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A particle swarm optimization for solving joint pricing and lot-sizing problem with fluctuating demand and trade credit financing

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ABSTRACT

Pricing is a major strategy for a retailer to obtain its maximum profit. Furthermore, under most market behaviors, one can easily find that a vendor provides a credit period (for example 30 days) for buyers to stimulate the demand, boost market share or decrease inventories of certain items. Therefore, in this paper, we establish a deterministic economic order quantity model for a retailer to determine its optimal selling price, replenishment number and replenishment schedule with fluctuating demand under two levels of trade credit policy. A particle swarm optimization is coded and used to solve the mixed-integer nonlinear programming problem by employing the properties derived in this paper. Some numerical examples are used to illustrate the features of the proposed model.

1. Introduction

In the inventory models developed, it is often assumed that payment will be made to the vendor for the goods immediately after receiving the consignment. Because the permissible delay in payments can provide economic sense for vendors, it is possible for a vendor to allow a certain credit period for buyers to stimulate the demand to maximize the vendors-owned benefits and advantage. Recently, several researchers have developed analytical inventory models with consideration of permissible delay in payments. Goyal (1985) first studied an EOQ model under the conditions of permissible delay in payments. Chung (1989) presented the discounted cash flows (DCF) approach for the analysis of the optimal inventory policy in the presence of the trade credit. Later, Shinn, Hwang, and Sung (1996) extended Goyal's (1985) model and considered quantity discounts for freight cost. Chung (1997) presented a simple procedure to determine the optimal replenishment cycle to simplify the solution procedure described in Goyal (1985). Teng (2002) provided an alternative conclusion from Goyal (1985), and mathematically proved that it makes economic sense for a well-established buyer to order less quantity and take the benefits of the permissible delay more frequently. Huang (2003) developed an EOQ model in which a supplier offers a retailer the permissible delay period M, and the retailer in turn provides the trade credit period N (with $N \leqslant M$) to his/her customers. He then obtained the closed-form optimal solution for the problem.

Jaber and Osman (2006) proposed a two-level supply chain model with delay in payments to coordinate the players' orders and minimize the supply chain costs. Jaber (2007) then incorporated the concept of entropy cost into the EOQ problem with permissible delay in payments. In real situations, "time" is a significant key concept and plays an important role in inventory models. Certain types of commodities deteriorate in the course of time and hence are unstable. As a result, while determining the optimal inventory policy for product of that type, the loss due to deterioration cannot be ignored. To accommodate more practical features of the real inventory systems, Aggarwal and Jaggi (1995) and Hwang and Shinn (1997) extended Goyal's (1985) model to consider the deterministic inventory model with a constant deterioration rate. Since the occurrence of shortages in inventory is a very nature phenomenon in real situations, Jamal, Sarker, and Wang (1997), Sarker, Jamal, and Wang (2000), Chang and Dye (2000), Chang, Hung, and Dye (2002) extended Aggarwal and Jaggi's (1995) model to allow for shortages and makes it more applicable in real world. Chang, Ouyang, and Teng (2003) then extended Teng's (2002) model, and established an EOQ model for deteriorating items in which the supplier provides a permissible delay to the purchaser if the order quantity is greater than or equal to a predetermined quantity. By considering the difference between unit selling price and unit purchasing cost, Ouyang, Chuang, and Chuang (2004) developed an EOQ model with noninstantaneous receipt under conditions of permissible delay in payments. Recently, Taso and Sheen (2007) developed a finite time horizon inventory model for deteriorating items to determine the most suitable retail price and appropriate replenishment cycle time with fluctuating unit purchasing cost

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and trade credit. Chang, Wu, and Chen (2009) established an inventory model to determine the optimal payment time, replenishment cycle and order quantity under inflation.

However, all the above models make an implicit assumption that the demand rate is constant over an infinite planning horizon. This assumption is only valid during the maturity phase of a product life cycle. During the introduction and growth phase of a product life cycle, the firms face increasing demand with little competition. Some researchers Resh, Friedman, and Barbosa (1976), Donaldson (1977), Dave and Patel (1981), Sachan (1984), Goswami and Chaudhuri (1991), Goyal, Morin, and Nebebe (1992), Chakrabarty, Giri, and Chaudhuri (1998) suggest that the demand rate can be well approximated by a linear form. A linear trend demand implies an uniform change in the demand rate of the product per unit time. This is a fairly unrealistic phenomenon and it seldom occurs in the real market. One can usually observe in the electronic market that the sales of items increase rapidly during the introduction and growth phase of the life cycle because there are few competitors in market. Recently, Yang, Teng, and Chern (2002) established an optimal replenishment policy for power-form demand rate and proposed a simple and computationally efficient method in a forward recursive manner to find the optimal replenishment strategy. Khanra and Chaudhuri (2003) advise that the demand rate should be represented by a continuous quadratic function of time in the growth stage of a product life cycle. They also provide a heuristic algorithm to solve the problem when the planning horizon is finite. To achieve maximum profit, Chen and Chen (2004) presented an inventory model for a deteriorating item with a multivariate demand function of price and time. Their model is solved with dynamic programming techniques by adjusting the selling price upward or downward periodically. Chen, Hung, and Weng (2007a, 2007b) dealt with the inventory model under the demand function following the product-life-cycle shape over a fixed time horizon. Skouri and Konstantaras (2009) studied an order level inventory model when the demand is described by a three successive time periods that classified time dependent ramp-type function.

In this paper, to obtain robust and general results, we will extend the constant demand to a generalized time varying demand, which is suitable not only for the growth stage but also for the maturity stage of a product life cycle. In addition, we assume that supplier offers retailer a trade credit period M, and retailer in turn provides a trade credit period N (with $N \leqslant M$) to his/her customers. The lot sizing problem is then to find the optimal pricing and replenishment strategy that will maximize the present value of total profit. A traditional particle swarm optimization is coded and used to solve the mixed-integer nonlinear programming problem by employing the properties derived in this paper. Finally, numerical examples will be used to illustrate the results.

2. Assumptions and notations

The mathematical model in this paper is developed on the basis of the following assumptions and notations.

2.1. Notations

$I(t)$ = the inventory level at time t.
A = ordering cost, \$/per order.
c = unit purchasing cost, \$/per unit.
p = unit selling price (a decision variable), \$/per unit, defined in the interval $[0,p_u]$.
$g(t,p)$ = the demand rate at time t and price p with $g(t,p) = \alpha(p)f(t)$, where $f(t)$ is positive in the planning horizon $[0,H]$ and $\alpha(p)$ is a non-negative, continuous, convex, decreasing function of the selling price in $[0,p_u]$.
r = the discount rate.
h = holding cost excluding interest charges, \$/unit/year.
I_e = interest which can be earned, \$/year.
I_r = interest charges which are invested in inventory, \$/year.
M = the retailer's trade credit period offered by supplier in years.
N = the customer's trade credit period offered by retailer in years, where $N \leqslant M$.
n = the number of replenishment periods during the planning horizon.
t_i = the ith replenishment time (a decision variable), $i = 1,2,\ldots,n$, with $0 = t_0 < t_1 < t_2 < \cdots < t_n = H$.
T_i = the length of ith replenishment period.
Q_i = the order quantity in the ith replenishment period.
$TP(n,p,\mathbf{t})$ = the present value of total profit, where $\mathbf{t} = \{t_1,t_2,\ldots,t_{n-1}\}$.

