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Optimal trade credit and lot size policies in economic production quantity models with learning curve production costs

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ABSTRACT

In reality, a seller (e.g., a supplier or a manufacturer) frequently offers his/her buyers trade credit (e.g., permissible delay in payment). Trade credit reduces the buyer's holding cost of inventory and hence attracts new buyers who consider it to be a type of price reduction. On the other hand, granting trade credit also increases the seller's opportunity cost (i.e., the loss of capital opportunity during the credit period) and default risk (i.e., the event in which the buyer will be unable to make the required payments on his/her debt obligation). In addition, it is a well-known fact of learning-by-doing that production cost of a new product declines by a factor of from 10 to 50 percent each time the accumulated production volume doubles. Therefore, we propose an economic production quantity model from the seller's prospective to determine his/her optimal trade credit period and production lot size simultaneously in which (i) trade credit increases not only sales but also opportunity cost and default risk, and (ii) production cost declines and obeys a learning curve phenomenon. Then the necessary and sufficient conditions to obtain the seller's optimal trade credit and order quantity are derived. Finally, we use some numerical examples to illustrate the theoretical results and to provide some managerial insights.

1. Introduction

In 1913, Harris proposed the classical economic order quantity (thereafter, EOQ) model by assuming that a buyer must pay for the items as soon as receiving them (Harris, 1913). In practice, a seller frequently offers his/her buyers a permissible delay period (i.e., credit period) for settling the amount owed to him/her. Usually, there is no interest charge to the buyer if the outstanding amount is paid within the permissible delay period. However, if the payment is not paid in full by the end of the permissible delay period, then the seller charge the buyer interest on the outstanding amount. Granting a permissible delay period attracts new buyers who may consider it to be a type of price reduction. Hence, from a seller's prospective, granting a permissible delay period increases sales. On the other hand, granting a permissible delay period increases not only the seller's opportunity cost but also the seller's default risk because the longer the permissible delay period, the higher the opportunity cost as well as the default risk. Consequently, it is an important and relevant issue for the seller to find an optimal trade credit such that the sales increase induced by trade credit can significantly overcome the cost increase of opportunity cost and default risk.

Goyal (1985) developed an EOQ model for the buyer when the seller offers a fixed permissible delay period. Shah (1993) considered a stochastic inventory model when delays in payments are permissible. Aggarwal and Jaggi (1995) then extended Goyal's model to consider the deteriorating items. Jamal et al. (1997) further generalized Aggarwal and Jaggi's model to allow for shortages. Hwang and Shinn (1997) added the pricing strategy to the model, and developed the optimal price and lot sizing for a retailer under the condition of permissible delay in payments. Teng (2002) amended Goyal's model by calculating interest earned based on sales revenue instead of purchase cost, and proved that it makes economic sense for a well-established buyer to order less quantity and take the benefits of the permissible delay more frequently. Huang (2003) extended Goyal's model to develop an EOQ model in which the supplier offers the retailer the permissible delay period M, and the retailer in turn provides the trade credit period N (with $N \leq M$) to his/her customers. Teng and Goyal (2007) complemented the shortcoming of Huang's model and proposed a generalized formulation. Teng (2009a) established an EOQ model for a retailer who offers distinct trade credits to its good and bad credit customers. Chang et al. (2010) presented the optimal manufacturer's replenishment policies in a supply chain with up-stream and down-stream trade credits. Hu and Liu (2010) investigated the optimal replenishment policy for the economic

