Group Decis Negot (2016) 25:537–565
DOI 10.1007/s10726-015-9451-9

A Mixed-Data Evaluation in Group TOPSIS with Differentiated Decision Power

Hsu-Shih Shih[1]

Published online: 18 September 2015

Abstract This main objective of this paper is to provide decision support for mixed data in group Technique for Order Preference by Similarity to Idea Solution (TOPSIS) with differentiated decision power. We use a signum function to compare the ordinal performance of alternatives on any qualitative criterion, or the partial information provided by decision makers. The proposed process for ordinal information is uniformly coherent with the traditional TOPSIS steps, preserving the characteristic of distance-based utilities. Ordinal weights are also considered herein, and the decision power of the group members is formulated by their weights under an agreement in the group. Two examples demonstrate that the proposed approach has some benefits and achieves robustness with two types of sensitivity analyses. Some discussions and their limitations to the approach are also provided.

Keywords TOPSIS · Ordinal information · Decision power · Group decision making · Mixed-data

1 Introduction

Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is a rather straightforward technique in dealing with multi-criteria or multi-attribute decision making (MCDM/MADM) problems that have cardinal information (Hwang and Yoon 1981), and it is popularly practiced in the Asia-Pacific region (Shih et al. 2007). Behind the use of this technique, there exist many assumptions that are sometimes hard to

✉ Hsu-Shih Shih
hshih@mail.tku.edu.tw

1 Department of Management Sciences, Tamkang University, Tamsui, New Taipei 25137, Taiwan, ROC

meet. One case occurs when there are often insufficient metric data for an evaluation. If there exists ordinal information in making a decision, which is easier to obtain, then we only require partial information from decision makers (DMs). Ordinal information is argued to be more reliable and available than cardinal (continuous) information (Larichev 1992), but few works in the literature directly address ordinal or qualitative information in the TOPSIS process without considering fuzzy or interval values. This study deals with the problem of ordinal information that exists in group TOPSIS, which is a generalization of TOPSIS in a group decision making environment. The differentiated decision power in group TOPSIS is addressed as well. Hence, we expect a coherent and complete procedure for a mixed-data multiple-criteria group decision support with differentiated decision power.

The basic concept of TOPSIS is in a straight line. It originates from the concept of a displaced ideal point from which the compromised solution has the shortest distance (Belenson and Kapur 1973; Zeleny 1974). Hwang and Yoon (1981) recommended that the ranking of alternatives is based on the shortest distance from the (positive) ideal solution (PIS) and the farthest distance away from the negative ideal solution (NIS) or anti-ideal solution in an n-dimensional Euclidean space. TOPSIS simultaneously considers these distances to both PIS and NIS and then ranks a preference order on account of their relative closeness, which is a combination of these two distance measures. Although there are multiple versions, the core thinking of the technique is that the distance function represents the DMs' preferences or utilities.

Because MCDM is a practical tool for selecting and ranking a number of alternatives, its applications are numerous. TOPSIS is deemed as a major analytic technique and has been successfully applied in numerous areas (Shih et al. 2007). Behzadian et al. (2012) collected 266 papers since 2000 and inferred that there are nine major application areas of TOPSIS: (1) supply chain management and logistics, (2) design, engineering and manufacturing systems, (3) business and marketing management, (4) health, safety, and environment management, (5) human resource management, (6) energy management, (7) chemical engineering, (8) water resources management, and (9) other topics. Of these TOPSIS applications, most are combined with other techniques, e.g., fuzzy sets (Langroudi et al. 2013), interval information (Tsaur 2011), analytic hierarchy/network process (AHP/ANP) (Torfi et al. 2010), entropy methods (Hung and Chen 2010), and mathematical programming (Garcia et al. 2010).

Since group TOPSIS is a major interest in the literature, we have to examine TOPSIS used in a group decision environment. Most research works involve fuzzy or interval data. Yue (2013) recently organized 15 papers for preference aggregation in group TOPSIS. Eleven of them are related to the manipulation of fuzzy or interval things, and only a few deal with crisp data. Two debates have arisen if a paper involved with fuzzy or interval data has a technical advantage compared to traditional approaches or if it can better match the decision making environment. The former is out of the scope of this study, while for the latter some scholars have found clues on the issue of the decision making environment in the past. Eckenrode (1965) mentioned that ranking by order is not only the easiest, but also is the most reliable. Kirkwood and Sarin (1985) also supported the argument for the ease of reaching an agreement with some kind of confidence. This situation could commonly happen in a group with a small number of DMs. Hence, from a practical aspect, this study investigates an uncovered area, i.e., group TOPSIS with partial information. Decision power is also posted for a possible better formulation for group decision support. Both issues motivate the present research.