2.2. Assumptions

1. The inventory system involves in only one item over a known and finite planning horizon H.
2. The replenishment occurs instantaneously at an infinite rate.
3. The items deteriorate at a constant rate of deterioration θ, where $0 < \theta \ll 1$. There is no repair or replacement of deteriorated units during the planning horizon. The items will be withdrawn from the warehouse immediately as they deteriorate.
4. Before the replenishment account is settled, the retailer can use the sales revenue to earn interest with an annual rate I_e. However, beyond the fixed credit period, the product still in stock is presumed to be financed with an annual rate I_r.
5. The retailer can accumulate revenue and earn interest after his/her customer pays for the amount of purchasing cost to the retailer until the end of the trade credit period offered by the supplier. That is, the retailer can accumulate revenue and earn interest during the period N to M with rate I_e under the condition of trade credit.

3. Model formulation

As shown in Fig. 1, the depletion of the inventory occurs due to the combined effects of the demand and deterioration in the interval $[t_{i-1},t_i]$. Hence, the variation of inventory level, $I(t)$, with respect to time can be described by the following differential equation:

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = -\theta I(t) - \alpha(p)f(t), \quad t_{i-1} \leqslant t < t_i, \tag{1}$$

with boundary condition $I(t_i) = 0$, $i = 1,2,\ldots,n$. The solution of (1) can be represented by

$$I(t) = e^{-\theta t}\int_t^{t_i} \alpha(p)f(t)e^{\theta u}\,\mathrm{d}u, \quad t_{i-1} \leqslant t < t_i. \tag{2}$$

Then, applying (2), the present value of the holding cost in the ith replenishment period, denoted by HC_i, $i = 1,2,\ldots,n$, can be written as

$$HC_i = h\int_{t_{i-1}}^{t_i} e^{-rt}e^{-\theta t}\int_t^{t_i} \alpha(p)f(t)e^{\theta u}\,\mathrm{d}u\,\mathrm{d}t. \tag{3}$$

The present value of the purchase cost during the ith replenishment period, denoted by PC_i, $i = 1,2,\ldots,n$, is

$$PC_i = ce^{-rt_{i-1}}\int_{t_{i-1}}^{t_i} \alpha(p)f(t)e^{\theta(t-t_{i-1})}\,\mathrm{d}t. \tag{4}$$

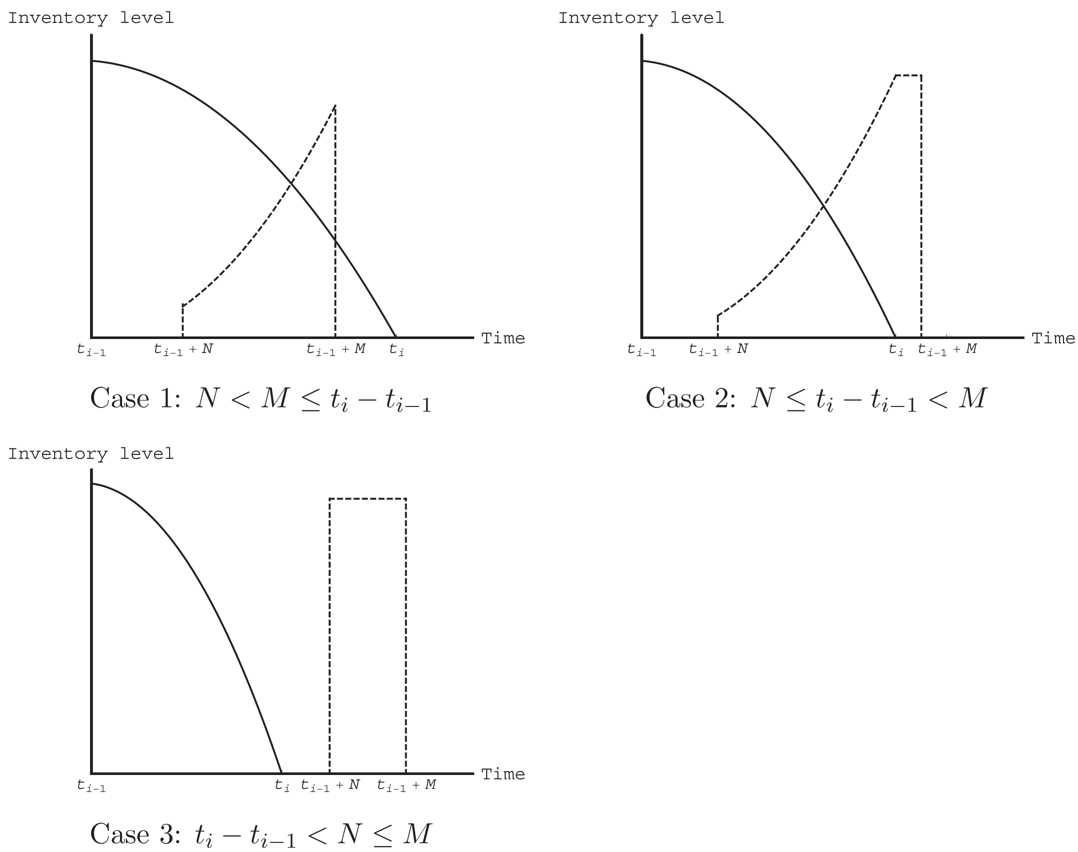

Fig. 1. The retailer's inventory level and accumulation of interest earned. A solid line denotes the inventory level at time t in the ith replenishment period, the area enclosed by dashed line represents the interest earned in the ith replenishment period.

The present value of the sales revenue in the ith replenishment period, denoted by SR_i, $i = 1,2,\ldots,n$, is

$$SR_i = p\int_{t_{i-1}}^{t_i} e^{-rt}\alpha(p)f(t)\mathrm{d}t, \quad i = 1,2,\ldots,n. \tag{5}$$

In this paper, the parameters M and N can be seen as exogenous variables. Regarding the exogenous variables, three possibilities may arise: *Case 1*: $N < M \leqslant t_i - t_{i-1}$, *Case 2*: $N \leqslant t_i - t_{i-1} < M$ or *Case 3*: $t_i - t_{i-1} < N \leqslant M$. The relationship between credit period and replenishment period is illustrated in Fig. 1. The present value of interest earned (*IE*) and interest charges (*IC*) for each case are presented in Appendix A.

As shown above, we can now formulate the present value of total profit for a given positive integer n as follows:

$$TP(p,\mathbf{t}|n) = \left\{ \begin{array}{c} \text{sales revenue} - \text{purchase cost} - \text{holding cost} \\ -\text{interest charges} + \text{interest earned} - \text{ordering cost} \end{array} \right\}$$
$$= \sum_{i=1}^{n}\left(SR_i - PC_i - HC_i - IC_i + IE_i - e^{-rt_{i-1}}A\right), \tag{6}$$

where

$$IE_i = \begin{cases} IE_{i1}, & t_i - t_{i-1} \geqslant M \\ IE_{i2}, & N \leqslant t_i - t_{i-1} < M \\ IE_{i3}, & t_i - t_{i-1} < N \end{cases}$$

and

$$IC_i = \begin{cases} IC_{i1}, & t_i - t_{i-1} \geqslant M \\ IC_{i2}, & N \leqslant t_i - t_{i-1} < M \\ IC_{i3}, & t_i - t_{i-1} < N. \end{cases}$$

The objective of this paper is to determine the optimal replenishment points t_i and the optimal selling price to maximize the present value of total profit of the inventory system. Hence, it is a n dimensional decision-making problem for a retailer and the problem can be mathematically formulated as follows:

$$\begin{array}{ll} \text{Maximize} & TP(p,\mathbf{t}|n) \\ \text{subject to} & c < p \leqslant p_u, \\ & t_{i-1} < t_i, \quad i = 1,2,\ldots,n, \\ & t_0 = 0, t_n = H. \end{array}$$

The formulated optimization model is a nonlinear programming problem with nonnegative constraints. Since it is difficult to solve analytically, we adopt an evolutionary computation algorithm to solve the problem. In this paper, an algorithm based on particle swarm optimization (PSO) is proposed to find the optimal pricing and replenishment schedule. The algorithm is similar to other population-based algorithms like Genetic algorithms but, there is no direct combination of individuals of the population. Instead, it relies on the social behavior of particle. In the next section, we will introduce how the PSO can be used to solve the problem.

