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Production Planning and Control

ISSN: 0953-7287 (Print) 1366-5871 (Online) Journal homepage: https://www.tandfonline.com/loi/tppc20

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**To cite this article:** Tsu-Pang Hsieh , Horng-Jinh Chang , Ming-Wei Weng & Chung-Yuan Dye (2008) A simple approach to an integrated single-vendor single-buyer inventory system with shortage, Production Planning and Control, 19:6, 601-604, DOI: 10.1080/09537280802462789

**To link to this article:** https://doi.org/10.1080/09537280802462789

Published online: 05 Dec 2008.

*Production Planning & Control*
Vol. 19, No. 6, September 2008, 601–604

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# A simple approach to an integrated single-vendor single-buyer inventory system with shortage

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(*Received 18 June 2008; final version received 4 September 2008*)

In this article, a simple approach with two basic inequalities (Cauchy–Schwarz inequality and arithmetic–geometric mean inequality) is used to solve the integrated single-vendor single-buyer inventory problem developed by Wu and Ouyang (Wu, K.-S. and Ouyang, L.-Y., 2003. An integrated single-vendor single-buyer inventory system with shortage derived algebraically. *Production Planning & Control*, 14 (6), 555–561). Without the method of completing perfect square, the proposed approach yields the global minimum of the integrated total cost per year more easily than the algebraic approach used by Wu and Ouyang (2003). In addition, for people without the background of calculus, it is more useful to determine the buyer's economic order quantity and the vendor's optimal number of deliveries.

**Keywords:** without derivatives; arithmetic–geometric mean inequality; Cauchy–Schwarz inequality; single-vendor single-buyer; shortage

## 1. Introduction

For many people who lack the knowledge of calculus, the method of completing perfect square is proposed to solve the economic order quantity (EOQ) or economic production quantity (EPQ) models in several research articles, for example Grubbström and Erdem (1999), Chang (2004), Ronald *et al*. (2004), Chang *et al*. (2005) and Sphicas (2006). In 2003, Wu and Ouyang developed an integrated single-vendor single-buyer inventory system with shortage. Without differential calculus, they extended Grubbström and Erdem's (1999) method to solve the three-variable problems algebraically. However, their method of completing perfect square is still complex.

In contrast to all papers mentioned above, Teng (in press) proposed a simple method by using the arithmetic–geometric mean inequality (or more briefly the AM–GM inequality) theorem to compute the global minimum economic order quantities. For EOQ or EPQ models to determine only one decision variable, i.e. the size of order, Teng's method yields the global minimum solution explicitly and immediately but fails to solve the multi-variable inventory problem.

In this article, we propose a simple approach with basic inequalities such as Cauchy–Schwarz inequality and AM–GM inequality to solve Wu and Ouyang's model (2003). Without taking differential calculus or using the method of completing the square, the solution procedure proposed by using basic inequalities is easier to find the optimal solutions (the buyer's lot size per order, maximum backorder level and the vendor's number of deliveries). In addition, the minimum integrated total cost of the proposed model is obtained more directly.

## 2. Model discussion

In contrast to the method of completing the square adopted by Wu and Ouyang (2003), two basic inequalities (Cauchy–Schwarz inequality and AM–GM inequality) are used to solve the integrated inventory problems including three decision variables: $Q$ (buyer's lot size per order), $B$ (maximum backorder level) and $n$ (the vendor's number of deliveries). First, these inequalities are shown shortly as follows:

**AM–GM mean inequality:** Let $x_1, x_2, \ldots, x_n$ be $n$ positive real numbers, then

$$\frac{x_1 + x_2 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 x_2 \cdots x_n},$$

with the equality holds if only $x_1 = x_2 = \cdots = x_n$.

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ISSN 0953–7287 print/ISSN 1366–5871 online

DOI: 10.1080/09537280802462789
http://www.informaworld.com

**Cauchy–Schwarz inequality:** Let $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ be two vectors in $n$ space, then

$$\left(a_1^2 + a_2^2 + \cdots + a_n^2\right)\left(b_1^2 + b_2^2 + \cdots + b_n^2\right) \geq (a_1 b_1 + a_2 b_2 + \cdots + a_n b_n)^2$$

with the equality holds if only $a_1/b_1 = a_2/b_2 = \cdots = a_n/b_n$.