The paper is organized as follows. In the next section the literature survey lists some basic works on ordinal information for decision making and decision power in multi-criteria group decision making. Section 3 focuses on the proposed integrated model for group TOPSIS in a step-by-step fashion. We then give illustrative examples with sensitivity analyses. Section 4 provides some discussions with their limitations. The final section draws concluding remarks.

2 Literature Survey

Many works have utilized TOPSIS as a tool for decision making, but few have presented the essential steps for tackling ordinal or qualitative information in a crisp domain. Most studies treated this information as pseudo-metric data. Hwang and Yoon (1981) suggested that a qualitative criterion could be converted into an interval scale by using a bipolar scale. For instance, decision makers might choose a 10-point scale and calibrate the qualitative attribute into assigned values. For the benefit criteria, 10 points might be the maximum value and 0 points the minimum value for performance measures, with the reverse direction of values measuring the cost criteria. Traditional TOPSIS thus could deal with both qualitative and quantitative criteria. Goh et al. (1996) utilized a 1- to 9-point scale for the level of achievement of the qualitative attribute in a robot selection problem. Parkan and Wu (1997) used a 5-point Likert scale for a process selection problem. No matter which scale is adopted, even for a 0–1.0 normalized scale (Rao 2006), the designed scales or categories are often believed to represent an interval level of measurement, and the above scales seem to be assigned in a subjective way. For instance, one conservative DM might think number 7 is good enough for evaluating the benefit criteria, whereas the other might assign 9 to the same case in a group decision environment. There should be a way for a more objective evaluation for group decision making. Herein the linguistic variables are commonly thought to have the same meanings for representing performances of the qualitative criteria.

In AHP, pairwise comparisons through a 9-point fundamental scale are used to manage tangible (objective, quantitative) and non-tangible (subjective, qualitative) factors (Saaty 1980), but most works employ the process of pairwise comparisons to obtain weights for criteria, e.g., Shyur and Shih (2006), in order to support TOPSIS. It appears that no work in the literature has used AHP to handle ordinal information for the TOPSIS process. Many other MCDM techniques have been proposed for dealing with qualitative criteria. Hinloopen et al. (1983) listed 16 examples of such methods. Voogd (1983) classified some of these into two categories: pure qualitative data and mixed qualitative and quantitative data. The former was further divided into frequency approaches and scaling models. The latter listed the geometric scaling model and three other analytical mixed-data evaluation techniques integrated into the EVAMIX (multicriteria EVAlation with MIXed qualitative and quantitative data) model (Voogd 1983). For mixed data, a debate arose as to whether the conversion of qualitative measures into quantitative form is worth it or not (Larichev et al. 1995) and concluded that some quantitative weighting estimates introduce inaccurate representations of a decision maker's preference.

In a similar vein, Xu and Lamond (2001) suggested a ranking procedure based on the distance between partial pre-orders without the need of conversion. Nonetheless, when handling mixed data within existing techniques, conversion seems unavoidable due to coherence with the quantitative data. For example, simple multi-attribute rating technique (SMART) allows decision makers (individual or group) to quantify qualitative information that can be categorized through performance scores (Edwards 1977). Measuring attractiveness by a categorical based evaluation technique (MACBETH) employs a non-numerical interactive questioning procedure, requesting a qualitative judgment about the difference of attractiveness, in order to build a quantitative value (Bana e Costa and Chagas 2004). Hence, a conversion of ordinal information to cardinal information in TOPSIS seems similarly possible.