4. Solution procedure

4.1. The background of particle swarm optimization

The PSO is an algorithm for finding optimal regions of complex search spaces through the interaction of individuals in a population of particles. It was proposed by Eberhart and Kennedy (1995) and Kennedy and Eberhart (1995) and has been widely used in finding solutions for optimization problems. The PSO algorithm is inspired by social behavior of bird flocking or fish schooling. In PSO, the potential solutions, called particles, fly through the problem space by following the current optimum particles. Assumed that our search space is d-dimensional, PSO is initialized with a population of random particles (solutions) and then searches for optimum by updating generations, where the population is the number of particles in the problem space. During every iteration, each particle is updated by following the two best values. The first one is the best solution so far reached by the particle, this best value is a personal best and called *pbest*. The other one is the current best solution, obtained so far by any particle in the population. This best value is a global best and called *gbest*. With *pbest* and *gbest* obtained, the particle will have velocity, which directs the flying of the particle. In each generation, a particle can update its velocity and position based on the following equations:

$$v_{k+1}^{j}=\chi\left[v_{k}^{j}+\varphi_{1}\times rand\times(pbest_{k}^{i}-x_{k}^{i})+\varphi_{2}\times rand\times(gbest_{k}-x_{k}^{i})\right] \tag{7}$$

and

$$x_{k+1}^{i}=x_{k}^{i}+v_{k+1}^{i}, \tag{8}$$

where v_k^i is the velocity of ith particle at the kth iteration, x_k^i is current the position of the ith particle, $pbest_k^i$ is the best searching experience so far of ith particle at the kth iteration, $gbest_k$ is best result obtained at the kth iteration, φ_1 and φ_2 are acceleration constants, *rand* is a random number between 0 and 1 and

$$\chi=\frac{2}{\left|-\varphi_{1}-\varphi_{2}-\sqrt{(\varphi_{1}+\varphi_{2}-4)(\varphi_{1}+\varphi_{2})}+2\right|}. \tag{9}$$

The parameters φ_1 and φ_2 in (7) are scalar constants that weight influence of particles' own experience and the social knowledge. The parameter χ in (7) is the so called constriction factor, which is used to prevent a particle from exploring too far into the search space. In general, the common value for $\varphi_1+\varphi_2$ is set to 4.1 and the constriction factor χ is approximately 0.729. Lastly, the algorithm will check the results every iteration until the best solution is found or termination conditions are satisfied.

In the PSO algorithm, velocities are clamped at each iteration to lie within $[-V_{\max}, V_{\max}]$ on each dimension, which is a parameter specified by the user. If the sum of accelerations causes the velocity on that dimension to exceed $V_{\max}$, then this velocity is limited to $V_{\max}$. This helps particles comb the search space rather than potentially take huge iterative steps that might cause some information to be missed. Then, the search procedure of the particle swarm optimization is summarized as follows:

Step 1 Initialize I particles with random positions and velocities on d-dimensions in the search space, where I is the number of particles.
Step 2 Evaluate the fitness of all particles.
Step 3 Keep track of the locations where each individual had its highest fitness so far.
Step 4 Keep track of the position with the global best fitness.
Step 5 Update the velocity of each particle, according to (7) and (9).
Step 6 Update the position of each particle, according to (8).
Step 7 Terminate if the standard deviation of fitness is less than ϵ (e.g. 10^{-5}) or the maximum number of iterations (e.g. 1000) is reached, otherwise go to Step 2.

4.2. Solving the pricing and replenishment scheduling problem

For any given feasible replenishment schedule, $0=t_0<t_1<t_2<\cdots<t_{n-1}<t_n=H$, to acquire optimal selling price that maximizes $TP(p|n,\mathbf{t})$, the value of p^* should be selected to satisfy

$$\frac{dTP(p|n,\mathbf{t})}{dp}=\frac{d}{dp}\sum_{i=1}^{n}\left\{SR_i-PC_i-HC_i+IE_i-IC_i-Ae^{-rt_{i-1}}\right\}=0. \tag{10}$$

After taking the first and second derivatives of $SR_i-PC_i-HC_i$, IE_i, IC_i and $Ae^{-rt_{i-1}}$ with respect to p yields

$$\begin{aligned}\frac{\mathrm{d}(SR_i-PC_i-HC_i)}{\mathrm{d}p}&=[\alpha(p)+p\alpha'(p)]\int_{t_{i-1}}^{t_i}e^{-rt}f(t)\mathrm{d}t\\&\quad-\alpha'(p)\left\{ce^{-rt_{i-1}}\int_{t_{i-1}}^{t_i}e^{\theta(t-t_{i-1})}f(t)\mathrm{d}t\right.\\&\quad\left.+h\int_{t_{i-1}}^{t_i}\int_t^{t_i}e^{-rt-\theta(t-u)}f(t)\mathrm{d}u\,\mathrm{d}t\right\},\end{aligned} \tag{11}$$

$$\begin{aligned}\frac{\mathrm{d}^2(SR_i-PC_i-HC_i)}{\mathrm{d}p^2}&=[2\alpha'(p)+p\alpha''(p)]\int_{t_{i-1}}^{t_i}e^{-rt}f(t)\mathrm{d}t\\&\quad-\alpha''(p)\left\{ce^{-rt_{i-1}}\int_{t_{i-1}}^{t_i}e^{\theta(t-t_{i-1})}f(t)\mathrm{d}t\right.\\&\quad\left.+h\int_{t_{i-1}}^{t_i}\int_t^{t_i}e^{-rt-\theta(t-u)}f(t)\mathrm{d}u\,\mathrm{d}t\right\},\end{aligned} \tag{12}$$

$$\frac{\mathrm{d}IE_i}{\mathrm{d}p}=\begin{cases}[\alpha(p)+p\alpha'(p)]I_e\int_{N+t_{i-1}}^{M+t_{i-1}}e^{-rt}(M-t+t_{i-1})f(t)\mathrm{d}t, & t_i-t_{i-1}\geqslant M\\ [\alpha(p)+p\alpha'(p)]I_e\{\int_{t_{i-1}}^{t_i}e^{-rt}(M+t_{i-1}-t_i)f(t)\mathrm{d}t & \\ +\int_{N+t_{i-1}}^{t_i}e^{-rt}(t_i-t)f(t)\mathrm{d}t\}, & N\leqslant t_i-t_{i-1}<M\\ [\alpha(p)+p\alpha'(p)]I_e\int_{t_{i-1}}^{t_i}e^{-rt}(M-N)f(t)\mathrm{d}t, & t_i-t_{i-1}<N\end{cases} \tag{13}$$

$$\frac{\mathrm{d}^2IE_i}{\mathrm{d}p^2}=\begin{cases}[2\alpha'(p)+p\alpha''(p)]I_e\int_{N+t_{i-1}}^{M+t_{i-1}}e^{-rt}(M-t+t_{i-1})f(t)\mathrm{d}t, & t_i-t_{i-1}\geqslant M\\ [2\alpha'(p)+p\alpha''(p)]I_e\left\{\int_{t_{i-1}}^{t_i}e^{-rt}(M+t_{i-1}-t_i)f(t)\mathrm{d}t\right. & \\ \left.+\int_{N+t_{i-1}}^{t_i}e^{-rt}(t_i-t)f(t)\mathrm{d}t\right\}, & N\leqslant t_i-t_{i-1}<M\\ [2\alpha'(p)+p\alpha''(p)]I_e\int_{t_{i-1}}^{t_i}e^{-rt}(M-N)f(t)\mathrm{d}t, & t_i-t_{i-1}<N\end{cases} \tag{14}$$

