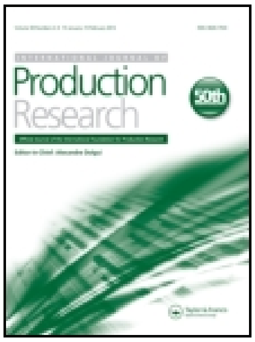

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Impacts of collaborative investment and inspection policies on the integrated inventory model with defective items

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For an imperfect production system, to reduce quality-related costs, a manager may consider investing capital in quality improvement. In general, the investment expense in reducing the defective rate of items is often paid by the vendor. On the other hand, the buyer may inspect the product quality as the order is received which implies it incurs an inspection cost. In a supply chain integrated system, to accomplish global optimisation, the vendor and buyer can agree to jointly invest capital to improve the imperfect production processes, and the buyer can remove the inspection programme as the defective rate reaches a certain low-level. Hence, this paper investigates the impacts of collaborative investment and inspection policies on an integrated inventory model with defective items. The objective of this study is to seek the optimal order quantity, shipping times from the vendor to the buyer per production run, and the defective rate that minimise the joint total cost per unit time. An algorithm is developed to find the optimal solution. Several numerical examples are presented to demonstrate the proposed model and solution procedure, and then several management insights are obtained from the numerical examples.

Keywords: inventory; integrated model; defective items; capital investment; non-inspect

1. Introduction and literature review

As we know, the objective of supply chain management is to be efficient and cost-effective across the entire system where total system-wide costs need to be minimised. To accomplish global optimisation in the field of inventory management, the concept of joint economic lot size (JELS) is introduced to refine the well-known classical economic order quantity (EOQ) model. The JELS model for a single vendor-single buyer was first developed and introduced by Goyal (1977). Later, Banerjee (1986) assumed that the vendor produces on a lot-for-lot basis in response the buyer's order, and developed the JELS model. Goyal (1988) extended Banerjee's (1986) model and assumed that the vendor's lot size is an integer multiple of the buyer's order size. Furthermore, Lu (1995) relaxed Goyal's (1988) assumption of a single vendor-single-buyer and proposed a model where the vendor can actually supply the buyer in a number of equal smaller lot-sizes, even before completing the entire lot. Further literatures in support of this issue include Aderohunmu, Mobolurin, and Bryson (1995), Goyal (1995), Hill (1997), Sarker and Khan (1999, 2003), Khan and Sarker (2002), Sarker (2002), Wu and Ouyang (2003), Hill and Omar (2006) and Lin (2009). Nevertheless, the majority of above research focused on the production shipment schedule between the vendor and buyer neglects the relationship between order lot and quality.

It is common yet unrealistic to assume that all units produced are good quality. Indeed, the production process may deteriorate and hence the defective or poor quality items will be produced. That is, product quality is not always perfect and actually dependent on the production process. Rosenblatt and Lee (1986) considered the effect of imperfect production processes on the economic production quantity (EPQ) model. At the same time, Porteus (1986) incorporated the effect of the defective items into the EPQ model and introduced the option of investing capital in production process quality improvement. Kim and Hong (1999) extended Rosenblatt and Lee's (1986) model and determined the optimal production run length in deteriorating production processes. Salameh and Jaber (2000) proposed a modified EPQ model by accounting for imperfect quality items where poor-quality items will be sold as a single batch in a discounted price at the end of the screening process. Following this, numerous studies on imperfect production processes have been published such as Goyal and Cárdenas-Barrón (2002), Chung and Hou (2003), Papachristos and Konstantaras (2006),

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Abdullah and Gultekin (2007), Halim, Giri, and Chaudhuri (2009) and their references. However, all articles described above are only from the perspective of the vendor.

Huang (2002) developed an integrated production-inventory model with imperfect quality where the number of defective items follows a given probability density function. Wee, Yu, and Wang (2006) investigated an integrated model for deteriorating items with imperfect quality in which shortages are allowed and completely backordered. Lo, Wee, and Huang (2007) developed an integrated model with imperfect production processes and Weibull distribution deterioration under inflation. Many other related articles can be found in Lin, Chen, and Kroll (2003), Goyal, Huang, and Chen (2003), Huang (2004), Ho et al. (2011), Sana (2011) and their references. Furthermore, for an imperfect production system, a manager may consider investing capital in quality improvement to reduce quality-related costs. That is, the production process is controllable and the defective rate of items can be reduced by investing capacity to improve manufacturing processes. Hong (1997) extended Rosenblatt and Lee (1986) to take the determination of production cycles, procurement schedules, and joint investment in set-up reduction and process quality improvement into consideration. Ouyang and Chang (2000) investigated the impact of quality improvement on the modified lot size reorder point model. Hou and Lin (2004) studied the effects of an imperfect production process on the optimal production run length when capital investment in process quality improvement is adopted. Yang and Pan (2004) proposed an integrated inventory model involving variable lead time and quality improvement investment with normal distributional demand. Other related studies that tackled quality improvement can be found in Keller and Noori (1988), Hwang, Kim, and Kim (1993), Hong and Hayya (1995), Tripathy, Wee, and Majhi (2003), Ouyang Wu, and Ho (2006, 2007) and Yoo, Kim, and Park (2012).

In general, the investment expense in reducing the defective rate of items should be paid by the vendor. However, if the expense is too high for the vendor to pay, it will be not feasible to achieve global optimisation for a supply chain integrated system. On the other hand, as the defective items are produced due to the imperfect production process, the buyer may assess the product quality as the order is received, before stocking it for immediate or later use. He/she may perform a complete inspection or inspect samples which implies it incurs an inspection cost. In a supply chain integrated system, to accomplish global optimisation, the vendor and buyer can agree to jointly invest capital to improve the imperfect production processes, and then the buyer will remove the inspection step upon receipt of the goods as the defective rate reaches a certain low-level (in this article we refer to this as 'non-inspect'). If the buyer does not inspect the received items, they will be treated as non-defective products to stock and sell to customers, resulting in a penalty cost for the defective items returned from customers. Consequently, the optimal investment strategy, by trading off the inspection cost against the penalty cost, is an import issue and needs to be incorporated in the field of inventory problems.

Therefore, this paper develops an integrated inventory model with defective items in which the defective rate can be improved through joint capital investment from the vendor and buyer. Mathematical analyses are utilised to seek the optimal order quantity, shipping times from the vendor to the buyer per production run, and defective rate that minimise the joint total cost per unit time. An algorithm is developed to find the optimal solution. Furthermore, several numerical examples are presented to demonstrate the proposed model and solution procedure, and then several management insights are obtained from the numerical examples.

