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An improved algorithm and solution on an integrated production-inventory model in a three-layer supply chain

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ABSTRACT

Ben-Daya et al. (2010) established a joint economic lot-sizing problem (JELP) for a three-layer supply chain with one supplier, one manufacturer, and multiple retailers, and then proposed a heuristic algorithm to obtain the integral values of four discrete variables in the JELP. In this paper, we first complement some shortcomings in Ben-Daya et al. (2010), and then propose a simpler improved alternative algorithm to obtain the four integral decision variables. The proposed algorithm provides not only less CPU time but also less total cost to operate than the algorithm by Ben-Daya et al. (2010). Furthermore, our proposed algorithm can solve certain problems, which cannot be solved by theirs. Finally, the solution obtained by the proposed algorithm is indeed a global optimal solution in each of all instances tested.

1. Introduction

To bear a better resemblance to practice, Ben-Daya et al. (2010) considered a joint economic lot-sizing problem (JELP) in a three-layer supply chain with one supplier, one manufacturer, and multiple retailers as follows: The retailers have a common basic cycle time T. The manufacturer has the cycle time $T_m = K_2T$ while the supplier has the cycle time $T_s = K_1T_m = K_1(K_2T)$. The supplier receives m_1 equal shipments of raw materials during its cycle time T_s, transforms them into semi-finished products, and delivers m_2 equal-sized batches to the manufacturer during the manufacturer's cycle time T_m. The manufacturer, in turn, transforms those semi-finished products into finished products and ships finished products to each retailer at its order quantity every T units of time. However, the order quantity received by a retailer might be different from those received by the others. Then they established the chain-wide annual total cost (a.k.a., the total cost) as a function of K_1, K_2, T, m_1, and m_2 using the sum of the costs incurred by the supplier, the manufacturer, and the retailers. They minimized the chain-wide annual total cost in which four variables (i.e., K_1, K_2, m_1, and m_2) are discrete positive integers, and the other T is a real number. Notice that their JELP is a nonlinear integer programming (NLIP) model, and thus is hard to find an optimal solution using an exact method. Furthermore, the JELP is complex and computationally intensive even using mathematical software such as LINGO to solve it. By relaxing all integral variables as continuous variables, Ben-Daya et al. (2010) derived a near optimal solution to the problem using an algebraic method of completing perfect square without classical differential calculus techniques. In general, most studies use the classical differential calculus method to obtain the optimal values of the continuous decision variables. However, an algebraic method of perfect squares has been used in optimization problems in the inventory field recently. Examples are Grubbström (1995), Grubbström and Erdem (1999), Cárdenas-Barrón (2001, 2007, 2008), and Sphicas (2006), just to name a few. For an up-to-date review on different optimization approaches in inventory lot-sizing problems, see Cárdenas-Barrón (2011).

Ben-Daya et al. (2010) solved the relaxed JELP (i.e., relaxing all discrete integral variables in JELP as continuous real-number

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variables) by an algebraic method of completing the square, then proposed an algorithm to find the integral values for those four discrete integral variables. However, their proposed integral procedure seems to be computationally expensive. In fact, their algorithm requires to compute the integral variables (K_1, K_2), the continuous variable (T), and the total cost function for several times. For simplicity, we set $\lceil w \rceil$ as the smallest integer which is greater than or equal to w. Then their algorithm requires evaluating the values of K_1, K_2, T, and the total cost TC for $4\lceil m_1 \rceil \lceil m_2 \rceil$ times, if both $\lceil m_1 \rceil$ and $\lceil m_2 \rceil$ are greater than one. If any of $\lceil m_1 \rceil$ or $\lceil m_2 \rceil$ is equal to 1, then the number of evaluations is less than or equal to $4\lceil m_1 \rceil \lceil m_2 \rceil$. For example, if $\lceil m_1 \rceil = 11$ and $\lceil m_2 \rceil = 13$, then their algorithm requires to compute each of K_1, K_2, T, and the total cost TC for 572 times.