$$\frac{\mathrm{d}IC_i}{\mathrm{d}p}=\begin{cases}cI_r\alpha'(p)\int_{M+t_{i-1}}^{t_i}\int_t^{t_i}e^{-rt-\theta(t-u)}f(u)\mathrm{d}u\mathrm{d}t, & t_i-t_{i-1}\geqslant M\\ 0, & N\leqslant t_i-t_{i-1}<M\\ 0, & t_i-t_{i-1}<N\end{cases} \tag{15}$$

$$\frac{\mathrm{d}^2IC_i}{\mathrm{d}p^2}=\begin{cases}cI_r\alpha''(p)\int_{M+t_{i-1}}^{t_i}\int_t^{t_i}e^{-rt-\theta(t-u)}f(u)\mathrm{d}u\,\mathrm{d}t, & t_i-t_{i-1}\geqslant M\\ 0, & N\leqslant t_i-t_{i-1}<M\\ 0, & t_i-t_{i-1}<N\end{cases} \tag{16}$$

and

$$\frac{\mathrm{d}Ae^{-rt_{i-1}}}{\mathrm{d}p}=\frac{\mathrm{d}^2Ae^{-rt_{i-1}}}{\mathrm{d}p^2}=0, \tag{17}$$

respectively.

Since $\alpha'(p)<0$ and $\alpha''(p)>0$, it is clear from (11), (13) and (15) that $dTP(p|n,\mathbf{t})/dp=0$ has a solution if $\alpha(p)+p\alpha'(p)<0$ (see Appendix B for details). Further, if the marginal revenue with respect to selling price is decreasing (i.e. $p\alpha(p)$ is a strictly concave function of p), it can be easily verified that

$$\frac{d^2TP(p|n,\mathbf{t})}{d^2p}=\frac{d^2}{d^2p}\sum_{i=1}^{n}\{SR_i-PC_i-HC_i+IE_i-IC_i-Ae^{-rt_{i-1}}\}<0,$$

from (12), (14) and (16) (see Appendix C for details). Consequently, $TP(p|n,\mathbf{t})$ is a strictly concave function of p, and there exits a unique solution that maximizes $TP(p|n,\mathbf{t})$. From this, we can obtain the following result: once $\mathbf{t}$ is known, the optimal selling price, p^*, can be uniquely determined as a function of $\mathbf{t}$. Thus, $p^* = p^{\text{opt}}(\mathbf{t})$ can be written as a function of $\mathbf{t}$. This results reduces the n dimensional problem of finding the optimal pricing and schedule to a $n-1$ dimensional problem as follows:

$$\begin{aligned}&\text{Maximize} \quad TP(\mathbf{t}|n)\\&\text{subject to} \quad c<p^{\text{opt}}(\mathbf{t})\leqslant p_u,\\&\qquad t_{i-1}<t_i,\quad i=1,2,\ldots,n,\\&\qquad t_0=0, t_n=H.\end{aligned}$$

Note that if marginal revenue is an increasing function of p, then selling price and revenue will always move in the same direction, hence the retailer can realize an infinite profit by setting an infinite p. It is impossible.

In this paper, the PSO with boundary constraints is adopted to solve the model. A pseudo-objective function is yielded using an exterior penalty function as follows:

$$\phi(\mathbf{t}|n)=TP(\mathbf{t}|n)-\mu\left\{\sum_{i=1}^{n}\{\max[0,p^{\text{opt}}-p_u]\}^2+\{\max[0,c-p^{\text{opt}}]\}^2+\{\max[0,t_{i-1}-t_i]\}^2\right\}, \tag{18}$$

where μ is a large positive number, known as the penalty number. (18) is then used to evaluate the fitness of individuals in a population. Thus, for any given integer of n, the problem becomes

$$\begin{aligned}&\text{Maximize} \quad \phi(\mathbf{t}|n)\\&\text{subject to} \quad t_0=0,\quad t_n=H,\end{aligned}$$

and the solution procedure for finding optimal pricing and replenishment schedule is provided as follows.

Algorithm 1

Step 1 Let dimension $d = n - 1$, population size $I = 10d$, $V_{\max} = H$, $\varphi_1 = \varphi_2 = 2.05$, $\mu = 10^9$, $\text{iter}_{\max} = 1000$ and $k = 0$.
Step 2 x_0^i: Randomly generate and sort d points in the range 0 to H, $i = 1,2,\ldots,I$.
Step 3 v_0^i: Randomly generate d points in the range $-V_{\max}$ and $V_{\max}$, $i = 1,2,\ldots,I$.
Step 4 Evaluate the fitness of all particles using (10) and (18).
Step 5 Compare the performance of each individual to its best performance so far

$$pbest_k^i=\begin{cases}x_k^i, & \text{if}\,\phi(x_k^i|n)>\phi\left(pbest_{k-1}^i|n\right)\\ pbest_{k-1}^i, & \text{otherwise}\end{cases},\quad i=1,2,\ldots,I$$

Step 6 Compare the performance of each particle to the global best particle

$$gbest_k=\begin{cases}\arg\max\limits_{1\leqslant i\leqslant I}\phi(x_k^i|n), & \text{if}\ \max\limits_{1\leqslant i\leqslant I}\phi(x_k^i|n)\\ & \quad >\phi\left(gbest_{k-1}^i|n\right)\\ gbest_{k-1}^i, & \text{otherwise.}\end{cases}$$

Table 1
Optimal time schedule for Example 1.

i	t_i	T_i	Q_i	Case
1	0.1824	0.1824	36.78	1
2	0.2987	0.1163	107.89	1
3	0.3941	0.0954	156.49	1
4	0.4665	0.0724	159.49	2
5	0.5329	0.0664	173.03	2
6	0.5958	0.0629	181.89	2
7	0.6569	0.0612	186.92	2
8	0.7181	0.0611	188.34	2
9	0.7814	0.0633	185.26	2
10	0.8506	0.0692	176.36	2
11	1.0000	0.1494	189.67	1

Step 7 Update v_k^i, $i = 1,2,\ldots,I$, according to (7) and (9).
Step 8 Update x_{k+1}^i, $i = 1,2,\ldots,I$, according to (8).
Step 9 Terminate if the standard deviation of $\phi(x_k|n) < 10^{-5}$ or $k = \text{iter}_{\max}$, otherwise $k = k + 1$ and go to Step 4.

Let n^* be the optimal replenishment number. To avoid using a brute force enumeration for finding n^*, we further simplify the search process by providing an intuitively good starting value for n^*. Because $\alpha'(p) < 0$, from Appendix B, $dTP(p|n,\mathbf{t})/dp = 0$ has a solution if and only if $\alpha(p) + p\alpha'(p) < 0$. Since marginal revenue, $\alpha(p) + p\alpha'(p)$, is a strictly decreasing function of p, the solution of $\alpha(p) + p\alpha'(p) = 0$, say p_l, is the lower bound for the optimal selling price. Moreover, the holding cost per unit (including inventory and deterioration costs) is $h + I_rc + \theta c$. Substituting the above results into classical EOQ formula, we obtain an estimate of the number of replenishments as

$$n=\text{round integer of}\ \sqrt{\frac{(h+I_rc+\theta c)H\int_0^H\alpha(p_l)f(t)dt}{2A}}. \tag{19}$$

It is obvious that searching for the optimal number of replenishments by starting with n in (19) instead of $n = 1$ will reduce the computational complexity significantly. Combining the above arguments, we propose the following algorithm to solve the pricing and replenishment scheduling problem.