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production quantity (thereafter, EPQ) model with permissible delay in payments and allowable shortages. Teng et al. (2011) obtained the retailer's optimal ordering policy when the supplier offers a progressive permissible delay in payments. Min et al. (2012) developed an EPQ model for deteriorating items with stock-dependent demand and permissible delay in payments. Teng et al. (2012) extended an EOQ model with trade credit financing from constant demand to non-decreasing demand. Recently, Sarkar (2012) established an EOQ model with permissible delay in payments and time varying deterioration rate. Many related articles can be seen in Chang et al. (2003, 2010), Chen et al. (in press-a, in press-b), Cheng et al. (2012), Goyal et al. (2007), Huang and Hsu (2008), Lou and Wang (2013), Min et al. (2010), Ouyang et al. (2005, 2006), Shinn and Hwang (2003), and their references. All inventory models described above are studied only from the perspective of the buyer whereas in practice the length of trade credit period is set by the seller. So far, how to determine the optimal length of trade credit period for the seller has received relatively little attention by the researchers except Abad and Jaggi (2003), Chern et al. (2013), Kim et al. (1995), Wang et al. (2014), Zhou et al. (2012), and others.

Arrow (1962), Hirschmann (1964), Rosen (1972), and The Boston Consulting Group (1972) observed that the total unit cost to produce a new product declines by a factor of from 10 to 50 percent each time the accumulated production volume doubles, due to learning by doing. In other words, when cost vs. production is plotted on a log–log scale, the graph is approximately a straight line with negative slope $-l$, where $0.1 \le l \le 0.5$. As noted the learning coefficient l in this learning-by-doing phenomenon can be estimated by plotting cost vs. production on a log–log scale. Many researchers have applied this learning-by-doing phenomenon into production-marketing model to obtain optimal pricing, advertising, quality, and other strategies, such as Teng and Thompson (1983, 1996), Thompson and Teng (1984), Tsai (2012), and others.

In this paper, we derive the seller's optimal trade credit and lot size policies in an EPQ model in which (1) the length of trade credit period increases not only demand rate (i.e., the longer the trade credit period, the higher the demand rate) but also the opportunity cost and the default risk (i.e., the longer the trade credit period, the higher the opportunity cost and the default risk), and (2) the production cost declines and obeys a learning curve phenomenon (i.e., the total unit production cost declines by a factor of 10 to 50 percent each time the accumulative production volume doubles). Then we establish the necessary and sufficient conditions for finding the optimal solution, characterize the impact of various parameters on the optimal solution, and provide some managerial insights. Due to the complexity of the problem, we are unable to obtain a closed-form solution to the seller's optimal credit period. Consequently, we propose an algorithm to obtain the seller's optimal trade credit. Finally, some numerical examples are provided to illustrate the theoretical results and obtain some managerial insights.

2. Notation and assumptions

The following notation and assumptions are used in the entire paper.

2.1. Notation

M	the seller's trade credit period to his/her buyers in years (decision variable)
Q	the seller's production lot size in units (decision variable)
o	the average ordering cost per order (or set-up cost per production run) in dollars
c_0	the learning curve production cost for making the first unit in dollars
s	the selling price per unit in dollars (with $s > c_0$)
h	the average stock holding cost per unit per year in dollars
r	the seller's annual compounded interest rate on opportunity cost
t	the time in years
$D(M)$	the annual demand rate in units as a function of the trade credit period M
P	the annual production rate in units (with $P > D(M)$)
$\Pi(M, Q)$	the seller's profit function per year in dollars
M^*	the seller's optimal trade credit period in years
Q^*	the seller's optimal production lot size in units
Π^*	the seller's optimal profit per year in dollars.

2.2. Assumptions:

Next, the following assumptions are made to establish the mathematical inventory model.

1. It is a well-known learning-by-doing phenomenon (e.g., see Arrow (1962), and Hirschmann (1964)) that the total unit production cost declines by a factor of from 10 to 50 percent each time the accumulative production volume doubles especially during the introduction phase of a new product. Mathematically, this is equivalent to the assertion that

$$c(t) = c(0)\left(\frac{X(0)}{X(t)}\right)^{l},$$

where $c(t)$ is the unit cost of production at time t, $X(t)$ is the accumulated production volume at time t, and l is the learning coefficient which usually falls in the range of $0.1 \le l \le 0.5$. For simplicity, we may assume that the learning curve production cost for making X units is as follows:

$$c(t)X(t) = c(0)\left[\frac{X(0)}{X(t)}\right]^{l} X(t) = c_0 X(t)^{1-l} = c_0[X(t)]^{u}, \tag{1}$$

where c_0, l, and u are positive constants, $0.1 \le l \le 0.5$, and hence $0.5 \le u \equiv 1 - l \le 0.9$. Note that if $u = 1$ then the total unit production cost is constant and there is no learning curve phenomenon.