In this paper, we first complement mathematical errors in Ben-Daya et al. (2010) on the optimal basic cycle time $T^* = \sqrt{W/Y}$ and the minimum value for the annual total cost $TC = 2\sqrt{WY}$. If α_2 in Ben-Daya et al. (2010) is negative then both $K_2 = \sqrt{\alpha_2 \phi_2 / (\psi_2 \sum O_r)}$ in (29) and $Y = (K_2\phi_2 + \alpha_2)/2$ are not real numbers. Consequently, neither optimal basic cycle time $T^* = \sqrt{W/Y}$ nor the annual total cost $TC = 2\sqrt{WY}$ is a real number. This contradicts to the facts that both T^* and TC are real numbers. Hence, for correctness and completeness, we need to discuss the case in which $\alpha_2 < 0$. For simplicity, we discuss and illustrate this case using a numerical example as Instance 14 in Section 3 later. We then rearrange the total cost in (27) in Ben-Daya et al. (2010), and then propose a simple integral procedure similar to that by García-Laguna et al. (2010) to obtain the integral values for those four discrete variables m_1, m_2, K_1, and K_2. In addition, the proposed integral procedure discriminates the situations in which there is only one solution and when there are two solutions for each discrete variable. Furthermore, we not only obtain the integral values for all discrete variables in simple-to-apply closed-form expressions, but also need to compute the value of the continuous variable (T) only once, instead of $4\lceil m_1 \rceil \lceil m_2 \rceil$ times using the algorithm in Ben-Daya et al. (2010).

2. Mathematical model and algorithm

For simplicity, we use the same notation and assumptions as in Ben-Daya et al. (2010). After some mathematical manipulations, the annual total cost for the entire supply chain in (27) in Ben-Daya et al. (2010) can be re-written as follows:

$$TC(m_1, m_2, K_1, K_2) = \sqrt{2}\left\{\sqrt{f_1 + f_2 + f_3 + f_4 + e}\right\} \tag{1}$$

where f_1, f_2, f_3, f_4, and e are given by

$$f_1 = LO_s m_1 + \frac{GA_s}{m_1},$$

$$f_2 = ZO_m m_2 + \frac{XA_m}{m_2},$$

$$f_3 = \psi_1 (A_m + O_m m_2) K_1 + \frac{\alpha_1 (A_s + O_s m_1)}{K_1},$$

$$f_4 = \psi_2 \left(\sum_{r=1}^{n_r} O_r\right) K_2 + \frac{\alpha_2 \phi_2}{K_2},$$

and

$$e = LA_s + GO_s + ZA_m + XO_m + \alpha_2 \sum_{r=1}^{n_r} O_r.$$

To avoid taking a square root of a negative number as shown in (29) in Ben-Daya et al. (2010), we examine the values of G, L, X, Z, ψ_1, ψ_2, α_1, α_2, and ϕ_2 as follows:

$$G = \frac{h_0 D^2}{P_s} > 0,$$

$$L = h_s D (1 - D/P_s) > 0,$$

$$X = \frac{2h_s D^2}{P_s} > 0,$$

$$Z = \frac{h_s D^2}{P_m} - h_s D - \frac{h_m D^2}{P_m} + h_m D = D\left(1 - \frac{D}{P_m}\right)(h_m - h_s),$$

$$\psi_1 = \frac{G}{m_1} + L > 0,$$

$$\psi_2 = K_1 \psi_1 + \alpha_1,$$

$$\alpha_1 = \frac{X}{m_2} + Z,$$

$$\alpha_2 = \frac{2h_m D^2}{P_m} - h_m D + \sum_{r=1}^{n_r} h_r D_r = h_m D\left(\frac{2D - P_m}{P_m}\right) + \sum_{r=1}^{n_r} h_r D_r,$$

and

$$\phi_2 = \frac{A_s + O_s m_1}{K_1} + A_m + O_m m_2 > 0.$$

To minimize (1) is equivalent to minimize $\sum_{i=1}^{4} f_i$. It is clear that each f_i, $i = 1$, 2, 3, and 4, has the similar mathematical form as $a_1 y + a_2/y$. For the following cost-minimizing problem:

Minimizing $a_1 y + a_2/y$ when both a_1 and a_2 are positive, and y is a positive integral decision variable,

García-Laguna et al. (2010) proved that the optimal integral solution is as follows:

$$y = \left\lceil -0.5 + \sqrt{0.25 + \frac{a_2}{a_1}} \right\rceil \text{ or}$$

$$y = \left\lfloor 0.5 + \sqrt{0.25 + \frac{a_2}{a_1}} \right\rfloor, \tag{2}$$

where $\lceil w \rceil$ and $\lfloor w \rfloor$ are the smallest integer greater than or equal to w, and the largest integer less than or equal to w, respectively. Furthermore, it is clear that $\lceil w \rceil = \lfloor w+1 \rfloor$ if and only if w is not an integral value. For this case the problem has a unique optimal solution for y, which is given by anyone of those two mathematical expressions in (2). Otherwise, the problem has two optimal solutions for y: both $y^* = y$ and $y^* = y+1$. This procedure is easy to understand and simple to apply.

