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An Optimal Demand Function Constructed by Consumer Willing-to-Pay Price and Transaction Cost

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Abstract

The demand function of a product is constructed as a bivariate normal distribution function with the consumer willing-to-pay price and the unit transaction cost as variables. Such a bivariate distributed demand function is much different from those of traditional demand functions, and can catch more practical situations. This study focuses on the reasoning process in constructing such a bivariate distributed demand function and its characteristics, as well as its feasible applications.

Keywords: Transaction Cost, Demand Function, Willing-to-Pay Price, Bivariate Normal Distribution.

1. Introduction

Based on a historical perspective, the theoretical evolution of demand functions can be roughly divided into three phases. The phase one theoretically features a consumer-driven demand model that is built on individual components of consumers' demand. The primary focal points of the above-mentioned model are to formulate or explore the contribution factors to consumers' demand [4].

The theoretical basis of phase two is the consumers' choice model; it assumes that a demand function is formulated by the lowest delivered price which is decided by consumers after all contribution factors have been carefully considered [3]. However, the consumers' choice model assumes that all factors taken into considerations are mutually distribution-independent, and thus we can neglect the possibility of factors without mutually distribution-independent [6]. Consequently, the application of the consumers' choice model is limited, especially for a case while the transaction cost of consumption behavior have treated travel cost as the sole cost item incurred on customers to obtain

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their purchase is discussed [1]. Neither phase two nor one model has been given due to the recognition of the fact that the convenience of delivery offered by producers to stimulate the consumption [5].

The theoretical basis of the proposed demand functions in phase three features the optimal consumer utility: given finite budget constraint, consumers will formulate their personal demand functions to maximize their personal utility after analyzing the effects of various price-quantity portfolios on their personal utility. With regard to a product under a given price, its demand function of the market can be obtained by aggregating those individual consumer demand functions. The distinctions of phase one and phase two are that all products and factors contributing to consumers' demand functions are no longer treated as mutually independent [2].

To construct a demand function in different historical phases, the theory of optimal consumer utility features the additional external flexible factors contributing to consumers' demand functions as required under specific decision-making conditions, and thus this flexibility has become the most widely adopted in economic studies today.

Before such a theory of the consumers' optimal utility function can be put into practical applications, two problems have to be solved: (1) how to estimate the value of utility functions (no matter it is the ordinal or cardinal of individual consumers' utility) and how to evaluate the budget, and then the demand function of individual consumers is derived, (2) how to derive the demand function for a specific product in the market by aggregating individual consumers demand functions.

In this paper, a new demand function for a specific product is constructed by means of a bivariate normal distribution of the willing-to-pay v and the unit transaction cost t. The rationale and the process of constructing such a demand function are different from those demand function models in the aforesaid three phases. In this study, the demand function is introduced in the direct sampling survey for estimating the distribution parameters (v, t) of consumer groups in the market, and thus a complete and concrete demand function can be derived. Furthermore, this demand function is related to practical marketing applications as well as development of microeconomic theories.

2. Notations

p : The selling price of a unit product.

t : The transaction cost of a unit product paid by a customer who demands that product, excluding selling price, but including transportation cost incurred on that customer who obtains the product, the time cost that consumer perceived on a delayed pick-up, and the setup cost for completing the purchase. Because the transaction cost paid for obtaining a product is different from customer to customer, so is the value of t. Generally speaking, consumers would have a positive value of t. However, the negative value of t exists for some consumer who treats the transaction process as his entertainment. The number of consumers with a negative value of t is related to the specific characteristics of a product, e.g., hairdressing. The esthetic utility of a haircut as well as the utility of comfort experienced in the service process may contribute to a negative value of t. In this study, u_t and σ_t are used to denote the expected value and the standard deviation of t, respectively.

v : The maximum price of a unit product that a consumer would be willing to pay. Since v includes the selling price p and the transaction cost t, we have $v \geq t$. Since different consumers may have different levels of preference for a product, the value of v is various. Most consumers have a positive value of v, but a minority of them may have a negative value of v: those consumers who do not need such a product, even though its price is zero, will have a negative value of v. In this study, u_v and σ_v are used to denote the expected value and the standard deviation of v, respectively; where $u_v > \mu_t$, and $\sigma_v > \sigma_t$.