Transforming ordinal information calls for further consideration. For instance, qualitative concordance analysis (Van Delft and Nijkamp 1977) utilizes a nominal scale as the basis for evaluation. The numerical interpretation method relaxes the limited number of ordinal levels for a wide range of representation (Voogd 1983). EVAMIX uses a signum function for measuring the dominance of any two alternatives on any criterion and weights these dominances together with cardinal evaluations (Voogd 1982). According to the guidelines of the MCDM methods by Guitouni and Martel (2006), EVAMIX and TOPSIS are grouped into the category of a single synthesizing criterion. Consequently, the essence of the process of EVAMIX should be able to help TOPSIS handle ordinal information, by proposing a coherent, simple, and operational procedure. While EVAMIX concentrates on the distance of the difference among the performance measures, TOPSIS reasons the distance between the one performance measure and PIS/NIS to be DMs' preference. Table 1 summarizes the comparisons of AHP, EVAMIX, and TOPSIS. Note that herein we do not take into account stochastic dominance (Lahdelma et al. 2003), which can be left for future study. Moreover, Xu et al. (2014) proposed a distance-based aggregation approach for group decision making with interval preference orderings, which minimizes the weighted arithmetic averaging operator of the interval preference of decision makers. It has no close relation with TOPSIS.

Another issue about ordinal information on criteria weights has not been discussed in detail. Many MCDM techniques, especially in compensatory models, require information about the relative importance of each criterion (Hwang and Yoon 1981). Elicitation of criteria weights can roughly be classified as direct and indirect estimations (Horsky and Rao 1984). The former is the main interest due to its straightforward support of TOPSIS. Barron and Barrett (1996) examined four approximate rank-order weights—rank order centroid (ROC), rank sum, rank reciprocal, and equal weights—and found that ROC weights are the most accurate and efficacious based on their designed simulation study. Alfares and Duffuaa (2009) reviewed past works that inferred criteria weights from ranks and proposed a method to determine the relative weights for any set of criteria. Although there are many other methods for assigning a weight from ordinal information, e.g., distance-metric methodology by Jones and Mardle (2004) and ordered weighting averaging (OWA) method by Ahn (2011), this

Table 1 Comparison of the characteristics of three common MCDM techniques

Item	Characteristic	AHP	EVAMIX	TOPSIS
1	Category	Cardinal information as well as information on the criterion of MCDM	Cardinal and ordinal information as well as information on the criterion of MCDM	Cardinal information as well as information on the criterion of MCDM
		The ordinal information is implicitly represented by pairwise comparisons		However, the proposed approach can process ordinal information[a]
2	Core process	Pairwise comparison (9-point fundamental scale for the measurement)	1. Indirect use of distance function: the pairwise comparisons of the difference between performance measures of any two alternatives are accumulated by distance functions to be the dominance scores	1. Direct use of distance function: the performance measures of alternatives from PIS and NIS are measured by distance functions as the DM's preference
			2. A signum function is used to pre-process ordinal data on the qualitative criteria	2. The proposed approach uses the signum function to handle ordinal data[a]
			3. A weighting step for the combination of cardinal and ordinal information	3. No need for the extra weighting step
3	Criteria	Given	Given	Given
4	Weight elicitation	Pairwise comparison	Given	Given
5	Consistency check	Provided	None	None
6	No. of criteria accommodated	7 ± 2 or hierarchical decomposition	Many more	Many more
7	No. of alternatives accommodated	7 ± 2	Many more	Many more
8	Others	Compensatory operation	Compensatory operation	Compensatory operation

Please refer to Voogd (1982), Hwang and Yoon (1981), Saaty (1980), and Shih et al. (2007)
[a] Means the proposed approach will keep the feature

study chooses ROC weights, because they need much less input from DMs as concluded by Olson and Dorai (1992). However, we shall carry out an extra sensitivity analysis on weights to check the model's stability on ranks.

The third interest relates to group aggregation with differentiated decision power in multi-criteria group decision. Decision power is defined as the potential ability of a person or a group to exercise control or influence on another person or group for the final decision (Griffin and Moorhead 2013). Though there are many forms of this power, e.g., legitimate power or expertise power, it can be accounted for in the political or business environment based on the number of votes or the percentages of shares (Korhonen 1997; Widerén 1994). The decision power of the person can thus be quantified as his or her relative weight in that group based on his or her position, expertise, percentages of shares, etc. A differentiated decision power can then be identified. Ramanathan and Ganesh (1994) classified the approaches of assigning this weight for decision power as the supra DM approach and the participatory approach. The former assigns weights by a supra DM, which is usually at the top level of the organization. For example, Tsui and Wen (2014) investigated a selection issue for green suppliers in a TFT–LCD company, and the company considers the weights of four stokehold departments as 36, 27, 27, and 10 %, respectively, which is determined by one key department being the supra DM. The latter approach mentions that there exist different expertises in a group, which can be quantified by interperson comparisons or discussions among the group members to obtain the their power. This expertise power can then be simply interpreted as the weight of a member in the group utility function.