Algorithm 2

Step 1 Choose two initial trial values of n^*, say n as in (19) and $n + 1$. Use Algorithm 1 to obtain $\{t_i^*\}$, and compute the corresponding $TP(n)$ and $TP(n + 1)$, respectively.
Step 2 If $TP(n) \leqslant TP(n + 1)$, then compute $TP(n + 2), TP(n + 3),\ldots,$ until we find $TP(k) > TP(k + 1)$. Set $n^* = k$ and stop.
Step 3 If $TP(n) > TP(n + 1)$, then compute $TP(n - 1), TP(n - 2),\ldots,$ until we find $TP(k) > TP(k - 1)$. Set $n^* = k$ and stop.

5. Computational results

5.1. Numerical examples

To illustrate the results, let us apply the proposed algorithms to solve the following numerical examples. In Example 1, the demand function follows the shape of a product life cycle. In Example 2, we have a quadratic increasing demand and in Example 3 we have a exponential decreasing demand. Algorithms 1 and 2 are implemented on a personal computer with Intel Core 2 Duo under Mac OS X 10.5.6 operating system using Mathematica version 7.

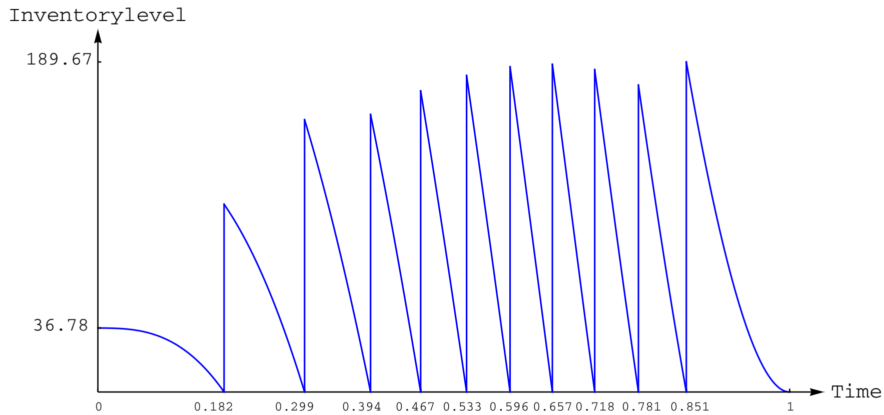

Fig. 2. Graphical representation of inventory system for Example 1.

Example 1. In this example, we consider the demand function for a product life cycle which has been presented by Chen et al. (2007a, 2007b):

$\alpha(p) = 5000 - 150p$	$f(t) = t^{3-1}(H-t)^{2-1}/\mathscr{B}(3,2)$	$\theta = 0.08$
$A = 50$	$h = 2$	$c = 10$
$H = 1$	$r = 0.2$	$I_r = 0.18$
$I_e = 0.12$	$M = 30/365$	$N = 15/365$

$\mathscr{B}(a,b) = \frac{(a-1)!(b-1)!}{(a+b-1)!}$.

Solving $\alpha(p) + p\alpha'(p) = 0$ first, we obtain $p_l = 16.6667$ and the estimated number of replenishments $n = 11$ from (19). Then, applying the Algorithms 1 and 2, we get $TP(11) = 17382.1$, $TP(12) = 17378.3$ and $TP(10) = 17378.8$. Therefore, the optimal number of replenishments is 11 and the corresponding optimal selling price is 21.7570. The optimum solution found after 162 iterations (109.903 s). The optimal time schedule is shown in Table 1. The behavior of the inventory system over the planning horizon and the convergence result of PSO algorithm for the optimal solution are depicted in Figs. 2 and 3, respectively.

Example 2. In this example, we consider the quadratic increasing demand function which is proposed by Khanra and Chaudhuri (2003):

$\alpha(p) = 100 - 3p$	$f(t) = 25 + 10t + t^2$	$\theta = 0.08$
$A = 50$	$h = 2$	$c = 10$
$H = 1$	$r = 0.2$	$I_r = 0.18$
$I_e = 0.12$	$M = 45/365$	$N = 15/365$

Solving $\alpha(p) + p\alpha'(p) = 0$ first, we obtain $p_l = 16.6667$ and the estimated number of replenishments $n = 8$ from (19). Then, applying the Algorithms 1 and 2, we get $TP(8) = 10572.5$, $TP(9) = 10575.3$ and $TP(10) = 10571.5$. Therefore, the optimal number of replenishments is 9 and the corresponding optimal selling price is 21.7811. The optimum solution found after 151 iterations (61.494 s). The optimal time schedule is shown in Table 2. The behavior of the inventory system over the planning horizon and the convergence result of PSO algorithm for the optimal solution are depicted in Figs. 4 and 5, respectively.

Example 3. In this example, we redo an inventory situation proposed by Chen and Chen (2004) while considering the trade credit financing:

$\alpha(p) = 300 - 120p$	$f(t) = e^{-0.06t}$	$\theta = 0.2$
$A = 40$	$h = 0.02$	$c = 1$
$H = 12$	$r = 0.02$	$I_r = 0.18/12$
$I_e = 0.12/12$	$M = 3/2$	$N = 1$

Note that the time unit is 1 month. The planning horizon is 1 year, which equals to 12 months. By applying (19), we obtain the estimated number of replenishments $n = 7$. Then, applying the Algorithms 1 and 2, we get $TP(8) = 105.7$, $TP(7) = 122.4$, $TP(6) = 133.1$, $TP(5) = 133.7$ and $TP(4) = 116.9$. Therefore, the optimal number of replenishments is 5 and the corresponding optimal selling price is 1.9150. The optimum solution found after 141 iterations (24.095 s). The optimal time schedule is shown in Table 3. The behavior of the inventory system over the planning horizon and the convergence result of PSO algorithm for the optimal solution are depicted in Figs. 6 and 7, respectively.

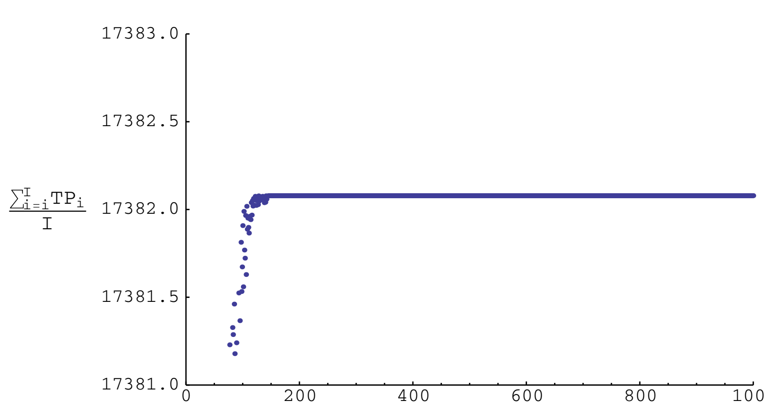

Fig. 3. The convergence result of PSO algorithm for $TP(11)$ of Example 1.