2. In practice, there are three simple ways to represent an increasing demand of the credit period M: linear, polynomial, or exponential. For simplicity, we assume that the demand rate $D(M)$ is a positive exponential function of the credit period M as

$$D(M) = Ke^{aM}, \tag{2}$$

where K and a are positive constants. For convenience, $D(M)$ and D will be used interchangeably.

3. Granting a longer credit period to the buyer induces a higher default risk to the seller. For example, the default risk of a 30-year mortgage is higher than that of a 15-year mortgage. In practice, there are three simple ways to represent an increasing of default risk with respect to the credit period M: linear, polynomial, or exponential. For simplicity, we may assume that the rate of default risk giving the credit period M is assumed here to be

$$F(M) = 1 - e^{-bM}, \tag{3}$$

where b is the coefficient of the default risk, which is a positive constant.

4. The seller offers the buyer a trade credit period of M. Since the seller's annual compounded interest rate is r, the future value

of \$1.00 received by the seller at time M is equivalent to the present value of \$$e^{-rM}$ received at time 0. Hence, the seller's net revenue, the present value, which received after default risk and opportunity cost is:

$$sD(M)[1-F(M)]e^{-rM} = sKe^{[a-(b+r)]M}. \tag{4}$$

5. Shortages are not allowed to occur.

3. Mathematical model and optimal solution

Based on the above assumptions, the inventory system considered here is as follows. The seller must decide his/her trade credit period M and production lot size Q of a single product simultaneously in order to maximize his/her profit per year. From the above assumptions and arguments, the annual profit can be expressed as

$\Pi(M, Q)$=net revenue after default risk and opportunity cost – learning production cost – set-up cost – holding cost

$$= sD(M)[1-F(M)]e^{-rM} - c_0[D(M)]^u - \frac{D(M)}{Q}o - \frac{Q}{2}\left[1-\frac{D(M)}{P}\right]h$$

$$= sKe^{[a-(b+r)]M} - c_0(Ke^{aM})^u - \frac{Ke^{aM}}{Q}o - \frac{Q}{2}\left(1-\frac{Ke^{aM}}{P}\right)h. \tag{5}$$

Then we discuss the seller's optimal solution to production lot size first, and then trade credit period next.

3.1. Optimal production lot size

To maximize the annual profit $\Pi(M, Q)$ with respect to Q is equivalent to minimize the annual total cost of the set-up cost and the holding cost, which is

$$TC(Q) = \frac{D(M)}{Q}o + \frac{Q}{2}\left[1-\frac{D(M)}{P}\right]h. \tag{6}$$

For simplicity, we apply an arithmetic–geometric inequality method (e.g., see Cárdenas-Barrón (2011) and Teng (2009b)) to obtain the optimal solution for (6). As we know, the arithmetic mean is always greater than or equal to the geometric mean. In short, for any two real positive numbers, say a and b, we have

$$\frac{a+b}{2} \geq \sqrt{ab} \tag{7}$$

the equation holds only if $a=b$. By applying (7), setting

$$\frac{D(M)}{Q}o = \frac{Q}{2}\left[1-\frac{D(M)}{P}\right]h > 0, \tag{8}$$

we know that the seller's optimal production lot size is

$$Q^* = \sqrt{\frac{2oD(M)}{h[1-D(M)/P]}}, \tag{9}$$

and the minimum total cost of the set-up cost and the holding cost is

$$TC(Q^*) = \sqrt{2ohD(M)\left[1-\frac{D(M)}{P}\right]} \tag{10}$$

consequently, the seller's inventory problem is reduced to a single decision variable M

$$\Pi(M) = \Pi(M, Q^*) = sD(M)[1-F(M)]e^{-rM} - c_0[D(M)]^u - \sqrt{2ohD(M)\left[1-\frac{D(M)}{P}\right]}$$

$$= sKe^{[a-(b+r)]M} - c_0(Ke^{aM})^u - \sqrt{2ohKe^{aM}\left(1-\frac{Ke^{aM}}{P}\right)} \tag{11}$$

next, we try to obtain the optimal trade credit period for the seller.