For group aggregation, social choice aims to combine individual opinions, preferences, interests, or welfares to reach a collective decision, but there is no procedure for aggregating individual ranking into group ranking without violating some reasonable assumptions (Arrow 1963). From a practical aspect, the aggregation of a group's ranking is still possible if compromise and agreement exist in the group, as it is reached through feedback and discussion among group members (Bezerra et al. 2014). We observe that many techniques can facilitate a compromise or an agreement (Hwang and Lin 1987). Baucells and Sarin (2003) developed a group utility function from additive individuals' utility functions over the multiple attributes, thus providing the foundation for applying MCDM in a group decision environment. Group preference is generally aggregated by taking the mean operation of individuals' preferences.

Ramanathan and Ganesh (1994) utilized the geometric mean and the weighted arithmetic mean for AHP and also took advantage of another hierarchy for deriving members' weights. Forman and Peniwati (1998) discussed several ways to aggregate group information for AHP and proposed weighted arithmetic and geometric means for aggregating individual judgments and individual priorities. Bernasconi et al. (2014) further examined empirical properties of group preference aggregation methods employed in AHP. On the other hand, Barzilai and Lootsma (1997) proposed a power relation and group aggregation in multiplicative AHP and SMART, however, their approach has drawn debate over its behavior realism (Korhonen 1997). Van den Honert (2001) developed two models for obtaining group members' weights of multiplicative AHP and SMART, respectively. For TOPSIS, Yue (2013) collected 15 works of preference aggregation in the group TOPSIS whereby most of them utilize arithmetical and geometric means for group aggregation. Since it seems not many works explicitly discussed decision power in the group, Baucells and Sarin (2003) offered some materials for further tackling group TOPSIS with differentiated decision power. We next suggest a mixed-data model for group TOPSIS with differentiated decision power.

3 Proposed Model

We first concentrate on the process of ordinal information and then deal with group aggregation with decision power in group TOPSIS.

3.1 Ordinal Information Process

There are two types of cardinal information involved in group TOPSIS: performance of alternatives on criteria and the weights of criteria. Most efforts are given toward measuring the performance on qualitative criteria. For the second part, centroid or ROC weights will be chosen herein for assigning weights to criteria due to their simplicity.

When there is ordinal information of performance measures on criteria, a simple process is proposed for group TOPSIS. This process considers the dominance of any alternative, by pairwise comparisons on the performance measure of this alternative with other alternatives, and sums up the dominances of this alternative with the others as accumulated dominances.

We present the decision matrix $\boldsymbol{D}^k$, $k = 1, \ldots, K$, representing the kth DM as follows:

$$D^k = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccccccc} X_1 & X_2 & \cdots & X_j & \cdots & X_n \end{array} \\ \left[\begin{array}{cccccc} x_{11}^k & x_{12}^k & \cdots & x_{1j}^k & \cdots & x_{1n}^k \\ x_{21}^k & x_{22}^k & \cdots & x_{2j}^k & \cdots & x_{2n}^k \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{i1}^k & x_{i2}^k & \cdots & x_{ij}^k & \cdots & x_{in}^k \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{m1}^k & x_{m2}^k & \cdots & x_{mj}^k & \cdots & x_{mn}^k \end{array}\right]_{m \times n} \end{array}, \tag{1}$$

where A_i denotes the alternative i, $i = 1, \ldots, m$; X_j represents the criterion j, $j = 1, \ldots, n$; and the number of quantitative criteria p and the number of qualitative criteria q are related by $p + q = n$. The element of $\boldsymbol{D}^k$, x_{ij}^k, indicates the performance rating of alternative A_i with respect to attribute X_j by DM k, $k = 1, \ldots, K$. Note that there should be K decision matrices for K members of the group.