2. Notation and assumptions

The following notation is used throughout this paper:

- D Demand rate of the market.
- P Production rate of the vendor.
- A The buyer's ordering cost per order.
- S The vendor's setup cost per setup.
- h_{b_1} The buyer's holding cost per non-defective item per unit time.
- h_{b_2} The buyer's holding cost per defective item per unit time, where $h_{b_2} < h_{b_1}$.
- h_{v_1} The vendor's holding cost per item per unit time.
- h_{v_2} The vendor's treatment cost per detective item.
- x The buyer's inspecting rate per order.
- C_s The buyer's inspecting cost per unit.
- C_p The buyer's penalty cost per defective item which is returned from the customer.
- C_T The vendor's fixed cost of transportation per shipment.

C_t The vendor's variable cost of transportation per unit.
θ The opportunity cost of the capital investment per dollar per unit time.
λ_U The proportion of defective items before improving production process, $\lambda_U < 1$ and is given.
λ_L The proportion of defective items which reaches 'non-inspect', where $\lambda_L < \lambda_U$ and is given.
λ The proportion of defective items, where $\lambda \in (0, \lambda_U]$, a decision variable.
Q The buyer's order quantity, a decision variable.
T The length of buyer's replenishment cycle, a decision variable.
m The number of shipment for the vendor to buyer per production cycle, an integer decision variable.
q Size of each shipment from the vendor to buyer in a production bath, a decision variable.
$JTC(\lambda, q, m)$ The joint total cost per unit time, which is a function of λ, q and m
$*$ The superscript represents optimal value.

In addition, the following assumptions are used throughout this paper:

(1) There is single-vendor and single-buyer for a single product in this model.
(2) The vendor's production rate of non-defective item is finite and greater than the demand rate, i.e. $P(1-\lambda) > D$.
(3) The buyer orders a lot of size Q (for non-defective items) per order. The vendor produces mq units in each production run, and delivers q units to the buyer in each shipment.
(4) The capital investment, $I(\lambda)$, in improving production process quality to reduce the defective rate of the product is given as a logarithmic function of λ, i.e.

$$I(\lambda) = \frac{1}{\delta} \ln\left(\frac{\lambda_U}{\lambda}\right), \quad 0 < \lambda \leq \lambda_U,$$

where λ_U is the proportion of defective items before improving production processes and δ denotes the percentage decrease in λ per dollar increase in $I(\lambda)$. This particular function is similar to Hall (1983), and has been widely used in the literature (see Porteus 1986; Keller and Noori 1988).
(5) The capital investment is shared between the vendor and buyer jointly. That is, the proportions of capital investments which the buyer and vendor should invest in machinery equipment are α and $1-\alpha$, respectively, $0 \leqslant \alpha \leqslant 1$.
(6) When the order is received, the buyer may perform a 100% inspection with inspect rate x to check the product quality before selling it. Defective items in each batch are discovered and returned to the vendor at the time of delivery of the next lot. Therefore, it incurs inspection and holding costs (including non-defective items and defective items) for the buyer. On the other hand, as the proportion of defective items reaches to or less than a certain low-level rate, λ_L, due to capital investment, the buyer does not need to conduct any cheques on the received items. We term this as 'non-inspect'. In this situation, all the received items from the vendor are sold to the customers directly. As a result, the defective items in quantity λq (where $\lambda \leqslant \lambda_L$) are sequentially returned from customers later. These defective items are stored and returned to the vendor at the end of each replenishment cycle.
(7) Inspection is non-destructive and error-free.

3. Model formulation

Based on above notation and assumptions, we develop an integrated inventory model with defective items. The integrated inventory system evolves as follows: the buyer orders Q units per order and the vendor delivers q units to the buyer in each shipment. Each received lot contains a percentage, λ, of defective items. Hence, the number of non-defective items in each shipment is $(1-\lambda)q$ which equals to the buyer's order quantity Q, i.e. $Q = (1-\lambda)q$ and, therefore, the replenishment cycle length is $Q/D = (1-\lambda)q/D$. On the other hand, if the defective rate of the product is reduced to or less than λ_L by investing capital in improving the production process, the buyer will not inspect, and all received items will be treated as non-defective products to stock and sell. Hence, the vendor's shipment size q is equal to the buyer's order quantity Q, i.e. $Q = q$ and therefore, the replenishment cycle length is $Q/D = q/D$. Following, we first establish the total cost per unit time of the buyer and vendor, respectively. Then, the joint total cost per unit time of the integrated inventory system is developed.

3.1 *The buyer's total cost per unit time*

The buyer's total cost per replenishment cycle, given that there are defective items in an arriving shipment, consists of the following elements.

(a) Ordering cost
The buyer's ordering cost per replenishment cycle is A.

(b) Opportunity cost of capital investment
There is an opportunity cost due to capital investment in improving production process quality which is $\theta I(\lambda)$. Because the capital investment is shared out between the buyer and vendor where the proportion of the buyer's investment is α $(0 \leqslant \alpha \leqslant 1)$, the opportunity cost of capital investment in improving production process quality per cycle for the buyer is $\alpha\theta I(\lambda)T = \alpha(\theta/\delta)T \ln[\lambda_U/\lambda]$, where

$$T = Q/D = \begin{cases} (1-\lambda)q/D, & \text{if } \lambda_L < \lambda \leq \lambda_U, \\ q/D, & \text{if } 0 < \lambda \leq \lambda_L. \end{cases}$$

(c) Inspection cost
Prior to investing capital to improve the production process, the defective rate for the items received by the buyer is λ_U. After the capital investment in improving the production process quality, if $\lambda_L < \lambda \leqslant \lambda_U$, then the buyer will inspect all received items. The inspection cost per unit is C_s and the buyer receives quantity q in each shipment; thus, the inspection cost per cycle is $C_s q$. On the other hand, if $0 < \lambda \leqslant \lambda_L$, then the buyer will not need to inspect all received items and the inspection cost per cycle is zero. Hence, the inspection cost per replenishment cycle for the buyer is

$$\begin{cases} C_s q, & \text{if } \lambda_L < \lambda \leq \lambda_U, \\ 0, & \text{if } 0 < \lambda \leq \lambda_L. \end{cases}$$

(d) Holding cost of non-defective items
As the buyer receives an arriving lot q containing some defective items with defective rate λ from the vendor, he/she may inspect all received items with inspect rate x if $\lambda_L < \lambda \leqslant \lambda_U$ and the holding cost of non-defective items (include the defective items before they are identified) per cycle is

$$h_{b_1}[(1-\lambda)qT/2 + \lambda q^2/(2x)] = h_{b_1}q^2[(1-\lambda)^2/(2D) + \lambda/(2x)].$$

On the other hand, if $0 < \lambda \leqslant \lambda_L$, all received items will be treated as non-defective items to stock and sell and hence the holding cost per cycle is $h_{b_1}qT/2 = h_{b_1}q^2/(2D)$. The behaviour of the inventory level for the buyer is shown in Figures 1(a) and 1(b).