3.2. Optimal trade credit period

In order to find the optimal solution M^* of $\Pi(M)$, we derive the first-order necessary condition for $\Pi(M)$ in (9) to be maximized as

$$\frac{d\Pi(M)}{dM} = [a-(b+r)]sKe^{[a-(b+r)]M} - uac_0(Ke^{aM})^u - aohKe^{aM}\left(1-\frac{2Ke^{aM}}{P}\right)\Big/\sqrt{2ohKe^{aM}\left(1-\frac{Ke^{aM}}{P}\right)} = 0. \tag{12}$$

from (12) we can obtain the following results:

Theorem 1. *The seller's optimal trade credit period is zero (i.e., $M^*=0$) if (1) $a \leq b+r$ and $P \geq 2D$, or (2) $[a-(b+r)]sKe^{[a-(b+r)]M} \leq uac_0(Ke^{aM})^u$ and $P \geq 2D$.*

Proof. If $a \leq b+r$ and $P \geq 2D$, then we know from (12) that $(d\Pi(M)/dM) \leq 0$. Consequently, the seller's optimal trade credit period is set to be zero. Likewise, we can easily prove $M^*=0$ if $[a-(b+r)]sKe^{[a-(b+r)]M} \leq uac_0(Ke^{aM})^u$ and $P \geq 2D$. This completes the proof.

A simple economical interpretation of Condition 1 is as follows. If $a \leq b+r$, then the higher the trade credit period, the lower the net revenue after default risk and opportunity cost. In this case, the seller should not offer a permissible delay in payments to the buyer. Similarly, a simple interpretation of Condition 2 is as follow. If the marginal net revenue increase (i.e., $[a-(b+r)]sKe^{[a-(b+r)]M}$) is less than or equal to the marginal production cost increase (i.e., $uac_0(Ke^{aM})^u$), then it is no worth of offering a trade credit period from the seller to the buyer. Notice that if $D < P < 2D$, then we are unable to prove Theorem 1 is still valid.

Now we discuss the other condition in which $[a-(b+r)]sKe^{[a-(b+r)]M} > uac_0(Ke^{aM})^u$. For simplicity, let us define

$$\Sigma(M) = uac_0(Ke^{aM})^u + aohKe^{aM}\left(1-\frac{2Ke^{aM}}{P}\right)\Big/\sqrt{2ohKe^{aM}\left(1-\frac{Ke^{aM}}{P}\right)}. \tag{13}$$

then we know from (12) that

$$[a-(b+r)]sKe^{[a-(b+r)]M} = \Sigma(M),$$

which implies that the seller's optimal trade credit period is

$$M^* = \frac{1}{a-(b+r)} \ln \frac{\Sigma(M)}{[a-(b+r)]sK} > 0. \tag{14}$$

Notice that the right-hand side of (14) is also a function of M. Hence, Eq. (14) is not a closed-form solution. Due to the complexity of the problem, it seems not to be tractable to find a closed-form solution to the seller's optimal trade credit period. Hence, we suggest the reader use a computer software (e.g., Maple or Mathematica) to obtain the seller's optimal trade credit period.