$f(y, z)$: The probability density function of bivariate (y, z) for demand groups, where $y = \dfrac{v - \mu_v}{\sigma_v}$, $z = \dfrac{t - \mu_t}{\sigma_t}$.

r : The correlation coefficient between y and z.

e : Unit transaction cost invested by the supplier. Suppliers often offer convenience of delivery to consumers, such as home delivery, setting up extensive supply outlets, providing parking lots or nursery rooms affiliated to a supply outlet, or absorbing part of transaction cost incurred on consumers.

Generally speaking, the higher value of e the supplier has, the more convenient it will be for the demand groups to obtain a product, and thus the lower will be the value of u_t, i.e., the expected transaction cost. Hence u_t can be a strict decreasing function of e. The assumption of this study is as following:

$$\mu_t'(e) < 0, \quad \mu_t''(e) > 0, \qquad \text{for all} \ \ e. \tag{1}$$

N : Potential demand of demand groups for product per unit time. The potential demand means the would-be demand regardless the price level. If the demand of product for a consumer's experiences is increasing in his utility by obtaining that product, it is then by definition that consumer has a potential demand for that product. The necessary condition for a consumer to purchase a product is that the corresponding p and t of that consumer must meet the following inequality:

$$v \geq p + t. \tag{2}$$

The potential demand for that product can be realized into real demand only when inequality (2) is satisfied.

3. Sales Rate (Q) and Consumer Surplus (CS) of the Supplier

From (2), we have

$$\sigma_v y + \mu_v = v \geq p + t = p + \sigma_t z + \mu_t,$$

where $\mu_t = \mu_t(e)$. Hence the consumer surplus is $(\sigma_v y + \mu_v) - (p + \sigma_t z + \mu_t)$.

Once the supplier has determined their (p, e) value, if the corresponding coordinate (y, z) of the demand of a customer for a product falls within zone R in Figure 1, a customer will actually buy that product. The sales rate Q of the supplier is the product of N and the integral of distribution function f for zone, as the following equation:

$$Q(p,e) = N \cdot \iint_R f(y,z)dzdy = N \cdot \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\sigma_t^{-1}(\sigma_v \cdot y + \mu_v - p - \mu_t)} f(y,z)dz \right] dy, \tag{3}$$

where $\mu_t = \mu_t(e)$. Also, consumer surplus of the demand groups is

$$CS(p,e) = N \cdot \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\sigma_t^{-1}(\sigma_v \cdot y + \mu_v - p - \mu_t)} (\sigma_v y + \mu_v - p - \mu_t - \sigma_t z) f(y,z)dz \right] dy. \tag{4}$$

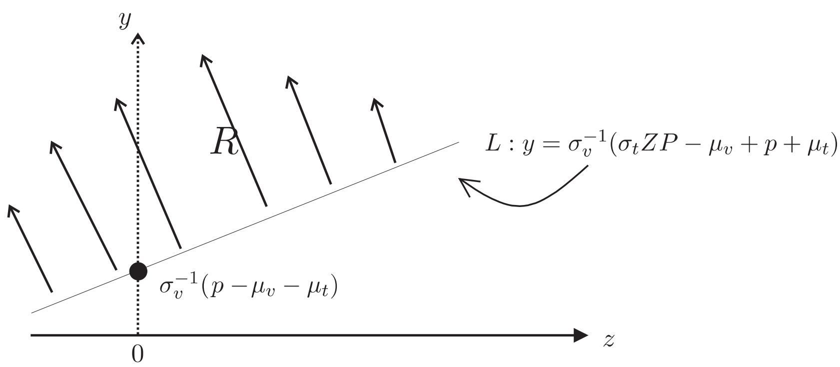

Figure 1. Relationship between sales rate Q and consumer surplus L.