Consequently, the holding cost of non-defective items per replenishment cycle is given by

$$\begin{cases} h_{b_1} q^2[(1-\lambda)^2/(2D) + \lambda/(2x)], & \text{if } \lambda_L < \lambda \leq \lambda_U, \\ h_{b_1} q^2/(2D), & \text{if } 0 < \lambda \leq \lambda_L. \end{cases}$$

(e) Holding cost of defective items
In every shipment, the buyer will receive λq defective items. When $\lambda_L < \lambda \leqslant \lambda_U$, these defective items will be discovered and returned to the vendor at the end of each shipment cycle, and hence the holding cost for defective items per replenishment cycle is $h_{b_2}[\lambda qT - \lambda q^2/(2x)] = h_{b_2}\lambda q^2[(1-\lambda)/D - 1/(2x)]$ (see Figure 1(a)). On the other hand, if $0 < \lambda \leqslant \lambda_L$, due to items not being inspected, the defective items are sequentially returned from customers later. Similar, these defective items are stored and returned to the vendor at the end of each cycle, and hence the holding cost for

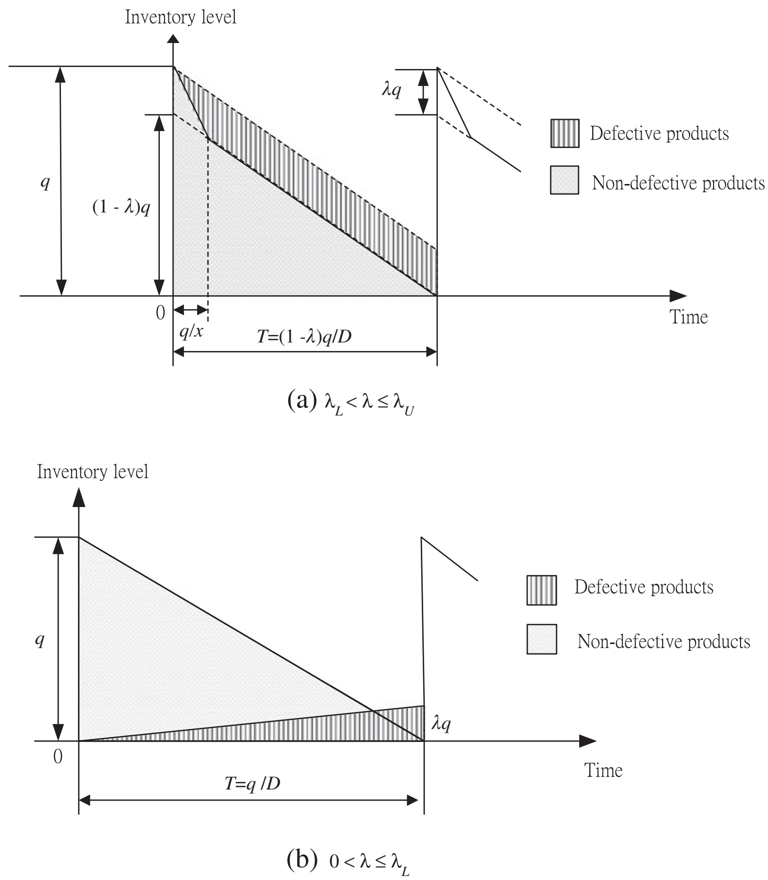

Figure 1. Inventory level of the buyer over time *T*.

defective items per cycle is $h_{b_2}\lambda qT/2 = h_{b_2}\lambda q^2/(2D)$ (see Figure 1(b)). Therefore, the holding cost for defective items is

$$\begin{cases} h_{b_2}\lambda q^2[(1-\lambda)/D - 1/(2x)], & \text{if } \lambda_L < \lambda \le \lambda_U, \\ h_{b_2}\lambda q^2/(2D), & \text{if } 0 < \lambda \le \lambda_L. \end{cases}$$

(f) Penalty cost of defective items (external failure cost)

As mentioned above, if $\lambda_L < \lambda \leqslant \lambda_U$, the buyer may inspect all received items, and hence there is no penalty cost. On the contrary, if $0 < \lambda \leqslant \lambda_L$, all units of received items will be treated as non-defective products to sell and then be sequentially returned by customers and incur a penalty cost of C_p per unit for the buyer. Hence, the penalty cost of defective items per cycle is

$$\begin{cases} 0, & \text{if } \lambda_L < \lambda \le \lambda_U, \\ C_p\lambda q, & \text{if } 0 < \lambda \le \lambda_L. \end{cases}$$

Consequently, the buyer's total relevant cost per unit time is the sum of the above elements divided by the length of the replenishment cycle $(1-\lambda)q/D$ or q/D. That is,

$$TC_b(\lambda,\ q) = \begin{cases} TC_{1b}(\lambda,\ q), & \text{if } \lambda_L < \lambda \le \lambda_U, \\ TC_{2b}(\lambda,\ q), & \text{if } 0 < \lambda \le \lambda_L, \end{cases}$$

where

$$TC_{1b}(\lambda,\ q) = \frac{D}{(1-\lambda)q}\left\{A + \frac{\alpha\theta(1-\lambda)q}{\delta D}\ln\left(\frac{\lambda_U}{\lambda}\right) + C_s q + h_{b_1}q^2\left[\frac{(1-\lambda)^2}{2D} + \frac{\lambda}{2x}\right] + h_{b_2}\lambda q^2\left[\frac{1-\lambda}{D} - \frac{1}{2x}\right]\right\}, \tag{1}$$

and

$$TC_{2b}(\lambda,\ q) = \frac{D}{q}\left[A + \frac{\alpha\theta\ q}{\delta D}\ln\left(\frac{\lambda_U}{\lambda}\right) + \frac{h_{b_1}q^2}{2D} + \frac{h_{b_2}\lambda\ q^2}{2D} + C_p\lambda q\right]. \tag{2}$$

3.2 *The vendor's total cost per unit time*

The vendor's total cost per production cycle consists of the following elements:

(a) Setup cost
The vendor's set-up cost per production cycle is S.