For the second-order sufficient condition, taking the derivative of (12) with respect to M, and re-arranging terms, we get

$$\frac{d^2\Pi(M)}{dM^2} = [a-(b+r)]^2 sKe^{[a-(b+r)]M} - (ua)^2 c_0(Ke^{aM})^u - (aohKe^{aM})^2\left[\left(\frac{2Ke^{aM}}{P}\right)^2 - \frac{6Ke^{aM}}{P} + 1\right] \times \left[2ohKe^{aM}\left(1-\frac{Ke^{aM}}{P}\right)\right]^{-3/2}. \tag{15}$$

consequently, if $[a-(b+r)]^2 sKe^{[a-(b+r)]M} \leq (ua)^2 c_0(Ke^{aM})^u$ and $((2Ke^{aM})/P)^2 - (6Ke^{aM}/P) + 1 > 0$, then we know that $(d^2\Pi(M)/dM^2) < 0$ in (15), and hence $\Pi(M)$ in (11) is a strictly concave function of M. From Eqs. (11)–(15), we can obtain the following theoretical results.

Theorem 2.

(1) If $[a-(b+r)]sK-uac_0(K)^u-aohK(1-(2K/P))/\sqrt{2ohK(1-(K/P))}>0$, $[a-(b+r)]^2sKe^{[a-(b+r)]M}\leq(ua)^2c_0(Ke^{aM})^u$, and $(2Ke^{aM}/P)^2-(6Ke^{aM}/P)+1>0$, then $\Pi(M)$ in (11) has a unique optimal solution $M^*>0$ as in (12).

(2) If $[a-(b+r)]sK-uac_0(K)^u-aohK(1-(2K/P))/\sqrt{2ohK(1-(K/P))}\leq 0$, $[a-(b+r)]^2sKe^{[a-(b+r)]M}\leq(ua)^2c_0(Ke^{aM})^u$, and $(2Ke^{aM}/P)^2-(6Ke^{aM}/P)+1>0$, then $\Pi(M)$ in (11) has a unique optimal solution $M^*=0$.

Proof. From (13), we know that

$$\frac{d^2\Pi(M)}{dM^2}<0 \text{ if } [a-(b+r)]^2sKe^{[a-(b+r)]M}\leq(ua)^2c_0(Ke^{a\,M})^u,$$

$$\text{and } \left(\frac{2Ke^{aM}}{P}\right)^2-\frac{6Ke^{aM}}{P}+1>0. \tag{16}$$

in addition, using the fact that

$$\lim_{M\to\infty}\frac{[a-(b+r)]sKe^{[a-(b+r)]M}}{e^{aM}}=0, \tag{17}$$

we get

$$\lim_{M\to\infty}\frac{d\Pi(M)}{dM}=-\infty. \tag{18}$$

Substituting $M=0$ into (12), we obtain

$$\frac{d\Pi(0)}{dM}=[a-(b+r)]sK-uac_0(K)^u-aohK\left(1-\frac{2K}{P}\right)\Big/\sqrt{2ohK\left(1-\frac{K}{P}\right)}. \tag{19}$$

Consequently, if $(d\Pi(0)/dM)>0$ then applying the Mean–Value Theorem we know that there exists a unique optimal trade credit period $M^*>0$ such that $(d\Pi(M^*)/dM)=0$. This proves Part (1) of the theorem.

However, if $(d\Pi(0)/dM)\leq 0$, then $(d\Pi(M)/dM)<0$ for all $M>0$, which implies that $\Pi(M)$ in (11) is a strictly decreasing function of M. Hence, if $(d\Pi(0)/dM)\leq 0$ then $M^*=0$ is the unique optimal solution to $\Pi(M)$ in (11). This completes the proof of Theorem 2.

Notice that the annual profit $\Pi(M, Q)$ has two decision variables M and Q. In fact, we need to prove the Hessian matrix with respect to the annual profit $\Pi(M, Q)$ at (M^*,Q^*) is negative definite. Hence, we prove the following result.