From the partial differential of (3) with respect to p and e, we have

$$\frac{1}{N}\cdot\frac{\partial Q}{\partial p} = -\sigma_t^{-1}\int_{-\infty}^{\infty} f(y,\sigma_t^{-1}(\sigma_v y+\mu_v-p-\mu_t))dy, \tag{5}$$

$$\frac{1}{N}\cdot\frac{\partial Q}{\partial e} = -\left[\sigma_t^{-1}\int_{-\infty}^{\infty} f(y,\sigma_t^{-1}(\sigma_v y+\mu_v-p-\mu_t))dy\right]\mu_t'(e_t). \tag{6}$$

Then, by the comparison of (5) and (6), we have

$$\frac{\partial Q}{\partial e} = \mu_t'(e)\frac{\partial Q}{\partial p}. \tag{7}$$

If q is given, let function $p_q(e)$ be determined by the equation $Q(p_q(e),e) = q$, the supplier's optimization problem, can be written as follows:

$$\begin{cases} \mathrm{Max}_e(p_q(e)-e) \\ \text{Subject to: } Q(p_q(e),e)=q, \quad \text{where } g \text{ is given} \end{cases}. \tag{8}$$

Let $e(q)$ be the optimum solution of model (8), it can be obtained that the necessary condition for an optimum solution is:

$$0 = \left.\frac{d[p_q(e)-e]}{de}\right|_{e^*} = p_q'(e(q))-1, \tag{9}$$

or

$$p_q'(e(q)) = 1, \quad \forall q. \tag{10}$$

Consider a partial differential of the constraint in (8) with respect to e, the following equation is obtained by the chain rule

$$0 = \frac{dq}{de} = \frac{dQ(p_q(e),e)}{de} = Q_p\cdot p_q'(e)+Q_e\cdot 1, \quad \forall q$$

then, from (7),

$$p_q'(e)\Big|_{e^*} = -\left.\frac{Q_e(p(e),e)}{Q_p(p(e),e)}\right|_{e^*} = -\mu_t'(e(q)), \quad \forall q \tag{11}$$

and from (10) and (11), we have $\mu_t'(e(q)) = -1$. This means that the value of $e(q)$ depends on only the function μ_t'. That is $e(\overline{q}) = e^*$, $\forall q$, where is determined by the equation: $\mu_t'(e^*) = -1$.

In the following, let q be a function of p and let $\mu^* = \mu_t(e^*)$, then from (3), the demand function $q = q(p)$ can be obtained as follows:

$$q = q(p) = N\int_{-\infty}^{\infty}\int_{-\infty}^{\sigma_t^{-1}(\sigma_v y+\mu_v-p-\mu^*)} f(y,z)dzdy. \tag{12}$$

4. Characteristics of the Demand Function $p = p(q)$ when $f(y, z)$ is a Bivariate Normal Distribution Function

An equation (12) yields that $q = q(p)$ is a strictly decreasing function of p. Let $p = p(q)$ be the inverse function of (12), then $p(q(p)) = p$, $\forall p$, and hence $p'(q) \cdot q'(p) = 1$. The differential of (12) is as follows:

$$p'(q) = \frac{1}{q'(p)} = \frac{-\sigma_t}{N \int_{-\infty}^{\infty} f(y, \sigma_t^{-1}(\sigma_v y + \mu_v - p - \mu^*))dy} < 0, \tag{13}$$

where

$$\begin{aligned}
&\int_{-\infty}^{\infty} f(y, \sigma_t^{-1}(\sigma_v p + \mu_v - p - \mu^*))dy \\
&= \frac{1}{2\pi\sqrt{1-\gamma^2}} \int_{-\infty}^{\infty} \exp\Bigg[\frac{-1}{2(1-\gamma^2)}\Big(y^2 + \sigma_t^{-2}(\sigma_v y + \mu_v - p - \mu^*)^2 \\
&\quad -2\gamma \cdot y \cdot \sigma_t^{-1}(\sigma_v y + \mu_v - p - \mu^*)\Big)\Bigg]dy \\
&= \frac{1}{2\pi\sqrt{1-\gamma^2}} \int_{-\infty}^{\infty} \exp\Bigg[\frac{-1}{2(1-\gamma^2)}\Bigg(\Big(1 + \frac{\sigma_v^2}{\sigma_t^2} - 2\gamma\frac{\sigma_v}{\sigma_t}\Big)y^2 \\
&\quad +2\Big(\frac{\sigma_v}{\sigma_t} - \gamma\Big)y \cdot \frac{\mu_v - p - \mu^*}{\sigma_t} + \Big(\frac{\mu_v - p - \mu^*}{\sigma_t}\Big)^2\Bigg)\Bigg]dy \\
&= \frac{1}{2\pi\sqrt{1-\gamma^2}} \int_{-\infty}^{\infty} e^{\frac{-y^2}{2}} \cdot e^{\frac{-\left[y(\frac{\sigma_v}{\sigma_t}-\gamma)+\frac{\mu_v-p-\mu^*}{\sigma_t}\right]^2}{2(1-\gamma^2)}} dy.
\end{aligned} \tag{14}$$