In order to apply the closed-form solution as shown in (2) to each discrete variable m_1, m_2, K_1, and K_2, we discuss the corresponding coefficients a_1 and a_2 to each of m_1, m_2, K_1, and K_2 separately as follows:

For m_1, both G and L are positive, which imply that both GA_s and LO_s are positive too. Thus, we have

$$m_1 = \left\lceil -0.5 + \sqrt{0.25 + \frac{GA_s}{LO_s}} \right\rceil \text{ or}$$

$$m_1 = \left\lfloor 0.5 + \sqrt{0.25 + \frac{GA_s}{LO_s}} \right\rfloor \tag{3}$$

For m_2, if h_m is greater than h_s, then both Z and ZO_m are positive. Since X is positive, we know that XA_m is positive too.

Hence, we know from (2) that

$$m_2 = \left[-0.5+\sqrt{0.25+\frac{XA_m}{ZO_m}}\right] \text{ or}$$

$$m_2 = \left\lfloor 0.5+\sqrt{0.25+\frac{XA_m}{ZO_m}}\right\rfloor. \tag{4}$$

On the other hand, if h_m is less than or equal to h_s, then Z is negative or zero, and the function f_2 is minimum at $m_2=\infty$. However, the value of the product in the second stage is higher than that in the first stage. Hence, it is obvious that the unit holding cost for the manufacturer h_m is greater than that for the supplier h_s. Consequently, we may assume without loss of generality that $h_m > h_s$ throughout the entire paper. As a result, we have $Z>0$, $\alpha_1>0$, and $\psi_2>0$. Thus, we have

$$K_1 = \left[-0.5+\sqrt{0.25+\frac{\alpha_1(A_s+O_s m_1)}{\psi_1(A_m+O_m m_2)}}\right] \text{ or}$$

$$K_1 = \left\lfloor 0.5+\sqrt{0.25+\frac{\alpha_1(A_s+O_s m_1)}{\psi_1(A_m+O_m m_2)}}\right\rfloor. \tag{5}$$

As we know, both ϕ_2 and ψ_2 are positive. If α_2 is negative, then it is obvious that f_4 is minimum at

$$K_2 = 1. \tag{6a}$$

Otherwise (if $\alpha_2>0$), then we have

$$K_2 = \left[-0.5+\sqrt{0.25+\frac{\alpha_2\phi_2}{\psi_2(\sum_{r=1}^{n_r} O_r)}}\right] \text{ or}$$

$$K_2 = \left\lfloor 0.5+\sqrt{0.25+\frac{\alpha_2\phi_2}{\psi_2(\sum_{r=1}^{n_r} O_r)}}\right\rfloor. \tag{6b}$$

Notice that the case of $\alpha_2<0$ is not discussed in Ben-Daya et al. (2010).

From the above results, we propose the following heuristic algorithm to obtain the integral values of m_1, m_2, K_1, and K_2:

An algorithm for finding integral values of m_1, m_2, K_1, and K_2

Step 1: Use Eqs. (3) and (4) to calculate the integral values of m_1 and m_2, respectively.
Step 2: Use Eqs. (5) and (6a) or (6b) to calculate the integral values of K_1 and K_2, respectively.
Step 3: Compute the total cost (TC) with the following equation:

$$TC = \sqrt{2\left(\frac{\phi_2}{K_2}+\sum_{r=1}^{n_r} O_r\right)(K_2\psi_2+\alpha_2)} \tag{7}$$

Therefore, the initial solution to the problem is (m_1, m_2, K_1, K_2, TC).
Step 4: Improving phase. In order to avoid a local optimum, we use a jumps strategy. In other words, the corresponding parallel multiple jumps from the current solution are needed to search for a better solution. The corresponding parallel multiple jumps for the integral solution of (m_1, m_2) are constructed as follows:

$$\begin{array}{ccc} (m_1-1,m_2-1) & (m_1,m_2-1) & (m_1+1,m_2-1) \\ (m_1-1,m_2) & & (m_1+1,m_2) \\ (m_1-1,m_2+1) & (m_1,m_2+1) & (m_1+1,m_2+1) \end{array}$$

Repeat **Step 2** and **Step 3** for all different jumps. Only consider jumps with $m_i-1 \geq 1$.
Step 5: Select the solution with the minimal total cost. If the solution can be improved by the corresponding parallel multiple jumps, then go to **Step 4**. Otherwise, a good solution is found. Then, determine T by

$$T = \sqrt{\frac{2(\phi_2/K_2+\sum_{r=1}^{n_r} O_r)}{K_2\psi_2+\alpha_2}}$$

and the solution is expressed as (m_1,m_2,K_1,K_2,T,TC).