Theorem 3. *If $Y\equiv[a-(b+r)]^2sKe^{[a-(b+r)]M}-(ua)^2c_0(Ke^{aM})^u+(a^2hKe^{aM}Q/2P)\leq 0$, then there exists a unique optimal solution (M^*,Q^*) that maximizes $\Pi(M, Q)$ globally.*

Proof. Taking the second-order partial derivatives of $\Pi(M, Q)$ in (5) with respect to M and Q, we get:

$$\frac{\partial^2\Pi(M)}{\partial M^2}=[a-(b+r)]^2sKe^{[a-(b+r)]M}-(ua)^2c_0(Ke^{aM})^u$$

$$-\frac{a^2oKe^{aM}}{Q}+\frac{a^2hKe^{aM}Q}{2P}<0, \tag{20}$$

$$\frac{\partial^2\Pi(M,Q)}{\partial Q^2}=-\frac{2}{Q^3}oKe^{aM}<0, \tag{21}$$

$$\frac{\partial^2\Pi(M,Q)}{\partial Q\partial M}=\frac{a}{Q^2}oKe^{aM}, \tag{22}$$

and

$$\left[\frac{\partial^2\Pi(M,Q)}{\partial M^2}\right]\left[\frac{\partial^2\Pi(M,Q)}{\partial Q^2}\right]-\left[\frac{\partial^2\Pi(M,Q)}{\partial Q\partial M}\right]^2$$
$$=\left(\frac{a}{Q^2}oKe^{am}\right)^2-\left(\frac{2}{Q^3}oKe^{am}\right)Y>0. \tag{23}$$

hence, the Hessian matrix associated with $\Pi(M, Q)$ is negative definite. Applying Theorem 2, we know that the unique solution (M^*,Q^*) is the global maximum solution. This completes the proof.

4. Numerical examples

In this section, we provide a couple of numerical examples to illustrate the theoretical results above.

Example 1. Let $a=0.2$, $b=0.1$, $r=0.05$, $u=0.9$, $s=\$15$ per unit, $c_0=\$8$ for the first production unit, $o=\$5$ per order, $h=\$1$ per unit per year, $K=1000$ units per year, and $P=10{,}000$ units per year. We first check the condition

$$[a-(b+r)]sK-uac_0(K)^u-aohK\left(1-\frac{2K}{P}\right)\Big/\sqrt{2ohK\left(1-\frac{K}{P}\right)}=19.8576>0. \tag{24}$$

then we substitute the values of the parameters into (12), and use computer software such as Mathematica 9.0, Maple 16.0, and others to obtain a trade credit period $M=0.2086$.

Check the concavity conditions with $M=0.2086$

$$37.89319=[a-(b+r)]^2sKe^{[a-(b+r)]M}\leq(ua)^2c_0(Ke^{aM})^u=134.8785,$$

and

$$\left(\frac{2Ke^{aM}}{P}\right)^2-\frac{6Ke^{aM}}{P}+1=0.4179>0.$$

Applying Theorem 2, we know that the unique optimal trade credit period

$M^*=0.2086$.

Substituting $M^*=0.2086$ into (9), we get the optimal production lot size

$Q^*=107.8868$.

Substituting $M^*=0.2086$ and $Q^*=107.8868$ into (5), we obtain the optimal annual profit for the seller

$\Pi^*(M^*,Q^*)=\$10,897.7216$.

Example 2. Using the same data as those in Example 1, we study the sensitivity analysis on the optimal solution with respect to each parameter in appropriate unit. The computational results are shown in Table 1.

The sensitivity analysis reveals that: (1) a higher value of u, b, r, c_0, h, and P causes lower values of M^*, Q^* and $\Pi^*(M^*, Q^*)$, (2) in contrast, a higher value of a, s, and K causes higher values of M^*, Q^* and $\Pi^*(M^*, Q^*)$, and (3) a higher value of o causes lower values of M^*, and $\Pi^*(M^*, Q^*)$ while a higher value of Q^*. A simple economic interpretation of the first numerical result is as follow. The higher the learning coefficient l (i.e., the lower the value of u), the lower the production cost. Hence, the seller can afford to offer longer credit period M^* (i.e. more sales), and hence produce larger lot-size Q^*, and obtain higher annual total profit $\Pi^*(M^*, Q^*)$. By contrast, the higher the coefficient of the default risk b, the lower annual total profit $\Pi^*(M^*, Q^*)$. Thus the seller cannot offer longer credit period M^* so that the seller produces less lot-size Q^*. Similarly, the reader can interpret the rest of the first numerical conclusion. For the second numerical conclusion, a higher value of

Table 1
Sensitivity analysis on parameters.

