of optimizing supplier–buyer lot sizing and trade credit policies is beneficial for channel performance.

Example 4. This example was carried out to evaluate the relative performances for various values of the problem parameters. The study was carried for different values of $\rho, S_B/S_V$ and r_B/r_V. With the exception of the selected parameters, the values of other parameters have been kept the same as in Example 1. In addition, a credit term "2/10 net 30" is offered by the supplier. The optimal policies maximizing the channel's profit for the various parameters are reported in Table 4.

Table 4 shows that, as the parameter ρ increases, the buyer will order a smaller quantity within a shorter replenishment cycle in order to more frequently take advantage of a trade credit. In addition, the buyer decreases the retail price to improve customer demand for the product, which in turn results in an increase in the supplier's production size (n^*Q^*). Simultaneously, the entire channel's expected total profit rises as the parameter ρ increases.

Table 3
Summary of the channel profit under different scenarios

	Independent	Coordinated	Improvement
Cash on delivery	\$101 464	\$108 361	\$6897 (6.80%)
Trade credit (2/10, net 30)	\$102 116	\$109 063	\$6947 (6.80%)
Improvement	\$652 (0.64%)	\$702 (0.65%)	\$7599 (7.50%)

Table 4
The results of sensitivity analysis

S_B/S_V	r_B/r_V	n^*			p^* ($)			T^* (days)			Q^*			$\Pi(n^*, p^*, T^*)$		
		$\rho = 0.1$	$\rho = 0.5$	$\rho = 0.9$	$\rho = 0.1$	$\rho = 0.5$	$\rho = 0.9$	$\rho = 0.1$	$\rho = 0.5$	$\rho = 0.9$	$\rho = 0.1$	$\rho = 0.5$	$\rho = 0.9$	$\rho = 0.1$	$\rho = 0.5$	$\rho = 0.9$
0.01	1.0	27	39	90	10.53	10.41	10.23	10.45	9.70	9.25	378	355	346	110753	111371	112407
	1.5	28	40	93	10.53	10.42	10.23	10.10	9.45	8.99	365	346	336	110732	111351	112387
	2.0	29	41	95	10.54	10.43	10.24	9.77	9.23	8.80	352	337	329	110712	111331	112369
	2.5	30	42	97	10.55	10.43	10.24	9.47	9.01	8.61	341	329	322	110692	111313	112350
	3.0	31	43	99	10.55	10.44	10.25	9.18	8.81	8.44	331	321	315	110673	111294	112332
0.1	1.0	8	12	29	10.60	10.51	10.34	35.85	31.73	29.18	1284	1149	1078	109235	109753	110698
	1.5	8	13	29	10.62	10.53	10.36	35.16	29.70	28.68	1257	1073	1057	109164	109690	110638
	2.0	9	13	30	10.64	10.54	10.37	31.78	29.32	27.97	1133	1057	1029	109098	109630	110579
	2.5	9	13	31	10.65	10.56	10.39	31.26	28.95	27.32	1113	1042	1003	109035	109571	110522
	3.0	10	14	31	10.67	10.58	10.41	28.99	27.72	26.90	1029	995	986	108975	109513	110466
0.5	1.0	2	4	9	10.84	10.79	10.66	131.44	102.18	94.13	4577	3577	3348	107467	107687	108348
	1.5	3	4	10	10.89	10.83	10.70	105.45	98.82	89.17	3652	3443	3158	107245	107493	108168
	2.0	3	4	10	10.93	10.87	10.73	102.17	95.79	86.31	3522	3323	3044	107047	107307	107997
	2.5	3	4	10	10.97	10.91	10.76	99.20	93.04	83.73	3403	3213	2941	106856	107127	107832
	3.0	3	4	11	11.01	10.95	10.80	96.50	90.53	80.16	3296	3113	2805	106672	106952	107674

Furthermore, it is observed that as the value of S_B/S_V increases (i.e., the relative ordering cost for the buyer increases), the buyer will order a greater lot size within a longer inventory cycle in order to save cost while selling the items to customers at a higher retail price. This results in substantially fewer replenishments (lower value of n) for each production run and lower product demand. Conversely, as the value of r_B/r_V increases, i.e., as the relative carrying cost rate (excluding interest charge) for the buyer increases, the buyer will order a smaller lot size within a shorter inventory cycle, so more replenishments for each production run (higher value of n) are required. Also, the entire channel's expected total profit reduces as the ratio S_B/S_V and r_B/r_V increase.