(b) Transportation cost
The vendor's transportation cost per shipment is the sum of a fixed per-lot transportation cost C_T and a variable transportation cost $C_t q$, hence, the total transportation cost per production cycle is $m(C_T + C_t q)$, where

$$m \equiv \begin{cases} m_1, & \text{if } \lambda_L < \lambda \le \lambda_U, \\ m_2, & \text{if } 0 < \lambda \le \lambda_L. \end{cases} \tag{3}$$

(c) Opportunity cost of capital investment
The capital investment is shared out between the buyer and vendor, with the proportion of the vendor's investment being $1-\alpha$ $(0 \le \alpha \le 1)$. From the above, we know that the length of each production cycle for the vendor is $mT = m_1(1-\lambda)q/D$ if $\lambda_L < \lambda \leqslant \lambda_U$, while is $mT = m_2 q/D$ if $0 < \lambda \leqslant \lambda_L$. Therefore, the opportunity cost of capital investment per production cycle for the vendor is

$$(1-\alpha)\theta I(\lambda)mT = \begin{cases} m_1(1-\alpha)(1-\lambda)(\theta/\delta)(q/D)\ln[\lambda_U/\lambda] & \text{if } \lambda_L < \lambda \le \lambda_U, \\ m_2(1-\alpha)(\theta/\delta)(q/D)\ln[\lambda_U/\lambda] & \text{if } 0 < \lambda \le \lambda_L. \end{cases}$$

(d) Holding cost
When the first q units are produced, the vendor will deliver them to the buyer. After the first shipment, the vendor will schedule successive deliveries every $(1-\lambda)q/D$ units of time until the inventory level falls to zero if $\lambda_L < \lambda \leqslant \lambda_U$. The behaviour of the inventory level for the vendor is shown in Figures 2(a). Consequently, the cumulative inventory per production cycle for the vendor is

$$\left[m_1 q\left(\frac{q}{P} + \frac{(m_1-1)(1-\lambda)q}{D}\right) - \frac{m_1^2 q^2}{2P}\right] - \left[\frac{(1-\lambda)q^2}{D}(1+2+\cdots+(m_1-1))\right] = m_1 q^2\left[\frac{1}{P} + \frac{(m_1-1)(1-\lambda)}{2D} - \frac{m_1}{2P}\right].$$

On the other hand, if $0 < \lambda \leqslant \lambda_L$, then the buyer will not inspect and all received items will be treated as non-defective products to stock (see Figure 2(b)). In this case, the cumulative inventory per production cycle for the vendor is

$$\left[m_2 q\left(\frac{q}{P} + (m_2 - 1)\frac{q}{D}\right) - \frac{m_2^2 q^2}{2P}\right] - \left[\frac{q^2}{D}(1 + 2 + \cdots + (m_2 - 1))\right] = m_2 q^2\left[\frac{1}{P} + \frac{m_2 - 1}{2D} - \frac{m_2}{2P}\right].$$

Therefore, the holding cost per production cycle is

$$\begin{cases} h_{v_1} m_1 q^2\left[\dfrac{1}{P} + \dfrac{(m_1 - 1)(1 - \lambda)}{2D} - \dfrac{m_1}{2P}\right], & \text{if } \lambda_L < \lambda \leq \lambda_L \\ h_{v_1} m_2 q^2\left[\dfrac{1}{P} + \dfrac{(m_2 - 1)}{2D} - \dfrac{m_2}{2P}\right], & \text{if } 0 \leq \lambda \leq \lambda_L. \end{cases}$$

(e) Treatment cost for defective items

In each shipment with size q, λq defective items will be returned by the buyer at the end of shipment cycle. The treatment cost for returned defective items per production cycle is $h_{v_2} m \lambda q$, where m is shown as in Equation (3).

Consequently, the vendor's total relevant cost per unit time is the sum of the above elements divided by the length of production cycle $m_1(1 - \lambda)q/D$ or $m_2 q/D$. That is,

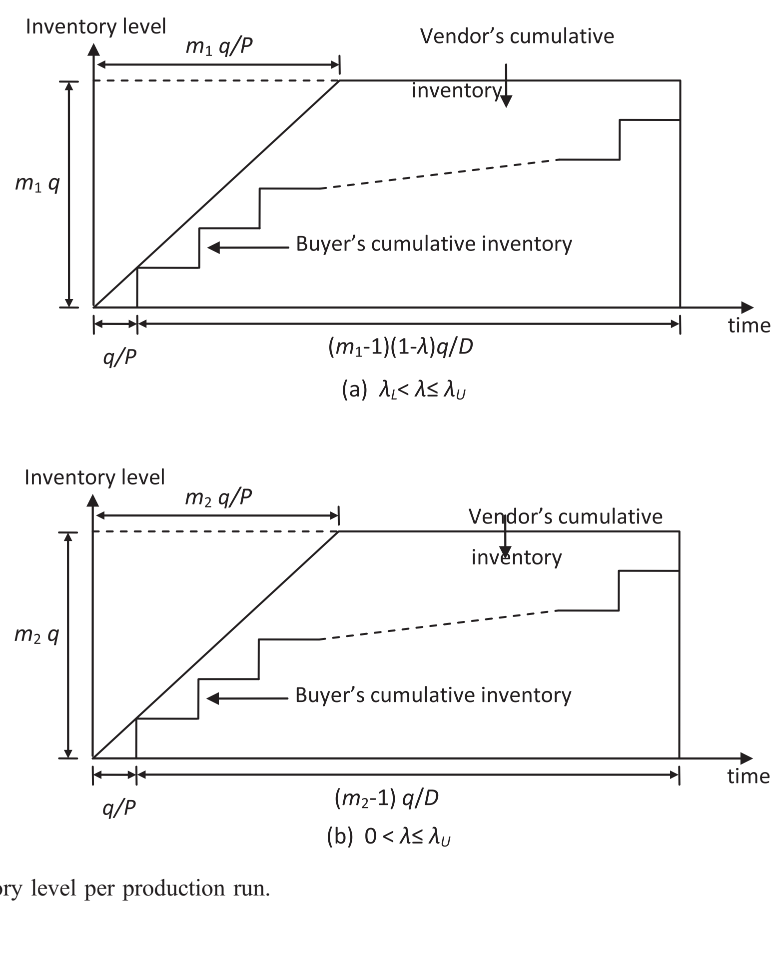

Figure 2. Vendor's inventory level per production run.