In the next section, we apply the proposed algorithm to the numerical examples of Ben-Daya et al. (2010) and additional instances.

3. Numerical examples

Ben-Daya et al. (2010) proved the efficiency of their algorithm with 12 instances. We apply our proposed algorithm for the same 12 instances. The results in Table 1 show that our proposed algorithm has the same solutions as theirs for 11 instances and obtains a smaller total cost for the other 1 instance (i.e., Instance 10). It is worth mentioning that the initial solution in 9 out of 12 instances cannot be improved by the corresponding parallel multiple jumps, and thus the initial solution is the final solution. Therefore, we can conclude that the initial solution of our proposed algorithm is a remarkably good heuristic solution to the problem. In addition, our proposed algorithm requires only one evaluation to find the total cost.

In order to show that the proposed algorithm obtains a less total cost to operate than the algorithm by Ben-Daya et al. (2010), we present 13 additional instances and their results are shown in Table 2. The data for those 13 instances are available upon request to the corresponding author.

Also, we calculate: (1) the number of iterations to obtain the total cost, (2) the CPU time to get the solution, and (3) the total

Table 1
Results for the 12 instances of Ben-Daya et al. (2010) with the proposed algorithm.

Instance	O_s	O_m	h_o	h_s	h_m	m_1	m_2	K_1	K_2	T	TC	Was the initial solution improved?	% of improvement with respect to the initial solution
1	600	300	0.08	0.8	2	1	2	1	7	0.0277	47,940.35	No	0
2	100	50	0.08	0.8	2	1	4	1	6	0.0274	41,307.40	Yes	0.25
3	200	100	0.08	0.8	2	1	3	1	6	0.0279	43,002.69	Yes	0.20
4	300	200	0.08	0.8	2	1	2	1	6	0.0280	45,186.45	Yes	0.13
5	1000	500	0.08	0.8	2	1	2	1	8	0.0280	51,762.23	No	0
6	2000	800	0.08	0.8	2	1	2	1	10	0.0278	58,099.39	No	0
7	3000	1000	0.08	0.8	2	1	1	2	6	0.0276	62,749.57	No	0
8	1000	500	0.2	0.8	2	1	2	1	8	0.0276	52,354.95	No	0
9	1000	500	0.4	0.8	4	1	1	2	4	0.0259	57,731.96	No	0
10	1000	500	0.8	0.8	5	1	1	2	4	0.0244	61,361.47	No	0
11	1000	500	0.4	2	4	1	2	1	6	0.0241	70,528.00	No	0
12	1000	500	0.8	2	5	1	2	1	6	0.0230	73,664.10	No	0

Table 2
Results for Instances 13–25.

Instance	m_1	m_2	K_1	K_2	T	TC	Was the initial solution improved?	% of improvement with respect to the initial solution
13	3	6	3	3	0.0114	559,655.45	Yes	0.002
14	2	2	2	1	0.2540	24,416.33	No	0
15	2	2	1	1	0.0115	79,625.92	Yes	7.6
16	10	10	2	7	0.0087	200,897.95	Yes	0.04
17	5	7	1	8	0.0125	272,688.52	Yes	6.8
18	4	5	1	3	0.0135	112,180.47	Yes	2.8
19	2	1	1	6	0.0062	868,395.07	Yes	26.4
20	13	8	1	15	0.0019	1,353,416.70	Yes	0.30
21	18	13	2	4	0.0151	76,226.62	Yes	0.004
22	6	4	8	2	0.0072	199,354.02	No	0
23	1	11	2	7	0.0043	1,706,102.10	Yes	0.06
24	1	3	1	1	0.0122	77,674.01	Yes	5.1
25	2	2	1	4	0.0115	106,334.35	Yes	1.9

Table 3
Number of evaluations of total cost, CPU times, and differences in total cost for both algorithms.