It can be obtained from (13) and (14) that:

$$p''(q) = \frac{\sigma_t^{-1} \cdot 2\pi\sqrt{1-\gamma^2}}{N\left[\int_{-\infty}^{\infty} e^{\frac{-\gamma^2}{2}} \cdot e^{\frac{-\left[y(\frac{\sigma_v}{\sigma_t}-\gamma)+\frac{\mu_v-p(q)-\mu^*}{\sigma_t}\right]^2}{2(q-\gamma^2)}}\right]^2} \left[\frac{d}{dq}\int_{-\infty}^{\infty} e^{\frac{-y^2}{2}} \cdot e^{\frac{-\left[y(\frac{\sigma_v}{\sigma_t}-\gamma)+\frac{\mu_v-p(q)-\mu^*}{\sigma_t}\right]^2}{2(1-\gamma^2)}} dy\right], \tag{15}$$

where

$$\frac{d}{dq}\int_{-\infty}^{\infty} e^{\frac{-y^2}{2}} \cdot e^{\frac{-\left[y(\frac{\sigma_v}{\sigma_t}-\gamma)+\frac{\mu_v-p-\mu^*}{\sigma_t}\right]^2}{2(1-\gamma^2)}} dy$$

$$= \frac{-p'(q)}{\left(\dfrac{\sigma_v}{\sigma_t}-\gamma\right)\cdot\sigma_t}\int_{-\infty}^{\infty} e^{\frac{-y^2}{2}}\, de^{\frac{-\left[y(\frac{\sigma_v}{\sigma_t}-\gamma)+\frac{\mu_v-p-\mu^*}{\sigma_t}\right]^2}{2(1-\gamma^2)}}.$$

Using integral by parts, then

$$= \frac{p'(q)}{\left(\dfrac{\sigma_v}{\sigma_t}-\gamma\right)\cdot\sigma_t}\int_{-\infty}^{\infty} e^{\frac{-\left[y(\frac{\sigma_v}{\sigma_t}-\gamma)+\frac{\mu_v-p-\mu^*}{\sigma_t}\right]^2}{2(1-\gamma^2)}}\, de^{\frac{-y^2}{2}}$$

$$= \frac{-p'(q)}{\left(\dfrac{\sigma_v}{\sigma_t}-\gamma\right)\cdot\sigma_t}\int_{0}^{\infty} e^{\frac{-y^2}{2}}\cdot y\cdot\left[e^{\frac{-\left[y(\frac{\sigma_v}{\sigma_t}-\gamma)+\frac{\mu_v-p-\mu^*}{\sigma_t}\right]^2}{2(1-\gamma^2)}} - e^{\frac{-\left[-y(\frac{\sigma_v}{\sigma_t}-\gamma)+\frac{\mu_v-p-\mu^*}{\sigma_t}\right]^2}{2(1-\gamma^2)}}\right]dy$$

$$= \frac{-p'(q)}{(\sigma_v-\sigma_t\gamma)}\int_{0}^{\infty} e^{\frac{-y^2}{2}}\cdot y\cdot\left[e^{\frac{-\left[y(\frac{\sigma_v}{\sigma_t}-\gamma)+\frac{\mu_v-p-\mu^*}{\sigma_t}\right]^2}{2(1-\gamma^2)}} - e^{\frac{-\left[y(\frac{\sigma_v}{\sigma_t}-\gamma)-\frac{\mu_v-p-\mu^*}{\sigma_t}\right]^2}{2(1-\gamma^2)}}\right]dy. \quad (16)$$

Condition 1: For any given p for $p < \mu_v - \mu^*$, i.e., $q(p) > q_I$ (shown in Figure 4), since $e^{\frac{-\left[y(\frac{\sigma_v}{\sigma_t}-\gamma)+\frac{\mu_v-p-\mu^*}{\sigma_t}\right]^2}{2(1-\gamma^2)}} < e^{\frac{-\left[y(\frac{\sigma_v}{\sigma_t}-\gamma)-\frac{\mu_v-p-\mu^*}{\sigma_t}\right]^2}{2(1-\gamma^2)}}$, $\forall\ y > 0$ (shown in Figure 2), and $p'(q) < 0$ (see (13), if (16) has a negative value, $p''(q) < 0$, $\forall\ q > q_I$ can be obtained by (15). It means that p, $p = p(q)$, in the interval q, $q \in [q_I, \overline{q}]$, is a strictly concave function.