6. Conclusion

In this paper, we first formulated an integrated supplier–buyer inventory model with the assumptions that the market demand is sensitive to the retail price and the supplier offers two payment options: trade credit and early-payments with discount price to the buyer. By analyzing the total channel profit function, we then developed a solution algorithm to determine the best payment method, the optimal retail price, order quantity and the number of shipment per production run from the supplier to the buyer. Numerical examples are presented to illustrate this model. Comprehensive sensitivity analyses for the effects of the parameters on the decisions are also offered.

Based on our analysis, it is found that a two-part trade credit term can increase profits not only for the supplier but also for the buyer and the entire supply chain. However, to achieve this goal the supplier should comprehensively estimate credit terms before adopting a two-part trade credit strategy. We also detect that as the supplier and buyer work in a cooperative manner to synchronize supply with customer demand, the channel's profit will improve significantly. Supply chain integration is found to be helpful in the supplier's profit gain and the buyer's cash flow management. To achieve jointly optimized supplier–buyer lot sizing and trade credit policies, which is known to be helpful for the supply chain in terms of profit, it is recommended that the supplier should share additional profits due to the joint procedure to encourage the buyer to cooperate.

As for future research, our model can be extended to more general supply chain networks, for example, multi-echelon or assembly supply chains. Also, it is interested to consider deteriorating items into the proposed model and considered the order quantity as a function of credit period.

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Appendix. Supplier and buyer make decision independently

Without cooperating with each other, the buyer will react to the supplier's credit terms by maximizing his/her own expected total profit per unit time $\mathrm{TBP}_j(p_B, T_B)$. So the problem becomes to determine the optimal values of p_B and T_B such that $\mathrm{TBP}_j(p_B, T_B)$, $j = 1, 2$ in (2) is maximized.

First, for fixed p_B, it can be easily determined that $\mathrm{TBP}_{ji}(p_B, T_B)$, $i = 1, 2$, is a concave function in T_B. Thus, for fixed p_B, the optimal solution of T_B (denoted by $T_{B_{ji}}(p_B)$) which maximize the expected total profit for the buyer will satisfy $\frac{\partial \mathrm{TBP}_{ji}(p_B, T_B)}{\partial T_B} = 0,\ i = 1, 2$. Solving these equations, we obtain

$$T_{B_{j1}}(p_B) = \sqrt{\frac{2S_B}{a p_B^{-\delta}[(1 - k_j\beta)v r_B + p_B I_{Be}]}} \quad \text{and}$$

$$T_{B_{j2}}(p_B) = \sqrt{\frac{2S_B + a p_B^{-\delta} M_j^2[(1 - k_j\beta)I_{Bp} - p_B I_{Be}]}{a p_B^{-\delta}[(1 - k_j\beta)v(r_B + I_{Bp})]}}.$$

To ensure $T_{B_{j1}}(p_B) < M_j$ and $T_{B_{j2}}(p_B) \geqslant M_j$, substituting the solutions into inequalities $T_{B_{j1}}(p_B) < M_j$ and $T_{B_{j2}}(p_B) \geqslant M_j$ respectively, we get

$$\text{if and only if} \quad S_B < \frac{ap_B^{-\delta}M_j^2}{2}[(1-k_j\beta)vr_B + p_BI_{Be}], \text{ then } T_{B_{j1}}(p_B) < M_j, \tag{A.1}$$

and

$$\text{if and only if} \quad S_B \geqslant \frac{ap_B^{-\delta}M_j^2}{2}[(1-k_j\beta)vr_B + p_BI_{Be}], \text{ then } T_{B_{j2}}(p_B) \geqslant M_j. \tag{A.2}$$