Instance	Ben-Daya et al. (2010)'s algorithm			Proposed algorithm			
	No. of evaluations of total cost	CPU time (seconds)	Total cost	No. of evaluations of total cost	CPU time (seconds)	Total cost	Difference
1	10	0.000011	47,940.35	6	0.000009	47,940.35	0
2	18	0.000012	41,307.40	10	0.000007	41,307.40	0
3	14	0.000009	43,002.69	8	0.000006	43,002.69	0
4	10	0.000008	45,186.45	8	0.000006	45,186.45	0
5	6	0.000006	51,762.23	6	0.000005	51,762.23	0
6	6	0.000006	58,099.39	6	0.000005	58,099.39	0
7	8	0.000006	62,749.57	4	0.000004	62,749.57	0
8	6	0.000006	52,354.95	6	0.000005	52,354.95	0
9	6	0.000006	57,731.96	4	0.000004	57,731.96	0
10	6	0.000006	61,491.74	4	0.000004	61,361.47	130.27
11	6	0.000007	70,528.00	6	0.000005	70,528.00	0
12	6	0.000006	73,664.10	6	0.000005	73,664.10	0
13	80	0.000104	559,667.28	12	0.000008	559,655.45	11.83
14	*	*	*	9	0.000007	24,416.33	*
15	12	0.000028	79,937.52	23	0.000009	79,625.92	311.59
16	480	0.000383	200,897.95	18	0.000009	200,897.95	0
17	200	0.000286	277,951.50	57	0.000015	272,688.52	5262.97
18	124	0.000164	112,180.47	33	0.00001	112,180.47	0
19	12	0.000029	892,861.69	20	0.000008	868,395.07	24,466.62
20	286	0.000395	1,359,146.16	25	0.000011	1,353,416.70	5729.46
21	1008	0.00098	76,226.62	13	0.000008	76,226.62	0
22	120	0.000106	199,354.02	9	0.000006	199354.02	0
23	40	0.000043	1,706,396.20	10	0.000006	1,706,102.10	294.10
24	14	0.000035	77,674.01	15	0.000008	77,674.01	0
25	20	0.000042	106,897.17	14	0.000007	106,334.35	562.81

* Ben-Daya et al. (2010)'s algorithm cannot solve this instance.

cost using their algorithm first and then our proposed algorithm. We then compare the difference of the total cost between theirs and ours in each instance. The results are shown in Table 3. Table 3 reveals that our proposed algorithm obtains cheaper total cost than theirs in 8 instances (i.e., Instances 10, 13, 15, 17, 19, 20, 23, and 25). For the other instances, we obtain the same total cost as theirs except for Instance 14. It is worth mentioning that Instance 14 cannot be solved using the algorithm in Ben-Daya et al. (2010) because α_2 is negative, and thus both $K_2 = \sqrt{\alpha_2\phi_2/(\psi_2 \sum O_r)}$ and $Y = (K_2\phi_2 + \alpha_2)/2$ are not real numbers. However, we can solve Instance 14 by our proposed algorithm. In addition, our proposed algorithm uses not only significantly less number of iterations but also less CPU time than theirs.

To verify our solutions are indeed optimal, we also solve all the 25 instances with LINGO optimizer. The results reveal that the solutions for the 25 instances by LINGO are exactly the same as the solutions obtained using the proposed algorithm. Furthermore, LINGO optimizer states that a global optimal solution is found in each of those 25 instances. This empirical experimentation shows that the proposed algorithm performs very well since it obtains the optimal solutions for all 25 instances. Furthermore, the initial solution obtained by the proposed algorithm is the optimal solution in 11 out of 25 instances.

Finally, it is important to mention that all instances in this paper were solved using a lap-top computer with the following technical characteristics: Intel ® Core™ 2 Duo CPU, P8700 @ 2.53 GHz, 3.45 GB of RAM.

4. Conclusions

We have complemented some shortcomings in Ben-Daya et al. (2010). For example, if α_2 is negative, then neither the optimal basic cycle time T^* nor the annual total cost TC in their algorithm is a real number. We have proposed a simple-to-apply alternative algorithm to obtain 4 discrete integral decision variables by an explicitly closed-form solution. In addition, the proposed algorithm not only needs less CPU time but also less total cost than the algorithm by Ben-Daya et al. (2010). Furthermore, our proposed algorithm has solved some problems, which cannot be solved by the algorithm in Ben-Daya et al. (2010). Finally, it is worth mentioning that the proposed algorithm is remarkably good because it has obtained the global optimal solution for all the 25 instances and its initial solution has been the global optimal solution in 11 out of 25 instances.

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