Parameter	M^*	Q^*	$\Pi^*(M^*,Q^*)$
$u=0.90$	0.2086	107.8868	\$10,897.7216
$u=0.89$	0.8385	115.8097	\$11,195.9304
$u=0.88$	1.4899	124.7744	\$11,513.0131
$a=0.20$	0.2086	107.8868	\$10,897.7216
$a=0.21$	1.2569	122.3510	\$10,983.9181
$a=0.22$	2.1180	137.6815	\$11,183.0530
$b=0.100$	0.2086	107.8868	\$10,897.7216
$b=0.099$	0.3654	109.7956	\$10,902.0919
$b=0.098$	0.5216	111.7380	\$10,908.9005
$r=0.050$	0.2086	107.8868	\$10,897.7216
$r=0.049$	0.3654	109.7956	\$10,902.0919
$r=0.048$	0.5216	111.7380	\$10,908.9005
$s=15$	0.2086	107.8868	\$10,897.7216
$s=16$	0.7103	114.1409	\$11,921.1250
$s=17$	1.1815	120.4257	\$12,969.7044
$c_0=\$8$	0.2086	107.8868	\$10,897.7216
$c_0=\$7$	1.2355	121.1734	\$11,468.1005
$c_0=\$6$	2.4228	139.2158	\$12,164.1028
$h=1$	0.2086	107.8868	\$10,897.7216
$h=2$	0.1723	75.9791	\$10,857.7568
$h=4$	0.1206	53.4166	\$10,801.4575
$o=5$	0.2086	107.8868	\$10,897.7216
$o=6$	0.2002	118.0733	\$10,888.5013
$o=8$	0.1854	136.1144	\$10,872.1472
$K=1000$	0.2086	107.8868	\$10,897.7216
$K=1200$	0.3598	121.6741	\$13,180.1958
$K=1600$	0.5981	148.3389	\$17,789.6754
$P=10,000$	0.2086	107.8868	\$10,897.7216
$P=9000$	0.2106	108.6159	\$10,898.3487
$P=7000$	0.2165	110.7856	\$10,900.1658

trade credit elasticity on demand (i.e., a) causes a longer credit period M^* (i.e., more sales), a larger lot-size Q^*, and a higher annual total profit $\Pi^*(M^*, Q^*)$. Again, one can interpret the rest of the second numerical conclusion by using an analogous argument. Finally, for the last numerical conclusion, if the set-up cost (i.e., o) is higher, then the seller produces more lot-size Q^* to reduce the number of set-up reqired. Of course, the higher the set-up cost, the lower the credit period M^* as well as the annual total profit $\Pi^*(M^*, Q^*)$.

5. Conclusions

How to determine the optimal length of trade credit for the seller has received relatively little attention by the researchers. In this paper, we have originally developed an EPQ model to reflect the facts: (1) the total unit production cost declines by a factor of from 10 to 50 percent each time the accumulative production volume doubles due to learning by doing, (2) trade credit reduces the buyer's inventory holding cost and attracts new buyers, and (3) the longer the credit period, the higher the opportunity cost and default risk. Then we have derived the necessary and sufficient conditions to obtain the optimal solution. In addition, we have characterized the influence of the parameters to the optimal solution. For example, if the learning coefficient l is higher, then the production cost is lower, and the seller can afford to offer a longer credit period M^* (i.e. more sales), and hence produce larger lot-size Q^*, and obtain higher annual total profit $\Pi^*(M^*, Q^*)$. Finally, we have provided numerical examples and sensitivity analysis to illustrate the proposed model and understand managerial insights. Hence, we have made some innovational and significant contributions for a seller to determine his/her optimal trade credit and lot size simultaneously.

For further research, this paper can be extended in several ways. For instance, we may add the constant deterioration rate for the items. Also, we could generalize the model to allow for shortages. Finally, we could consider the effect of inflation rates on the economic order quantity.

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