Condition 2: For any given p for $p > \mu_v - \mu^*$, i.e., $q(p) < q_I$ (shown in Figure 4), since $e^{\frac{-\left[y(\frac{\sigma_v}{\sigma_t}-\gamma)+\frac{\mu_v-p-\mu^*}{\sigma_t}\right]^2}{2(1-\gamma^2)}} > e^{\frac{-\left[y(\frac{\sigma_v}{\sigma_t}-\gamma)-\frac{\mu_v-p-\mu^*}{\sigma_t}\right]^2}{2(1-\gamma^2)}}$, $\forall\ y > 0$ (shown in Figure 3), $p''(q) > 0$, $\forall\ q > q_I$ can be obtained by (15) and (16). It means that p, $p = p(q)$, in the interval q,

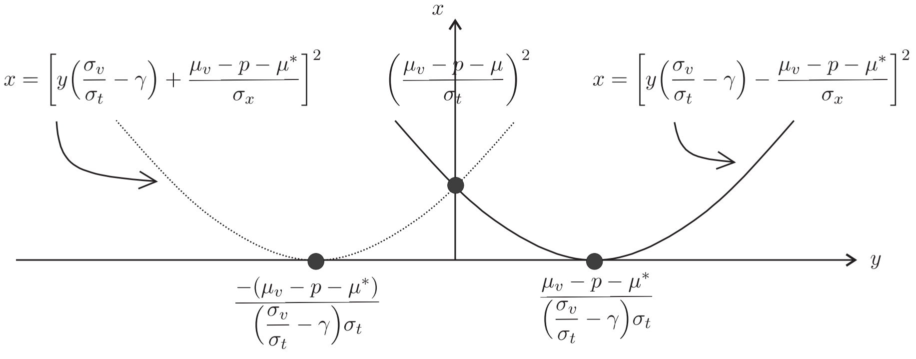

Figure 2. Condition 1.

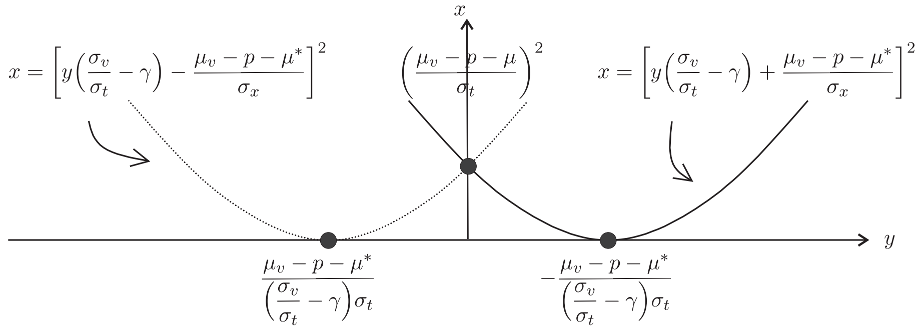

Figure 3. Condition 2.

$q \in [0, q_I]$, is a strictly convex function.

Condition 3: Assume $\mu_v - p(q) - \mu^* = 0$, i.e., $q = p^{-1}(\mu_v - \mu^*)$. Since the integral value of (16) is zero, $p''(q) = 0$ can be obtained by (15).

Combine Conditions 1, 2 and 3, that $q_I = p^{-1}(\mu_v - \mu^*)$ is the only inflection point of the graph of demand function can be verified. This means that $q_I = p^{-1}(\mu_v - \mu^*)$ is the only maximum point of function $p'(q)$, $q \in [0, \infty]$. Hence, $p'(q_I) = \underset{q\geq 0}{\text{Max}}\, p'(q)$, or $q'(\mu_v - \mu^*) = \underset{q\geq 0}{\text{Min}}\, q'(p)$ (shown in Figure 4′). By (12), in Figure 4 $\overline{q}$,

$$\begin{aligned}\overline{q} &= N\int_{-\infty}^{\infty}\int_{-\infty}^{\sigma_t^{-1}(\sigma_v y+\mu_v-\mu^*)} f(y,z)dzdy\\ &= \frac{N}{2\pi\sqrt{1-\gamma^2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\sigma_t^{-1}(\sigma_v y+\mu_v-\mu^*)} \exp\left[-\frac{y^2+z^2-2\gamma yz}{2(1-\gamma^2)}\right]dzdy. \qquad (17)\end{aligned}$$

Also, by (12), it can be obtained that $\lim_{p\to\infty} q(p) = 0$, and thus the vertical axis $q \equiv 0$ is the vertical approximation line for demand function $p = p(q)$.