Table 2
Optimal time schedule for Example 2.

i	t_i	T_i	Q_i	Case
1	0.1342	0.1342	120.03	1
2	0.2505	0.1163	109.21	2
3	0.3642	0.1137	111.54	2
4	0.4755	0.1113	113.82	2
5	0.5845	0.1090	116.04	2
6	0.6914	0.1069	118.20	2
7	0.7962	0.1048	120.32	2
8	0.8990	0.1028	122.39	2
9	1.0000	0.1010	124.41	2

In order to test the performance of PSO algorithm in our problem, we have performed 100 runs of the algorithm for each example. The computational results are summarized in Table 4 and Fig. 8. We can find that the maximal difference between the solutions is within 0.006%, and thus, we are confident that using PSO results in good solutions for our problem. The following inferences can be made from the results in Examples 1–3.

1. When the demand is increasing with time, the length of the ith replenishment cycle, T_i, is decreasing. Otherwise, the length of the ith replenishment cycle, T_i, is increasing.
2. For the duration of the increasing demand, since the length of the ith replenishment cycle is decreasing, the order quantity is boosted as the trade credit policy changes from Case 1 to Case 2. But the order quantity increases with time under the same trade credit policy.

Table 3
Optimal time schedule for Example 3.

i	t_i	T_i	Q_i	Case
1	2.1073	2.1073	174.08	1
2	4.3432	2.2359	164.31	1
3	6.7238	2.3806	154.62	1
4	9.2684	2.5446	145.02	1
5	12.0000	2.7316	135.51	1

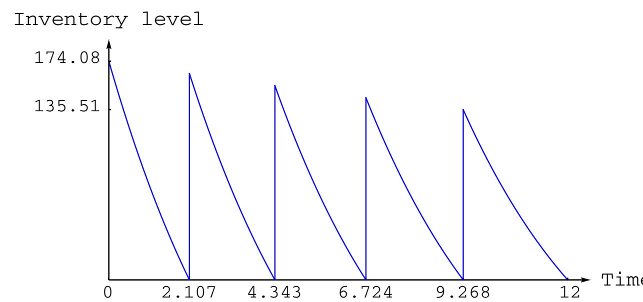

Fig. 6. Graphical representation of inventory system for Example 3.

3. For the duration of the decreasing demand, since the length of the ith replenishment cycle is increasing, the order quantity is boosted as the trade credit policy changes from Case 2 to Case 1. But the order quantity decreases with time under the same trade credit policy.

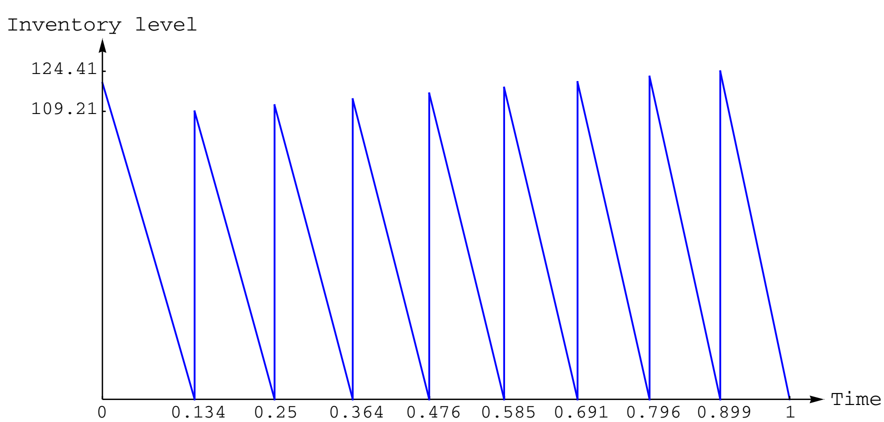

Fig. 4. Graphical representation of inventory system for Example 2.

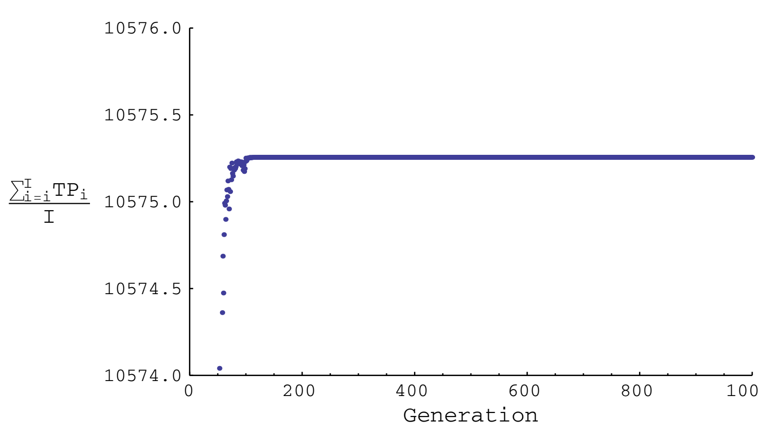

Fig. 5. The convergence result of PSO algorithm for $TP(9)$ of Example 2.

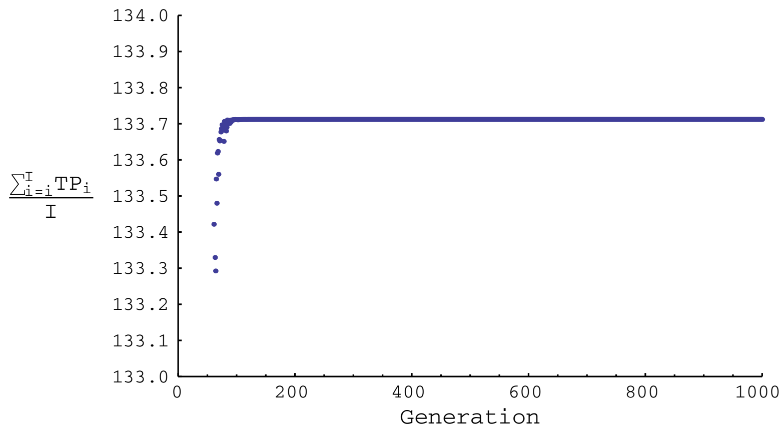

Fig. 7. The convergence result of PSO algorithm for $TP(5)$ of Example 3.

Table 4
Experimental results for Examples 1–3.

Example	Best	Worst	Mean	Std
Example 1	17382.1	17382.1	17382.1	0.000027
Example 2	10575.3	10574.7	10575.2	0.116877
Example 3	133.712	133.712	133.712	0.000000

5.2. Sensitivity analysis

Next, we study the sensitivity of the optimal solution to change in the values of the different parameters associated with the model. Since we focus on the effect of trade credit in this paper, we will ignore the effect of varying A and H. Applying the algorithm procedures yields the results reported in Tables 5–7. The results obtained for illustrative examples provide certain insights about the problem studies. Some of them are as follows:

1. The net present value of the total profit increases if M and I_e increase. However, it decreases if c, h, θ, r, N and I_r increases.
2. The optimal replenishment number increases if h, θ, r, M and I_e increase. However, it is insensitive on the change in N and I_r.
3. The net present value of the total profit is more sensitive on the change in c, h, r and θ. It implies that the effects of c, h, r and θ on the discounted total profit are significant.
4. The effect of varying M and I_e is negative correlated with varying N and I_c.
5. Large variation in the input parameters hardly have an effect on the value of the number of orders made in most cases. This implies that the algorithm developed in the paper is robust.