$$TC_v(\lambda,\ m) = \begin{cases} TC_{1v}(\lambda,\ m_1), & \text{if } \lambda_L < \lambda \leq \lambda_U, \\ TC_{2v}(\lambda,\ m_2), & \text{if } 0 < \lambda \leq \lambda_L, \end{cases}$$

where

$$TC_{1v}(\lambda,\, m_1) = \frac{D}{m_1(1-\lambda)q}\left\{S + m_1(C_T + C_t q) + \frac{(1-\alpha)\theta\,(1-\lambda)m_1 q}{\delta D}\ln\left[\frac{\lambda_U}{\lambda}\right] + h_{v_1} m_1 q^2\left[\frac{1}{P} + \frac{(m_1-1)(1-\lambda)}{2D} - \frac{m_1}{2P}\right] + h_{v2} m_1 \lambda q\right\}, \tag{4}$$

and

$$TC_{2v}(\lambda,\, m_2) = \frac{D}{m_2 q}\left\{S + m_2(C_T + C_t q) + \frac{(1-\alpha)\theta m_2 q}{\delta D}\ln\left[\frac{\lambda_U}{\lambda}\right] + h_{v_1} m_2 q^2\left[\frac{1}{P} + \frac{(m_2-1)}{2D} - \frac{m_2}{2P}\right] + h_{v2} m_2 \lambda q\right\}. \tag{5}$$

3.3 *The joint total cost per unit time*

Once the buyer and vendor have built up a long-term strategic partnership, they can determine together what the best policy is for one another. Therefore, the joint total cost per unit time can be obtained as the sum of the buyer's and vendor's total costs per unit time, which leads to

$$JTC(\lambda,\, q,\, m) = TC_b(\lambda,\, q) + TC_v(\lambda,\, m) \;=\begin{cases} JTC_1(\lambda,\, q,\, m_1), & \text{if } \lambda_L < \lambda \leq \lambda_U, \\ JTC_2(\lambda,\, q,\, m_2), & \text{if } 0 < \lambda \leq \lambda_L, \end{cases}$$

where

$$JTC_1(\lambda,\, q,\, m_1) = \frac{D}{(1-\lambda)q}\left\{\frac{m_1 A + S + m_1 C_T}{m_1} + \frac{\theta(1-\lambda)q}{\delta D}\ln\left[\frac{\lambda_U}{\lambda}\right] + (C_t + C_s + h_{v_2}\lambda)\, q + h_{b_1} q^2\left[\frac{(1-\lambda)^2}{2D} + \frac{\lambda}{2x}\right]\right.$$
$$\left. + h_{b_2}\lambda q^2\left[\frac{1-\lambda}{D} - \frac{1}{2x}\right] + m_1(1-\lambda)^2 q^2\left[\frac{1}{P} + \frac{(m_1-1)}{2D} - \frac{m_1}{2P}\right]\right\}, \tag{6}$$

and

$$JTC_2(\lambda,\, q,\, m_2) = \frac{D}{q}\left\{\frac{m_2 A + S + m_2 C_T}{m_2} + \frac{\theta q}{\delta D}\ln\left[\frac{\lambda_U}{\lambda}\right] + \frac{(h_{b1} + h_{b_2}\lambda)q^2}{2D} + (C_t + C_p\lambda + h_{v_2}\lambda)\, q\right.$$
$$\left. + h_{v_1} m_2(1-\lambda)^2 q^2\left[\frac{1}{P} + \frac{(m_2-1)}{2D} - \frac{m_2}{2P}\right]\right\}. \tag{7}$$

4. Theoretical results

The objective of this study is to determine the optimal batch quantity, q^*, the proportion of defective items, λ^*, and the number of shipment per production cycle, m^*, to minimise the joint total cost per unit time.

In order to solve this problem, we consider the following two cases: (i) $\lambda_L < \lambda \leq \lambda_U$ and (ii) $0 < \lambda \leq \lambda_L$.

Case 1. $\lambda_L < \lambda \leqslant \lambda_U$

Firstly, for fixed q and $\lambda \in (\lambda_L,\, \lambda_U]$, checking the effect of m_1 on the joint total cost per unit time $JTC_1(\lambda,\, q,\, m_1)$ in Equation (6). Taking second-order derivative of $JTC_1(\lambda,\, q,\, m_1)$ with respect to m_1, it gets

$$\frac{d^2 JTC_1(\lambda, q, m_1)}{dm_1^2} = \frac{2DS}{(1-\lambda)q m_1^3} > 0.$$

Hence, $JTC_1(\lambda, q, m_1)$ is a convex function of m_1. Consequently, the search for the optimal number of shipments m_1 (denoted by m_1^*) is reduced to find a local minimum.

Next, for a given m_1, the necessary conditions for the joint total cost per unit time $JTC_1(\lambda, q, m_1)$ to be minimised are $\partial JTC_1(\lambda, q, m_1)/\partial q = 0$ and $\partial JTC_1(\lambda, q, m_1)/\partial \lambda = 0$, simultaneously. These imply

$$h_{b1}q^2\left[\frac{(1-\lambda)^2}{2D}+\frac{\lambda}{2x}\right]+h_{b_2}\lambda q^2\left[\frac{1-\lambda}{D}-\frac{1}{2x}\right]+h_{v_1}q^2\left[\frac{1}{P}+\frac{(m_1-1)(1-\lambda)}{2D}-\frac{m_1}{2P}\right]-\frac{m_1A+S+m_1C_T}{m_1}=0, \tag{8}$$

and

$$\frac{m_1A+S+m_1C_T}{m_1}+(C_t+C_s+h_{v_2})q+\frac{(2h_{b_2}-h_{b_1})(1-\lambda)^2q^2}{2D}+\frac{(h_{b_1}-h_{b_2})q^2}{2x}-\frac{\theta\,(1-\lambda)^2q}{\delta D\lambda}+\frac{(2-m_1)h_{v_1}q^2}{2P}=0. \tag{9}$$

It is not easy to find the closed-form solutions of q and λ from Equations (8) and (9) for a given m_1. Besides, due to the high-power expression of the polynomial function, the convexity property of the total cost per unit time cannot be proved by using the Hessian matrix. Instead, we solve the problem by using the following search procedure:

For any given m_1 and $\lambda \in (\lambda_L, \lambda_U]$, the necessary condition for the joint total cost per unit time $JTC_1(\lambda, q, m_1)$ to be minimised is $dJTC_1(\lambda, q, m_1)/dq = 0$, which implies:

$$\frac{h_{b1}q^2(1-\lambda)^2}{2D}+\frac{h_{b_2}\lambda\,(1-\lambda)q^2}{D}+\frac{(h_{b1}-h_{b_2})\lambda q^2}{2x}+h_{v_1}q^2\left[\frac{1}{P}+\frac{(m_1-1)(1-\lambda)}{2D}-\frac{m_1}{2P}\right]=\frac{m_1A+S+m_1C_T}{m_1}. \tag{10}$$