So, the demand function $p = p(q)$ can be shown as the following figure:

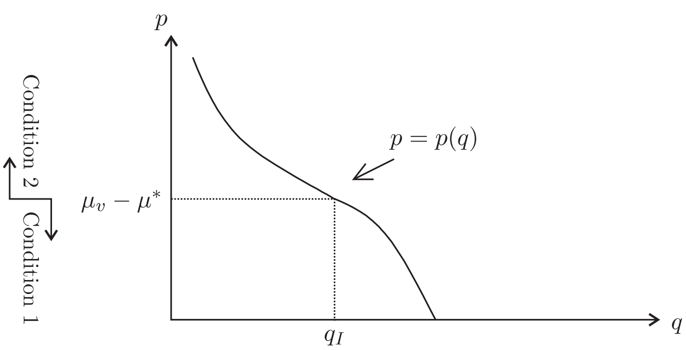

Figure 4. Demand function $p = p(q)$.

Differentiating (13) yields that $p''(q) = \dfrac{-1}{q/(p)^2} \cdot q''(p) \cdot p'(q)$. Since $p''(q) > 0$ if and only if $q < q_1$ (c.f., Figure 4), so $q''(p) > 0$ if and only if $p > \mu_v - \mu^*$. Therefore, the graph of inverse function of $p = p(q)$, or $q = q(p)$, is as follows:

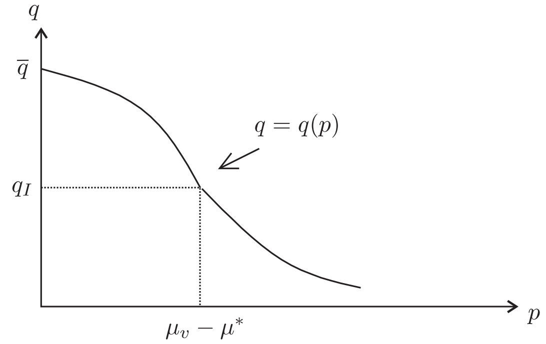

Figure 4′. Demand function $q = q(p)$.

From (12), q_I in Figure 4 can be expressed as

$$q_I = N \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\sigma_t^{-1}(\sigma_v y)} f(y,z)dz \right] dy. \tag{18}$$

It can be obtained by (18) that q_I is determined only by $\dfrac{\sigma_v}{\sigma_t}$, and is not related to μ_v, μ_t, μ^*. (19)

Eq. (12) can also be applied to the sensitivity analysis of the demand function with respect to its associated parameters.

5. Sensitivity Analyses

The effects of related parameters on the demand function are as follows:

(1) The effects of changes in μ_v on the demand function and other conditions remaining unchanged: if μ_v increases to $\overline{\mu}_v$, the integral area of equation (12) will increase. Consequently, the inequality $q_{\overline{\mu}_v}(p) > q_{\mu_v}(p)$ is satisfied to any given p, i.e., the graph of demand function $p_{\mu_v}(q)$ will move upward to the graph of $p_{\overline{\mu}_v}(q)$ (Figure 5). In addition, from (18) it can be obtained that the inflection point of the graph for $p_{\mu_v}(q)$ and the inflection point of the graph for $p_{\overline{\mu}_v}(q)$ have the same horizontal coordinate (as q_I in Figure 5).