6. Concluding remarks

In this paper, we consider a retailer's optimal pricing and lot-sizing problem for deteriorating items with fluctuating demand under trade credit financing. We have successfully formulated the problem as a mixed-integer nonlinear programming model and proposed a solution algorithm associated with it. In contrast to the classical fixed selling price policy under trade credit, the pricing policy in this model provides more flexibility by changing price upward or downward. We can also use similar derivations as in Appendix C to prove that $\partial^2 TP(\mathbf{p}|n,\mathbf{t})/\partial p_i^2 > 0$ where $\mathbf{p} = \{p_1, p_2, \ldots, p_n\}$ and p_i denotes the selling price per unit in the ith replenishment cycle. Hence, the model in this paper not only

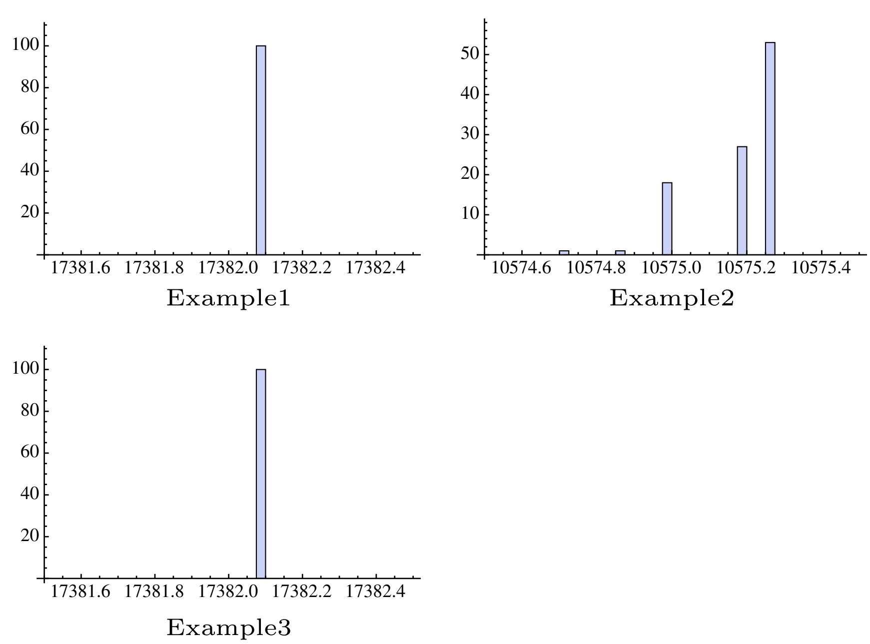

Fig. 8. The histogram of the optimum solutions obtained from 100 runs of PSO.

Table 5
Sensitivity analysis on TP^* and n^* for Example 1.

	−50%	−40%	−30%	−20%	−10%	0%	10%	20%	30%	40%	50%
c	26032.5	24165.7	22367.7	20637.7	18975.9	17382.1	15856.3	14398.6	13009.0	11687.4	10433.8
	10	11	11	11	11	11	11	11	11	11	11
h	17449.1	17435.0	17420.6	17407.5	17394.8	17382.1	17369.4	17356.7	17344.0	17331.9	17320.3
	10	10	10	11	11	11	11	11	11	12	12
θ	17407.9	17402.7	17397.6	17392.4	17387.2	17382.1	17376.9	17371.8	17366.6	17361.4	17356.3
	11	11	11	11	11	11	11	11	11	11	11
r	18524.4	18288.5	18056.6	17828.2	17603.6	17382.1	17163.6	16948.1	16735.5	16526.4	16320.6
	10	10	10	11	11	11	11	11	11	12	12
M	17295.4	17307.7	17321.0	17335.3	17354.4	17382.1	17411.1	17441.7	17472.2	17503.1	17534.5
	10	10	10	10	11	11	11	11	11	11	11
N	17424.2	17413.9	17404.5	17396.0	17388.6	17382.1	17376.6	17372.1	17368.6	17366.1	17364.2
	11	11	11	11	11	11	11	11	11	11	11
I_r	17385.5	17384.8	17384.1	17383.4	17382.7	17382.1	17381.4	17380.8	17380.2	17379.6	17379.0
	11	11	11	11	11	11	11	11	11	11	11
I_e	17351.9	17356.9	17362.3	17368.1	17375.1	17382.1	17389.2	17396.3	17403.7	17412.2	17420.8
	10	10	10	11	11	11	11	11	12	12	12

Table 6
Sensitivity analysis on TP^* and n^* for Example 2.

	−50%	−40%	−30%	−20%	−10%	0%	10%	20%	30%	40%	50%
c	15913.0	14760.2	13650.9	12583.6	11558.4	10575.3	9634.3	8735.2	7878.7	7064.1	6291.6
	8	8	9	9	9	9	9	9	9	9	9
h	10633.0	10620.9	10608.6	10596.7	10586.0	10575.3	10564.5	10553.9	10543.2	10533.1	10523.5
	8	8	8	9	9	9	9	9	9	10	10
θ	10597.1	10592.2	10588.3	10583.9	10579.6	10575.3	10570.6	10566.5	10562.1	10557.8	10553.4
	8	8	9	9	9	9	9	9	9	9	9
r	11202.1	11072.6	10944.9	10819.0	10696.0	10575.3	10455.9	10338.3	10222.0	10108.6	9996.8
	8	8	8	8	9	9	9	9	9	10	10
M	10479.3	10495.5	10512.6	10530.7	10549.7	10575.3	10605.4	10635.9	10666.5	10697.0	10727.6
	8	8	8	8	8	9	9	9	9	9	9
N	10611.5	10603.5	10595.8	10588.4	10581.6	10575.3	10569.2	10563.5	10558.2	10553.4	10549.0
	9	9	9	9	9	9	9	9	9	9	9
I_r	10575.3	10575.3	10575.3	10575.3	10575.3	10575.3	10575.2	10575.2	10575.2	10575.2	10575.2
	9	9	9	9	9	9	9	9	9	9	9
I_e	10537.2	10544.2	10551.2	10558.3	10566.6	10575.3	10583.5	10592.0	10601.7	10611.8	10621.9
	8	8	8	8	9	9	9	9	10	10	10

Table 7
Sensitivity analysis on TP^* and n^* for Example 3.

	−50%	−40%	−30%	−20%	−10%	0%	10%	20%	30%	40%	50%
c	584.8	478.9	380.8	290.6	208.2	133.7	67.1	8.3	-42.7	-85.7	-109.4
	5	5	5	5	5	5	5	5	5	5	3
h	141.2	139.7	138.2	136.7	135.2	133.7	132.2	130.8	129.3	128.1	126.8
	5	5	5	5	5	5	5	5	6	6	6
θ	226.8	205.1	186.8	169.2	151.5	133.7	118.3	103.3	88.3	73.3	58.3
	4	4	5	5	5	5	6	6	6	6	7
r	150.1	146.7	143.4	140.1	136.9	133.7	130.6	127.6	124.8	122.1	119.5
	5	5	5	5	5	5	5	5	6	6	6
M	129.8	130.6	131.4	132.2	132.9	133.7	134.5	135.3	136.1	137.1	138.5
	5	5	5	5	5	5	5	5	5	6	6
N	135.4	135.0	134.6	134.3	134.0	133.7	133.5	133.4	133.2	133.2	133.2
	5	5	5	5	5	5	5	5	5	5	5
I_r	134.4	134.2	134.1	134.0	133.8	133.7	133.6	133.5	133.3	133.2	133.1
	5	5	5	5	5	5	5	5	5	5	5
I_e	133.4	133.5	133.5	133.6	133.7	133.7	133.8	133.8	133.9	133.9	134.0
	5	5	5	5	5	5	5	5	5	5	5

can be easily extended the single price policy to change selling prices upward or downward periodically, but is ideal for managers to design marketing strategies to stay ahead of the challenges their products are likely to face. Furthermore, the PSO algorithm is selected in this paper because of its robustness, simplicity and ease of implementation. The computational results indicated that the PSO algorithm offers acceptable efficiency and accurate search capability.