Next, we take the second-order derivative of $JTC_1(\lambda, q, m_1)$ with respect to q and obtain

$$\frac{d^2JTC_1(\lambda,\, q\,,\, m_1)}{dq^2}=\frac{2D(m_1A+S+m_1C_T)}{m_1(1-\lambda)q^3}>0.$$

Consequently, for any given m_1, and $\lambda \in (\lambda_L, \lambda_U]$, $JTC_1(\lambda, q, m_1)$ is a convex function of q. Thus, there exists a unique value of q (say $q_{m_1,\lambda}$) which minimises $JTC_1(\lambda, q, m_1)$ as:

$$q_{m_1,\lambda}=\sqrt{\left(\frac{m_1A+S+m_1C_T}{m_1}\right)\left\{\frac{h_{b1}(1-\lambda)^2}{2D}+\frac{h_{b_2}\lambda\,(1-\lambda)}{D}+\frac{(h_{b1}-h_{b_2})\lambda}{2x}+h_{v_1}\left[\frac{1}{P}+\frac{(m_1-1)(1-\lambda)}{2D}-\frac{m_1}{2P}\right]\right\}^{-1}}. \tag{11}$$

Case 2. $0 < \lambda \le \lambda_L$

Similarly, for fixed q and $\lambda \in (0, \lambda_L]$, $JTC_2(\lambda, q, m_2)$ in Equation (7) can also be proved to be a convex function of m_2. Consequently, the search for the optimal shipment number m_2 (denoted by m_2^*) is also reduced to find a local minimum.

Next, for any given m_2 and $\lambda \in (0, \lambda_L]$, by using a similar approach as above, it can be easily found that $d^2JTC_2(\lambda, q\,, m_2)/d\,q^2 > 0$, and hence the optimal solution of q (denoted by $q_{m_2,\lambda}$) that minimises the joint total cost per unit time $JTC_2(\lambda, q, m_2)$ can be obtained by solving the equation $dJTC_2(\lambda, q\,, m_2)/dq = 0$ as follows:

$$q_{m_2,\lambda}=\sqrt{\left(\frac{m_2A+S+m_2C_T}{m_2}\right)\left\{\frac{(h_{b1}+h_{b_2}\lambda)}{2D}+h_{v_1}\left[\frac{1}{P}+\frac{(m_2-1)}{2D}-\frac{m_2}{2P}\right]\right\}^{-1}}. \tag{12}$$

Finally, for any given m_1 or m_2 and q, since both $JTC_1(\lambda, q, m_1)$ and $JTC_2(\lambda, q, m_2)$ are smooth curves of $\lambda \in (\lambda_L, \lambda_U]$ and $\lambda \in (0, \lambda_L]$, respectively, we can develop the following iterative algorithm to find the optimal solution (λ^*, q^*, m^*) for the whole problem.

Algorithm

Step 1: For given λ_L and λ_U,

Step 1–1. Set $m_1 = 1$.

Step 1–2. Divide the interval $(\lambda_L, \lambda_U]$ into n equal subintervals and let $\lambda_i = \lambda_L + i\,(\lambda_U - \lambda_L)/n$, $i = 1, 2, ..., n$, where n is large enough.

Step 1–3. For each λ_i, $i = 1, 2, ..., n$, we find q_{m_1,λ_i} from Equation (11), and then calculate the corresponding joint total cost per unit time $JTC_1(\lambda_i, q_{m_1,\lambda_i}, m_1)$ from Equation (6).

Step 1–4. Find $\underset{i=1,2,...,n}{Min} JTC_1(\lambda_i, q_{m_1,\lambda_i}, m_1)$ and let $JTC_1(\lambda_{(m_1)}, q_{m_1,\lambda_{(m_1)}}, m_1) = \underset{i=1,2,...,n}{Min} JTC_1(\lambda_i, q_{m_1,\lambda_i}, m_1)$.

Step 1–5. Set $m_1 = m_1 + 1$, and repeat Steps 1–2 to 1–4 to get $JTC_1(\lambda_{(m_1)}, q_{m_1,\lambda_{(m_1)}}, m_1)$.

Step 1–6. If $JTC_1(\lambda_{(m_1)}, q_{m_1,\lambda_{(m_1)}}, m_1) > JTC_1(\lambda_{(m_1-1)}, q_{m_1-1,\lambda_{(m_1-1)}}, m_1-1)$, then $JTC_1(\lambda_1^*, q_1^*, m_1^*) = JTC_1(\lambda_{(m_1-1)}, q_{m_1-1,\lambda_{(m_1-1)}}, m_1-1)$, and hence $(\lambda_1^*, q_1^*, m_1^*) = (\lambda_{(m_1-1)}, q_{m_1-1,\lambda_{(m_1-1)}}, m_1-1)$ is the optimal solution for Case 1. Otherwise, return to Step 1–5.

Step 2: For given λ_L,

Step 2–1. Set $m_2 = 1$.

Step 2–2. Divide the interval $(0, \lambda_L]$ into n equal subintervals and let $\lambda_j = j\,\lambda_L/n$, $j = 1, 2, ..., n$, where n is large enough.

Step 2–3. For each λ_j, $j = 1, 2, ..., n$, we find q_{m_2,λ_j} from Equation (12), and then calculate the corresponding joint total cost per unit time $JTC_2(\lambda_j, q_{m_2,\lambda_j}, m_2)$ from Equation (7).

Step 2–4. Find $\underset{j=1,2,...,n}{Min} JTC_2(\lambda_j, q_{m_2,\lambda_j}, m_2)$ and let $JTC_2(\lambda_{(m_2)}, q_{m_2,\lambda_{(m_2)}}, m_2) = \underset{j=1,2,...,n}{Min} JTC_2(\lambda_j, q_{m_2,\lambda_j}, m_2)$.

Step 2–5. Set $m_2 = m_2 + 1$, and repeat Steps 2–2 to 2–4 to get $JTC_2(\lambda_{(m_2)}, q_{m_2,\lambda_{(m_2)}}, m_2)$.

Step 2–6. If $JTC_2(\lambda_{(m_2)}, q_{m_2,\lambda_{(m_2)}}, m_2) > JTC_2(\lambda_{(m_2-1)}, q_{m_2-1,\lambda_{(m_2-1)}}, m_2-1)$, then $JTC_2(\lambda_2^*, q_2^*, m_2^*) = JTC_2(\lambda_{(m_2-1)}, q_{m_2-1,\lambda_{(m_2-1)}}, m_2-1)$, and hence $(\lambda_2^*, q_2^*, m_2^*) = (\lambda_{(m_2-1)}, q_{m_2-1,\lambda_{(m_2-1)}}, m_2-1)$ is the optimal solution for Case 2. Otherwise, return to Step 2–5.