Same as in Section 4, motivated by (A.1) and (A.2), we can find an unique value $p_{B_{j0}}$ such that $S_B = \frac{ap_{B_{j0}}^{-\delta}M_j^2}{2}[(1-k_j\beta)vr_B + p_{B_{j0}}I_{Be}]$. Thus, (A.1) and (A.2) reduces to

$$\text{if and only if} \quad p_B < p_{B_{j0}}, \text{ then } T_{B_{j1}}(p_B) < M_j, \tag{A.3}$$

and

$$\text{if and only if} \quad p_B \geqslant p_{B_{j0}}, \text{ then } T_{B_{j2}}(p_B) \geqslant M_j. \tag{A.4}$$

Consequently, from (A.1)–(A.4), our problem becomes to find the optimal value of p_B which maximizing the following expected total profit function,

$$\text{TBP}_j(p_B) = \begin{cases} \text{TBP}_{j1}(p_B) & \text{if } p_B < p_{B_{j0}}, \\ \text{TBP}_{j2}(p_B) & \text{if } p_B \geqslant p_{B_{j0}}, \end{cases} \quad j = 1, 2. \tag{A.5}$$

Taking the first partial derivative of $\text{TBP}_{j1}(p_B)$ and $\text{TBP}_{j2}(p_B)$ with respect to p_B and setting the results to be zero, we have

$$\frac{\partial \text{TBP}_{j1}(p_B)}{\partial p_B} = ap_B^{-\delta}\left[\frac{\delta v}{p_B} + (1-\delta)(1+M_jI_{Be})\right] + \frac{\sqrt{ap_B^{-\delta}S_B}\left[\delta(1-k_j\beta)vr_B + (\delta-1)p_BI_{Be}\right]}{p_B\sqrt{2\left[(1-k_j\beta)vr_B + p_BI_{Be}\right]}} = 0 \tag{A.6}$$

and

$$\begin{aligned}\frac{\partial \text{TBP}_{j2}(p_B)}{\partial p_B} = {} & ap_B^{-\delta}\left[\frac{\delta}{p_B}(1-k_j\beta)v(M_jI_{Bp}-1) - (\delta-1)\right] \\ & + \left\{\frac{\delta}{p_B}\left\{S_B + ap_B^{-\delta}M_j^2\left[(1-k_j\beta)vI_{Bp} - p_BI_{Be}\right]\right\}\right. \\ & \left. + \frac{1}{2}ap_B^{-\delta}M_j^2I_{Be}\right\}\sqrt{\frac{ap_B^{-\delta}[(1-k_j\beta)v(r_B+I_{Bp})]}{2S_B + ap_B^{-\delta}M_j^2[(1-k_j\beta)vI_{Bp} - p_BI_{Be}]}} = 0.\end{aligned} \tag{A.7}$$

Next, as the second-order condition $\frac{\partial^2 \text{TBP}_{ji}(p_B)}{\partial p_B^2} < 0$ holds, $j = 1, 2$, $\text{TBP}_{ji}(p_B)$ is a concave function in p_B. By using the similar steps 2–3 in Algorithm 1, we can get (p_B^*, T_B^*) such that $\text{TBP}(p_B^*, T_B^*) = \max_{j=1,2;i=1,2}\text{TBP}_{ji}(p_B, T_B)$. Hence, the optimal solution for the buyer is (p_B^*, T_B^*).

Furthermore, for given (p_B^*, T_B^*), we examine the effect of n on the supplier's expected total profit per unit time. With $\frac{\partial^2 \text{TVP}_j(n_V)}{\partial n_V^2} = -\frac{2S_V}{n_V^3 T_B^*} < 0$, $\text{TVP}_j(n_V)$ is a concave function in n_V. Therefore, the search for the optimal shipment number, n_V^*, is reduced to find a local optimal solution.

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