(2) The effects of changes in σ_v on the demand function and other conditions remaining unchanged: if σ_v increases to $\overline{\sigma}_v$, the integral area of (12) will increase. Consequently, the inequality $q_{\overline{\sigma}_v}(p) > q_{\sigma_v}(p)$ for any given p, i.e., the graph of demand function $p_{\sigma_v}(q)$

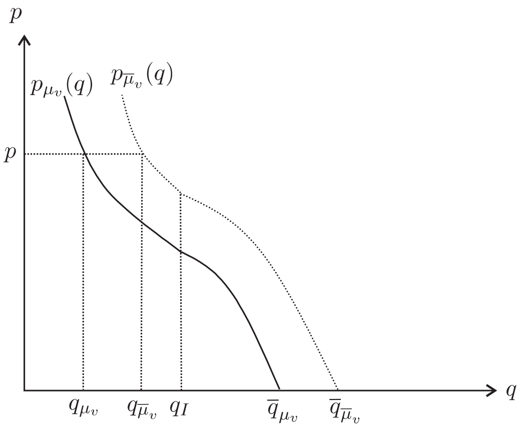

Figure 5. μ_v increasing to $\overline{\mu}_v$.

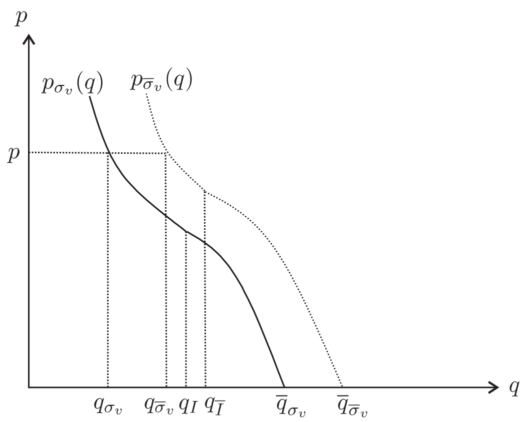

Figure 6. σ_v increasing to $\overline{\sigma}_v$.

will move upward to the graph of $p_{\overline{\sigma}_v}(q)$ (Figure 6). Also, from (18) it can be obtained that the inflection point q_I of the graph for $p_{\sigma_v}(q)$ has a smaller horizontal coordinate small than that of the inflection point $q_{\overline{I}}$ of the graph for $p_{\overline{\sigma}_v}(q)$ (as q_I and $q_{\overline{I}}$ in Figure 6).

(3) Assume that $\mu_t(e)$ increases to $\mu_t(e) + \sigma \forall\ e$, $v = p + t$ and μ_v also increases. From (11), $q = q(p)$ remains unchanged if μ^* is constant. With other conditions are unchanged, if $\mu_t(e)$ increases to $\overline{\mu}_t(e)$, the integral area of (12) will increases, and then the inequality $q_{\overline{\mu}_t(e)}(p) > q_{\mu_t(e)}(p)$ is satisfied for any given p, i.e., the demand function $p_{\mu_t(e)}(q)$ will move upward to $p_{\overline{\mu}_t(e)}(q)$ (Figure 7). In addition, inflection points of the graphs for $p_{\mu_t(e)}(q)$ and $p_{\overline{\mu}_t(e)}(q)$ have the same horizontal coordinate which is obtained from (18).

(4) The effects of σ_t on demand function and the other conditions remaining unchanged: if σ_t increases to $\overline{\sigma}_t$, the integral area of (12) decreases, and then the inequality $q_{\overline{\sigma}_t}(p) < q_{\sigma_t}(p)$ will be satisfied for any given p, i.e., the graph of $p_{\sigma_t}(q)$ will move downward to the graph of $p_{\overline{\sigma}_t}(q)$ (Figure 8). Also, by (18), it can be obtained that the inflection points of the graphs of $p_{\sigma_t}(q)$ and $p_{\overline{\sigma_t}}(q)$ have the same horizontal coordinate (as q_I in Figure 8).

6. Conclusion

The demand function proposed in this work is expressed as a bivariate normal distribution function of variables v (the price which consumer groups would be willing to pay) and t (the unit transaction cost). This demand function distinguishes itself from other traditional approaches to construct a demand function into two aspects:

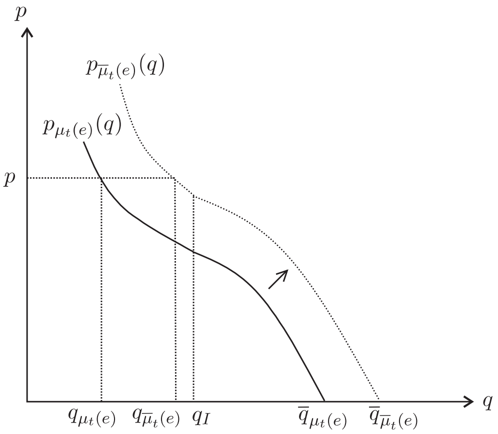

Figure 7. $\mu_t(e)$ increasing to $\overline{\mu}_t(e)$.