To further process the performance information of alternatives on qualitative criteria, we express the following qualitative decision matrix, which is a submatrix of the decision matrix $\boldsymbol{D}^k$:

$$D_{qual}^{k} = \begin{matrix} \\ A_1 \\ A_2 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{matrix} \begin{matrix} \begin{matrix} X_{p+1} & X_{p+2} & \cdots & X_j & \cdots & X_n \end{matrix} \\ \begin{bmatrix} x_{1(p+1)}^{k} & x_{1(p+2)}^{k} & \cdots & x_{1j}^{k} & \cdots & x_{1n}^{k} \\ x_{2(p+1)}^{k} & x_{2(p+2)}^{k} & \cdots & x_{2j}^{k} & \cdots & x_{2n}^{k} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{i(p+1)}^{k} & x_{i(p+2)}^{k} & \cdots & x_{ij}^{k} & \cdots & x_{in}^{k} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{m(p+1)}^{k} & x_{m(p+2)}^{k} & \cdots & x_{mj}^{k} & \cdots & x_{mn}^{k} \end{bmatrix}_{m \times q} \end{matrix}, \tag{2}$$

where there are q qualitative criteria, and their subscripts count from $p + 1$ to n.

Since the ordinal information of performance measures on criterion j is by a partial order set, we can compare the orders of the performance measures. Here, the preference order of $x_{1j}^k, x_{2j}^k, \ldots, x_{ij}^k, \ldots,$ and x_{mj}^k is represented by a function of their performances as $o(x_{1j}^k), o(x_{2j}^k), \ldots, o(x_{ij}^k), \ldots,$ and $o(x_{mj}^k)$, respectively, on criterion j for DM k. It is noted that the orders of the performance measures are predefined in the partial order set, and they can be linguistic or semantic terms, or levels of achievement. Thus, a comparison can be made.

Based on the presentation of the ordinal information process in EVAMIX, we introduce a signum function to count the dominance of the order comparison on each qualitative criterion for simplicity. Comparing any two orders of the alternatives, if one is preferred to the other, then the function will give 1; if one is indifferent to the other, then the function will give 0; if one is inferior to the other, then the function will give -1.

$$s_j^k\left(o\left(x_{ij}^k\right), o\left(x_{i'j}^k\right)\right) = \begin{cases} 1, & \text{if } o\left(x_{ij}^k\right) \succ o\left(x_{i'j}^k\right), \\ 0, & \text{if } o\left(x_{ij}^k\right) \approx o\left(x_{i'j}^k\right), \\ -1, & \text{if } o\left(x_{ij}^k\right) \prec o\left(x_{i'j}^k\right). \end{cases} \quad \forall i \text{ and } i', i \neq i', \tag{3}$$

where both i and $i' = 1, \ldots, m$ for m alternatives on criterion j. Note that there is no need to distinguish the criteria of benefits and costs in the above expression since the comparison is based on the orders that are the preferences in DMs' minds.

The dominance measure $s_j^k(o(x_{ij}^k), o(x_{i'j}^k))$ is a comparison of the performance orders of any two alternatives A_i and $A_{i'}$, respectively, on criterion j for DM k. Thus, we illustrate a dominance matrix $\boldsymbol{S}_j^k$ by the following pairwise comparisons on criterion j.

$$\boldsymbol{S}_j^k = \begin{matrix} \\ A_1 \\ A_2 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{matrix} \begin{matrix} \begin{matrix} A_1 & A_2 & \cdots & A_i & \cdots & A_m \end{matrix} \\ \begin{bmatrix} - & s_j^k(o(x_{ij}^k), o(x_{2j}^k)) & \cdots & s_j^k(o(x_{1j}^k), o(x_{ij}^k)) & \cdots & s_j^k(o(x_{ij}^k), o(x_{mj}^k)) \\ s_j^k(o(x_{2j}^k), o(x_{ij}^k)) & - & \cdots & s_j^k(o(x_{2j}^k), o(x_{ij}^k)) & \cdots & s_j^k(o(x_{2j}^k), o(x_{mj}^k)) \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ s_j^k(o(x_{ij}^k), o(x_{1j}^k)) & s_j^k(o(x_{ij}^k), o(x_{2j}^k)) & \cdots & - & \cdots & s_j^k(o(x_{ij}^k), o(x_{mj}^k)) \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ s_j^k(o(x_{mj}^k), o(x_{ij}^k)) & s_j^k(o(x_{mj}^k), o(x_{2j}^k)) & \cdots & s_j^k(o(x_{mj}^k), o(x_{ij}^m)) & \cdots & - \end{bmatrix}_{m \times m} \end{matrix} \tag{4}$$