Step 3: Find $\underset{k=1,2}{Min} JTC_k(\lambda_k^*, q_k^*, m_k^*)$. Let $JTC(\lambda^*, q^*, m^*) = \underset{k=1,2}{Min} JTC_k(\lambda_k^*, q_k^*, m_k^*)$, and then (λ^*, q^*, m^*) is the optimal solution.

The above algorithm can be implemented with the help of a computer-oriented numerical technique for a given set of parameter values. Once the optimal solution (λ^*, q^*, m^*) is obtained, we can get the joint capital investment $I(\lambda^*) = (1/\delta)\ln[\lambda_U/\lambda^*]$ and $Q^* = (1-\lambda^*)q^*$ or q^* according to the value of λ^* that belongs to the interval $(\lambda_L, \lambda_U]$ or $(0, \lambda_L]$. Furthermore, we have $T^* = Q^*/D$ and $JTC^* = JTC(\lambda^*, q^*, m^*)$.

5. Numerical examples

Example 1: The theoretical results and algorithm presented above can be illustrated by using the following numerical example. Let's consider an inventory system with the following data: $A = 50$, $P = 2000$, $D = 1000$, $S = 200$, $h_{b_1} = 2$, $h_{b_2} = 0.5$, $h_{v_1} = 1.5$, $h_{v_2} = 0.5$, $x = 3000$, $C_s = 0.3$, $C_T = 10$, $C_t = 0.3$, $C_p = 10$, $\theta = 0.01$, $\delta = 0.0005$, $\alpha = 0.5$, $\lambda_U = 0.05$ and $\lambda_L = 0.005$ in appropriate units. In addition, we set $n = 500$. Using the above algorithm, we obtain the computational results as shown in Table 1.

Table 1 reveals that the optimal number of shipment per production cycle is $m^* = 3$, the batch quantity per shipment is $q^* = 244.093$ units and the proportion of defective items is $\lambda^* = 0.00188 < 0.005 = \lambda_L$, which implies the buyer makes no inspection on the received items. In this situation, the buyer's optimal order quantity $Q^* = q^* = 244.093$ units and the optimal joint total cost per unit time JTC^* = \$1423.210. To see the effects of capital investment, we also

Table 1. Results of using the algorithm for Example 1.

m_1	$\lambda_{(m_1)}$	$q_{m_1,\lambda(m_1)}$	$JTC_1(\lambda_{(m_1)}, q_{m_1,\lambda_{(m_1)}}, m_1)$	m_2	$\lambda_{(m_2)}$	$q_{m_2,\lambda(m_2)}$	$JTC_2(\lambda_{(m_2)}, q_{m_2,\lambda(m_2)}, m_2)$
1	0.01112	437.051	1845.608	1	0.00187	434.698	1581.589
2	0.01256	304.558	1705.688	2	0.00188	302.290	1443.940
3	**0.01328**	**246.267**	**1683.857**	**3**	**0.00188**	**244.093**	**1423.210**
4	0.01382	211.817	1694.322	4	0.00189	209.722	1434.361

Note: Boldface type expresses the optimal solution of Case 1 and 2, respectively, in Example 1.

Table 2. Optimal solutions under different values of θ, C_s and C_p.

θ	C_s	C_p	m^*	λ^*	q^*	Q^*	JTC^*	Whether to invest	Whether to inspect
0.01	0.1	10	3	0.00188	244.093	244.093	$JTC_2(\lambda_2^*, q_2^*, m_2^*) = 1423.210$	Yes	No
		30	3	0.00065	244.129	244.129	$JTC_2(\lambda_2^*, q_2^*, m_2^*) = 1444.386$	Yes	No
		50	3	0.00039	244.136	244.136	$JTC_2(\lambda_2^*, q_2^*, m_2^*) = 1454.441$	Yes	No
	0.3	10	3	0.00188	244.093	244.093	$JTC_2(\lambda_2^*, q_2^*, m_2^*) = 1423.210$	Yes	No
		30	3	0.00065	244.129	244.129	$JTC_2(\lambda_2^*, q_2^*, m_2^*) = 1444.386$	Yes	No
		50	3	0.00039	244.136	244.136	$JTC_2(\lambda_2^*, q_2^*, m_2^*) = 1454.441$	Yes	No
	0.5	10	3	0.00188	244.093	244.093	$JTC_2(\lambda_2^*, q_2^*, m_2^*) = 1423.210$	Yes	No
		30	3	0.00065	244.129	244.129	$JTC_2(\lambda_2^*, q_2^*, m_2^*) = 1444.386$	Yes	No
		50	3	0.00039	244.136	244.136	$JTC_2(\lambda_2^*, q_2^*, m_2^*) = 1454.441$	Yes	No
0.03	0.1	10	3	0.04361	251.283	240.325	$JTC_1(\lambda_1^*, q_1^*, m_1^*) = 1503.37$	Yes	Yes
		30	3	0.04361	251.283	240.325	$JTC_1(\lambda_1^*, q_1^*, m_1^*) = 1503.37$	Yes	Yes
		50	3	0.04361	251.283	240.325	$JTC_1(\lambda_1^*, q_1^*, m_1^*) = 1503.37$	Yes	Yes
	0.3	10	3	0.00500	244.004	244.004	$JTC_2(\lambda_2^*, q_2^*, m_2^*) = 1528.891$	Yes	No
		30	3	0.00196	244.091	244.091	$JTC_2(\lambda_2^*, q_2^*, m_2^*) = 1591.989$	Yes	No
		50	3	0.00119	244.113	244.113	$JTC_2(\lambda_2^*, q_2^*, m_2^*) = 1622.150$	Yes	No
	0.5	10	3	0.00500	244.004	244.004	$JTC_2(\lambda_2^*, q_2^*, m_2^*) = 1528.891$	Yes	No
		30	3	0.00196	244.091	244.091	$JTC_2(\lambda_2^*, q_2^*, m_2^*) = 1591.989$	Yes	No
		50	3	0.00119	244.113	244.113	$JTC_2(\lambda_2^*, q_2^*, m_2^*) = 1622.150$	Yes	No
0.05	0.1	10	3	0.05000	252.372	239.753	$JTC_1(\lambda_1^*, q_1^*, m_1^*) = 1504.010$	No	Yes
		30	3	0.05000	252.372	239.753	$JTC_1(\lambda_1^*, q_1^*, m_1^*) = 1504.010$	No	Yes
		50	3	0.05000	252.372	239.753	$JTC_1(\lambda_1^*, q_1^*, m_1^*) = 1504.010$	No	Yes
	0.3	10	3	0.00500	244.004	244.004	$JTC_2(\lambda_2^*, q_2^*, m_2^*) = 1620.994$	Yes	No
		30	3	0.05000	252.372	239.753	$JTC_1(\lambda_1^*, q_1^*, m_1^*) = 1717.536$	No	Yes
		50	3	0.05000	252.372	239.753	$JTC_1(\lambda_1^*, q_1^*, m_1^*) = 1717.536$	No	Yes
	0.5	10	3	0.00500	244.004	244.004	$JTC_2(\lambda_2^*, q_2^*, m_2^*) = 1620.994$	Yes	No
		30	3	0.00327	244.053	244.053	$JTC_2(\lambda_2^*, q_2^*, m_2^*) = 1710.483$	Yes	No
		50	3	0.00198	244.090	244.090	$JTC_2(\lambda_2^*, q_2^*, m_2^*) = 1760.750$	Yes	No