Now we can begin to discuss the model developed by Wu and Ouyang (2003). The integrated total annual cost function simplified by Wu and Ouyang (2003) can be written in the form

$$\begin{aligned} TC &= \frac{dC_b}{Q} + \frac{Q}{2}\left[H_b\left(1-\frac{B}{Q}\right)^2 + S_b\left(\frac{B}{Q}\right)^2 + H_v\left(\frac{2d}{p}-1\right)\right] \\ &\quad + \frac{dC_v}{nQ} + \frac{nQH_v}{2}\left(1-\frac{d}{p}\right) \\ &= \frac{dC_b}{Q} + \frac{dC_v}{nQ} + \frac{QH_v}{2}\left[(n-1)\left(1-\frac{d}{p}\right)+\frac{d}{p}\right] \\ &\quad + \frac{Q}{2}\left[H_b\left(1-\frac{B}{Q}\right)^2 + S_b\left(\frac{B}{Q}\right)^2\right]. \end{aligned} \quad (1)$$

The Cauchy–Schwarz and AM–GM inequalities imply that

$$\begin{aligned} TC &= \frac{dC_b}{Q} + \frac{dC_v}{nQ} + \frac{QH_v}{2}\left[(n-1)\left(1-\frac{d}{p}\right)+\frac{d}{p}\right] \\ &\quad + \frac{Q}{2}\left\{\left[\sqrt{H_b}\left(1-\frac{B}{Q}\right)\right]^2 + \left[\sqrt{S_b}\left(\frac{B}{Q}\right)\right]^2\right\} \\ &\quad \times \left[\left(\frac{\sqrt{S_b}}{\sqrt{H_b+S_b}}\right)^2 + \left(\frac{\sqrt{H_b}}{\sqrt{H_b+S_b}}\right)^2\right] \\ &\geq \frac{dC_b}{Q} + \frac{dC_v}{nQ} + \frac{QH_v}{2}\left[(n-1)\left(1-\frac{d}{p}\right)+\frac{d}{p}\right] + \frac{Q}{2}\frac{H_b S_b}{H_b+S_b} \\ &= \frac{d}{Q}\left(C_b + \frac{C_v}{n}\right) + \frac{Q}{2}\left\{\frac{H_b S_b}{H_b+S_b} \right. \\ &\quad \left. + H_v\left[(n-1)\left(1-\frac{d}{p}\right)+\frac{d}{p}\right]\right\} \\ &\geq \sqrt{2d\left(C_b+\frac{C_v}{n}\right)\left\{\frac{H_b S_b}{H_b+S_b} + H_v\left[(n-1)\left(1-\frac{d}{p}\right)+\frac{d}{p}\right]\right\}}. \end{aligned}$$

The first inequality follows easily from Cauchy–Schwarz inequality and the second follows from AM–GM inequality, respectively. Consequently, $TC$ attains its minimum on

$$\sqrt{2d\left(C_b+\frac{C_v}{n}\right)\left\{\frac{H_b S_b}{H_b+S_b} + H_v\left[(n-1)\left(1-\frac{d}{p}\right)+\frac{d}{p}\right]\right\}} \quad (2)$$

with equality holds if only

$$\frac{\sqrt{H_b}(1-B/Q)}{\sqrt{S_b}/\sqrt{H_b+S_b}} = \frac{\sqrt{S_b}(B/Q)}{\sqrt{H_b}/\sqrt{H_b+S_b}}$$

and

$$\frac{d}{Q}\left(C_b+\frac{C_v}{n}\right) = \frac{Q}{2}\left\{\frac{H_b S_b}{H_b+S_b} + H_v\left[(n-1)\left(1-\frac{d}{p}\right)+\frac{d}{p}\right]\right\}.$$

After some algebraic manipulation, the optimal $Q$ and $B$ are given by

$$Q^* = \sqrt{\frac{2d(C_b + C_v/n)}{H_b S_b/(H_b+S_b) + H_v[(n-1)(1-d/p)+d/p]}} \quad (3)$$

and

$$B^* = \frac{H_b}{H_b+S_b} \times \sqrt{\frac{2d(C_b + C_v/n)}{H_b S_b/(H_b+S_b) + H_v[(n-1)(1-d/p)+d/p]}}, \quad (4)$$

respectively.