The proposed model can be extended in several ways. For instance, we may generalize the model to allow for shortages, quantity discounts and capacity constraint of owned warehouse. Also, we could extend the deterministic demand function to stochastic demand patterns. Finally, we could extend the sales environment to an advance booking system.

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Appendix A

Case 1: $N < M \leqslant t_i - t_{i-1}$

In this case, since the length of replenishment period is larger than the credit period, the retailer can continue to accumulate revenue and earn interest with an annual rate I_e on it. Hence, the present value of the interest earned in the ith replenishment period, denoted by $IE_{i1}, i = 1,2,\ldots,n$, is

$$IE_{i1} = pI_e \int_{t_{i-1}+N}^{t_{i-1}+M} e^{-rt}(t_{i-1} + M - t)\alpha(p)f(t)\mathrm{d}t. \tag{A1}$$

After the time that account is settled, the retailer starts to pay for the interest charges on the items in stocks with an annual rate I_r. The present value of interest charges in the ith period as denoted by IC_i, $i = 1,2,\ldots,n$, is

$$IC_{i1} = cI_r \int_{t_{i-1}+M}^{t_i} e^{-rt}e^{-\theta t} \int_t^{t_i} e^{\theta u}\alpha(p)f(t)\mathrm{d}u\,\mathrm{d}t. \tag{A2}$$

Case 2: $N \leqslant t_i - t_{i-1} < M$

As shown in Fig. 1, it is assumed that the length of replenishment period is shorter than the credit period, the retailer pays no interest charges ($IC_{i2} = 0$) and earns the interest during the period $[t_{i-1} + N, t_{i-1} + M]$. Thus the present value of the interest earned in the ith replenishment period, denoted by $IE_{i2}, i = 1,2,\ldots,n$, is

$$IE_{i2} = pI_e \int_{t_{i-1}+N}^{t_i} e^{-rt}(t_i - t)\alpha(p)f(t)dt + pI_e \int_{t_{i-1}}^{t_i} e^{-rt}(t_{i-1} + M - t_i)\alpha(p)f(t)dt. \tag{A3}$$

Case 3: $t_i - t_{i-1} < N \leqslant M$

From Fig. 1, it is assumed that the length of replenishment period is shorter than the credit period, the retailer pays no interest charges ($IC_{i3} = 0$) and earns the interest during the period $[t_{i-1} + N, t_{i-1} + M)$. Thus, the interest earned in the ith replenishment period, denoted by $IE_{i3}, i = 1,2,\ldots,n$, is given by

$$IE_{i3} = pI_e \int_{t_{i-1}}^{t_i} e^{-rt}(M - N)\alpha(p)f(t)\mathrm{d}t, \quad i = 1,2,\ldots,n. \tag{A4}$$

Appendix B

For any given feasible replenishment schedule, $0 = t_0 < t_1 < t_2 < \cdots < t_{n-1} < t_n = H$, to acquire optimal selling price that maximizes $TP(p|n,\mathbf{t})$, the value of p^* should be selected to satisfy

$$\frac{\mathrm{d}TP(p|n,\mathbf{t})}{\mathrm{d}p} = \frac{\mathrm{d}}{\mathrm{d}p}\sum_{i=1}^{n}\left\{SR_i - PC_i - HC_i + IE_i - IC_i - Ae^{-rt_{i-1}}\right\} = 0.$$

After rearranging the terms in previous equation, we thus get

$$[\alpha(p) + p\alpha'(p)]\sum_{i=1}^{n}\left\{\int_{t_{i-1}}^{t_i} e^{-rt}f(t)\,\mathrm{d}t + I_eW_i\right\} = \alpha'(p)\sum_{i=1}^{n}\left\{ce^{-rt_{i-1}}\int_{t_{i-1}}^{t_i} e^{\theta(t-t_{i-1})}f(t)\mathrm{d}t + \int_{t_{i-1}}^{t_i}\int_t^{t_i} e^{-rt-\theta(t-u)}f(t)\mathrm{d}u\,\mathrm{d}t + cI_rX_i\right\}, \tag{B1}$$

where

$$W_i = \begin{cases} \int_{N+t_{i-1}}^{M+t_{i-1}} e^{-rt}(M - t + t_{i-1})f(t)\mathrm{d}t, & t_i - t_{i-1} \geqslant M \\ \left\{\int_{t_{i-1}}^{t_i} e^{-rt}(M + t_{i-1} - t_i)f(t)\mathrm{d}t + \int_{N+t_{i-1}}^{t_i} e^{-rt}(t_i - t)f(t)\mathrm{d}t\right\}, & N \leqslant t_i - t_{i-1} < M \\ \int_{t_{i-1}}^{t_i} e^{-rt}(M - N)f(t)\mathrm{d}t & t_i - t_{i-1} < N \end{cases}$$

and

$$X_i = \begin{cases} \int_{M+t_{i-1}}^{t_i}\int_t^{t_i} e^{-rt-\theta(t-u)}f(u)\mathrm{d}u\mathrm{d}t, & t_i - t_{i-1} \geqslant M \\ 0, & N \leqslant t_i - t_{i-1} < M \\ 0, & t_i - t_{i-1} < N. \end{cases}$$

For any given feasible replenishment schedule, we have $W_i \geqslant 0$ and $X_i \geqslant 0$. Since $\alpha'(p) < 0$, it is obvious to see that (B1) holds if and only if $\alpha(p) + p\alpha'(p) < 0$.

Appendix C

From (12), (14) and (16), we have

$$\frac{\mathrm{d}^2TP(p|n,\mathbf{t})}{\mathrm{d}p^2} = [2\alpha'(p) + p\alpha''(p)]\sum_{i=1}^{n}\int_{t_{i-1}}^{t_i} e^{-rt}f(t)\mathrm{d}t + [2\alpha'(p) + \alpha''(p)]I_eY_i - \alpha''(p)\sum_{i=1}^{n}\left\{ce^{-rt_{i-1}}\int_{t_{i-1}}^{t_i} e^{\theta(t-t_{i-1})}f(t)\mathrm{d}t + \int_{t_{i-1}}^{t_i}\int_t^{t_i} e^{-rt-\theta(t-u)}f(t)\mathrm{d}u\mathrm{d}t + cI_rZ_i\right\},$$

where

$$Y_i = \begin{cases} \int_{N+t_{i-1}}^{M+t_{i-1}} e^{-rt}(M - t + t_{i-1})f(t)\mathrm{d}t, & t_i - t_{i-1} \geqslant M \\ \left\{\int_{t_{i-1}}^{t_i} e^{-rt}(M + t_{i-1} - t_i)f(t)\mathrm{d}t + \int_{N+t_{i-1}}^{t_i} e^{-rt}(t_i - t)f(t)\mathrm{d}t\right\}, & N \leqslant t_i - t_{i-1} < M \\ \int_{t_{i-1}}^{t_i} e^{-rt}(M - N)f(t)\mathrm{d}t, & t_i - t_{i-1} < N \end{cases}$$

and

$$Z_i = \begin{cases} \int_{M+t_{i-1}}^{t_i}\int_t^{t_i} e^{-rt-\theta(t-u)}f(u)\mathrm{d}u\,\mathrm{d}t, & t_i - t_{i-1} \geqslant M \\ 0, & N \leqslant t_i - t_{i-1} < M \\ 0, & t_i - t_{i-1} < N. \end{cases}$$

For any given feasible replenishment schedule, we have $Y_i \geqslant 0$ and $Z_i \geqslant 0$. Since $\alpha''(p) > 0$, if $2\alpha'(p) + p\alpha''(p) < 0$, then we have $\mathrm{d}^2TP(p|n,\mathbf{t})/\mathrm{d}p^2 < 0$.

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