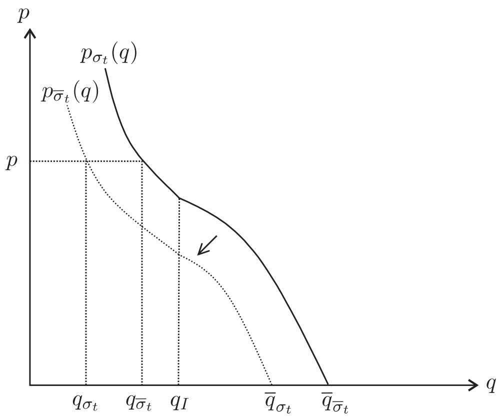

Figure 8. σ_t increasing to $\overline{\sigma}_t$.

(I) (1) A traditional demand function only shows the relationship between the quantity and the price of a product, and it has failed to consider the transaction cost factors that vary with people.

(2) The demand function constructed in the present study requires bivariate normal distribution variables, i.e., the price that a consumer would be willing to pay, and the transaction cost per unit. The resultant Equation (12), allows for a concrete and tangible expression of the demand function under the estimations of these two variables through sampled survey (as shown in Figure 4 or Figure 4′). Such a feature has significant implication for decision making on the marketing practice.

(II) The proposed demand function has following features:

(1) The vertical axis $q = 0$ is the vertical approximation line for the demand function $p = p(q)$ (Figure 4); the horizontal axis $p \equiv 0$ is the horizontal approximation line for the graph of demand function $q = q(p)$; the horizontal axis $p \equiv 0$ crosses the graph of demand function $p = p(q)$ at $\overline{q}$, and means that definition domain for q is $q \in [0, \overline{q}]$.

(2) $(\mu_v - \mu^*, q_I)$, of which $\mu_v - \mu^* = p(q_I)$, is the only inflection point of the graph for demand function $p = p(q)$.

(3) Demand function $p = p(q)$ is a convex function in the interval $[0, q_I]$, i.e., $p''(q) > 0$, $\forall q < q_I$, and a concave function in the interval $[q_I, \overline{q}]$, i.e., $p''(q) < 0$ and $q > q_I$, where $q_I = \operatorname*{Max}_q p'(q)$.

In addition, effects and characteristics of the expected value of μ_v, standard deviation of σ_v for consumer groups and other contributing factors are listed below:

(1) Effects of changes in μ_v for the demand function (Figure 5)

(2) Effects of changes in σ_v for the demand function (Figure 6)

(3) Effects of changes in μ_t for the demand function (Figure 7)

(4) Effects of changes in σ_t for the demand function (Figure 8)

All economic theories such as (1) the profit maximization for producers in a monopolistic or oligopolistic market, (2) the maximization of social well-being for non-profit organizations, (3) the market equilibrium are all concerned with the characteristics of demand functions. Therefore, the proposed demand function would result in outcomes which distinct from previous approaches and thus bear significant implications in the field of microeconomics.

References

[1] Bawa, K. and Ghost, A., *A model of household grocery shopping behavior*, Marketing Letters, Vol. 10, No. 2, pp.149-160, 1999.

[2] Eiselt, H. A. and Laporte, G., *Demand allocation functions*, Location Sci., Vol. 6, pp.175-187, 1998.

[3] Eiselt, H. A. and Laporte, G., *Trading areas of facilities with different sizes*, Recherche Operationnelle/Oper. Res., Vol. 22, pp.33-44, 1988.

[4] Elzinga, D. J. and Hearn, D. W., *Anote on a minimax location problem*, Transportation Sci., Vol. 7, pp.100-103, 1973.

[5] Kahn, B. E. and Schmittlein, D. C., *Shopping trip behavior: An empirical investigation*, Marketing Letters, Vol. 1, pp.55-70, 1989.

[6] Serra, D. and Colome, R., *Consumer choice and optimal locations models: Formulations and heuristics*, Regional Science, Vol. 80 pp.439-464, 2001.

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