show the optimal joint total cost per unit time without capital investment $JTC(\lambda_U, q^*, m^*)$ = \$ 1714.536. By comparing these results, we can obtain that it is benefit for the buyer and vendor to jointly invest capital in quality improvements of the product.

Example 2: From Equations (6) and (7), it is obvious that the change of value of C_s (i.e., the buyer's inspection cost per unit) only affects $JTC_1(\lambda, q, m)$ while the change of value of C_p (i.e. the buyer's penalty cost per defective item) only affects $JTC_2(\lambda, q, m)$. In this example, we discuss the influences of changes in these two cost parameters C_s and C_p on m^*, λ^*, q^*, Q^* and JTC^* of Example 1. We take the values of C_s and C_p as $C_s \in \{0.1, 0.3, 0.5\}$ and $C_p \in \{10, 30, 50\}$, respectively. In addition, we consider $\theta \in \{0.01, 0.03, 0.05\}$. Using the proposed algorithm above, we obtain the computational results for different values of θ, C_s and C_p as shown in Table 2.

From Table 2, we find that the optimal order, investment and inspection strategies are determined by trading off the opportunity cost of the capital investment and examining cost against the penalty cost (external failure cost). When the opportunity cost of the capital investment is low enough (for example, $\theta = 0.01$ in Table 2), the optimal investment police is to invest capacity jointly to reduce the defective rate less than $\lambda_L(= 0.005)$ whether the buyer's inspection cost per unit and penalty cost per defective item are large or not. In this case, the buyer's optimal inspection policy is not to inspect the received items. When the opportunity cost of the capital investment is high (for example, $\theta = 0.05$ in Table 2), if the buyer's inspection cost per unit is high enough (for example, $C_s = 0.5$ in Table 2), the joint capital investment in improving production process quality still is considered by the buyer and vendor, and the buyer's optimal inspection policy is not to inspect the received items. Furthermore, if the buyer's inspection cost per unit is low (for example, $C_s = 0.1$ in Table 2), the capital investment in improving production process quality is usually not considered by the buyer nor vendor and the buyer should inspect all received items. On the other hand, if the buyer's inspection

Table 3. Compare buyer's and vendor's optimal total costs under different values of α with no capital investment

α	$TC_b(\lambda^*, q^*)$	$TC_v(\lambda^*, m^*)$	$JTC = C(\lambda^*, q^*, m^*)$
0.00	467.962	955.248	1423.210
0.25	484.366	938.844	1423.210
0.50	500.770	922.440	1423.210
0.75	517.174	906.036	1423.210
1.00	533.577	889.633	1423.210
No investment	773.720	940.816	1714.536

cost per unit is medium and the penalty cost per defective item is not low (for example, $C_s = 0.3$ and $C_p \geqslant 30$ in Table 2), the capital investment in improving procedure process quality is not considered by the buyer nor vendor and the buyer should inspect all received items. Finally, for a medium opportunity cost of the capital investment (for example, $\theta = 0.03$ in Table 2), the optimal investment policy is to invest capacity jointly, and the buyer's optimal policy is to inspect the received items as the buyer's inspection cost per unit is low (for example, $C_s = 0.1$ in Table 2). However, when the buyer's inspection cost is not low (for example, $C_s \geqslant 0.3$ in Table 2), the buyer does not inspect the received items.

Example 3: In order to understand the impact of the proportion of capital investment on the buyer's and vendor's total costs, we now compare the optimal total cost per unit time of the buyer with that of the vendor. The data is the same as Example 1 except $\theta = 0.01$ and $\alpha \in \{0, 0.25, 0.5, 0.75, 1\}$. Using the proposed algorithm above, we obtain the computational results for different values of α as shown in Table 3. In addition, for comparison, we also list the optimal solution without capital investment in Table 3.

From Table 3, we find that the buyer's optimal total cost per unit time increases while the vendor's optimal total cost per unit time decreases as the value of α increases. Note that when $\alpha = 1$ (i.e. capital investment in improving production process quality is completely paid by the buyer), the vendor has a minimum total cost per unit time $TC_v(\lambda^*, m^*) =$ \$889.633. On the contrary, when $\alpha = 0$ (i.e. capital investment in improving production process quality is completely paid by the vendor), the buyer has a minimum total cost per unit time $TC_b(\lambda^*, q^*) =$ \$467.962. Furthermore, we find that for the value of $\alpha \in \{0, 0.25, 0.5, 0.75, 1\}$, the buyer's or joint total cost with investment is less than that without investment, while the vendor's total cost with investment is greater than that without investment as $\alpha = 0$. That is, it is unfavourable to the integrated inventory system if the investment expense in reducing the defective rate of items is completely paid by the vendor. Consequently, even though the optimal joint total cost per unit time for the integrated model is not affected by the value of α, it still play a significant role in an integrated production and inventory model with defective items, and is a critical coordination factor to accomplish global optimisation in the field of inventory management. It is why our paper thinks that the buyer and vendor should agree to jointly invest capital to improve the production processes.

6. Conclusions

This study develops an integrated production-inventory model with defective items. The defective rate of the product can be improved through joint capital investment from the vendor and buyer. We then provide an algorithm to find the optimal solution procedure. Furthermore, several numerical examples are presented to demonstrate our model and solution. Numerical examples reveal that (1) the optimal order, investment and inspection policies are determined by trading off the opportunity cost of the capital investment and inspecting cost against the penalty cost, and (2) the proportion of vendor's/buyer's capital investment in quality improvement plays a significant role in an integrated production-inventory model with imperfect production processes, and is a critical coordination factor to accomplish the target of global optimisation in the field of inventory management. We believe that our work will provide a basic foundation for further study of this kind of integrated inventory model with defective items.

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