From Equation (2), because

$$\begin{aligned} &\left(C_b+\frac{C_v}{n}\right)\left\{\frac{H_b S_b}{H_b+S_b} + H_v\left[(n-1)\left(1-\frac{d}{p}\right)+\frac{d}{p}\right]\right\} \\ &= C_b\left[\frac{H_b S_b}{H_b+S_b} + H_v\left(\frac{2d}{p}-1\right)\right] + C_v H_v\left(1-\frac{d}{p}\right) \\ &\quad + \frac{C_v}{n}\left[\frac{H_b S_b}{H_b+S_b} + H_v\left(\frac{2d}{p}-1\right)\right] + C_b H_v n\left(1-\frac{d}{p}\right), \end{aligned}$$

if $H_b S_b/(H_b+S_b) + H_v(2d/p-1) \leq 0$, it can be easily observed that the integrated total cost per year $TC$ has a global minimum as $n = 1$. Therefore, we have

$$\begin{aligned} TC &\geq \sqrt{2d\left(C_b+\frac{C_v}{n}\right)\left\{\frac{H_b S_b}{H_b+S_b} + H_v\left[(n-1)\left(1-\frac{d}{p}\right)+\frac{d}{p}\right]\right\}} \\ &\geq \sqrt{2d(C_b+C_v)\left(\frac{H_b S_b}{H_b+S_b} + H_v\frac{d}{p}\right)}, \end{aligned}$$

and consequently,

$$Q^* = \sqrt{\frac{2d(C_b+C_v)}{H_b S_b/(H_b+S_b) + H_v d/p}} \quad \text{and} \quad B^* = \frac{H_b}{H_b+S_b} Q^*. \quad (5)$$

On the other hand, if $H_b S_b/(H_b+S_b) + H_v(2d/p-1) > 0$, then $H_b S_b/(H_b+S_b) + H_v(2d/p-1)$ and $(1-d/p)$ are both positive real numbers. We recall

Equation (2) and apply AM–GM inequality again, it is easy to get

$$\begin{aligned}\frac{TC^2}{2d} &\geq C_b\left[\frac{H_bS_b}{H_b+S_b}+H_v\left(\frac{2d}{p}-1\right)\right]+C_vH_v\left(1-\frac{d}{p}\right)\\ &\quad+\frac{C_v}{n}\left[\frac{H_bS_b}{H_b+S_b}+H_v\left(\frac{2d}{p}-1\right)\right]+C_bH_vn\left(1-\frac{d}{p}\right)\\ &\geq C_b\left[\frac{H_bS_b}{H_b+S_b}+H_v\left(\frac{2d}{p}-1\right)\right]+C_vH_v\left(1-\frac{d}{p}\right)\\ &\quad+2\sqrt{C_vC_bH_v\left(1-\frac{d}{p}\right)\left[\frac{H_bS_b}{H_b+S_b}+H_v\left(\frac{2d}{p}-1\right)\right]}.\end{aligned}$$

Hence in this case, we can determine the optimal $n$ that minimises $TC$ as

$$\frac{C_v}{n}\left[\frac{H_bS_b}{H_b+S_b}+H_v\left(\frac{2d}{p}-1\right)\right]=C_bH_vn\left(1-\frac{d}{p}\right)$$

and so,

$$n=\sqrt{\frac{C_v[H_bS_b/(H_b+S_b)+H_v(2d/p-1)]}{C_bH_v(1-d/p)}}. \qquad (6)$$

Since the number of deliveries must be a positive integer, the solution obtained in Equation (6) will be a good approximation to find the vendor's optimal number of deliveries in order to avoid using a brute force enumeration.

## 3. Conclusions

In this article, we provide a simple approach to solve Wu and Ouyang's (2003) model by using two basic inequalities (Cauchy–Schwarz inequality and AM–GM inequality). There are a lot of ways to solve this problem, but this is one of the most elementary methods, requiring only basic knowledge of inequalities. Without taking complex differential calculus or using complicated quadratic expression derived by algebraic manipulations, we can obtain the global minimum solutions much more easily and simply.

## Acknowledgements

We would like to thank the associate editor and two anonymous reviewers for their valuable and constructive comments, which have led to a significant improvement in the manuscript. This research was partially supported by the National Science Council of the Republic of China under Grant NSC-97-2221-E-156-005 and NSC-97-2221-E-366-006-MY2.

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