The accumulated dominance measure t_{ij}^k is the summation of the comparison of the performance of alternative A_i to all other alternatives $A_{i'}$ on criterion j.

$$t_{ij}^k = \sum_{i\prime=1}^{m} s_j^k \left(o\left(x_{ij}^k\right), o\left(x_{i\prime j}^k\right)\right), \quad i \neq i', \forall j. \tag{5}$$

Thus, an accumulated dominance vector on criterion j by DM k is:

$$\boldsymbol{T}_j^k = \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{matrix} \begin{bmatrix} t_{1j}^k \\ t_{2j}^k \\ \vdots \\ t_{ij}^k \\ \vdots \\ t_{mj}^k \end{bmatrix}_{m\times 1} . \tag{6}$$

We next obtain an adjusted performance value with respect to each criterion by the following expression within the maximum and minimum values of the above vector.

$$q_{ij}^k = \frac{t_{ji}^k - \min_{i=1}^m \left[t_{ji}^k\right] + \delta}{\max_{i=1}^m \left[t_{ji}^k\right] - \min_{i=1}^m \left[t_{ji}^k\right] + \delta}. \tag{7}$$

Here, we add a small value δ for normalization in order to avoid the zero performance for further computation. In traditional TOPSIS, a small value δ is equal to 1 for the worst value of the benefit criteria (Hwang and Yoon 1981). A normalized value is then obtainable by linear or vector normalization as done by TOPSIS.

The adjusted performance values can be linear or vector normalized so that the process is coherent with the traditional TOPSIS procedure. For vector normalization, the equation is:

$$r_{ij}^k = \frac{q_{ij}^k}{\sqrt{\sum_{i=1}^m q_{ij}^k}}. \tag{8}$$

Here, $i = 1, \ldots, m$; $j = 1, \ldots, n$; $k = 1, \ldots, K$. In such a way, the ordinal information on the qualitative criteria can be transformed into cardinal information and be integrated into the traditional TOPSIS process. Here, Eq. (8) provides a consistent form of information as shown at Step 2 of Shih et al. (2007).

It is quite common for DMs to only provide ordinal information on the weight of each criterion. We choose ROC weights, an approximating weight set, because of their simplicity (Edwards and Barron 1994). Given the ranks of criteria are $w_1 = w_2 = \ldots = w_n > 0$ for n criteria, the weight of j^{th} criterion can thus be calculated as follows.

$$w_j(\text{ROC}) = \frac{1}{m}\sum_{i=j}^{m}\left(\frac{1}{i}\right), \tag{9}$$

where $j = 1, \ldots, n$ by a descending rank order and $\sum_{j=1}^{n} w_j = 1$.

Note that the weights of the criteria depend on their ranks, and their values have no difference for any member in the group. Compared to Step 4 of Shih et al. (2007), we can re-define the weight $w_j = w_j^1 = w_j^2 = \ldots = w_j^k$, for criterion $j = 1, \ldots, n$ and for each DM $k = 1, \ldots, K$.

Edwards and Barron (1994) offered ROC weights by the number of criteria up to 16. For the case of four criteria, their weights are 0.5208, 0.2708, 0.1458, and 0.0625, respectively, in a descending ranked order and without a tie. If the second and the third ranks are a tie, then based on the concept of corner points with duplicated points at the tie, the modified weights are 0.4792, 0.2292, 0.2292, and 0.0625, respectively, after modifying Eq. (9). These default weights are considered for further operation. As the extra routine has now been outlined, we will integrate the operation into the proposed procedure in Sect. 3.3.

3.2 Group Aggregation with Differentiated Decision Power

The influence or power of one member on the other in a group or an organization is this paper's interest. The power is usually implicitly expressed in a group and can be many types (Griffin and Moorhead 2013). However, it is uneasy to quantify the power. As Baucells and Sarin (2003) proposed a group utility function, group preference can be aggregated from individuals' utility functions. Following the same stream, a differentiated power can be thought of as another kind of weight to the individual's utility function. In group TOPSIS, Shih et al. (2007) considered group measures or utility, $\overline{S_i^+}$ and $\overline{S_i^-}$, as aggregates of the separation measures or individuals' utilities of each DM, S_i^{k+} and S_i^{k-}, $k = 1, \ldots, K$, respectively, by taking the geometric or arithmetical means of these values without considering differentiated decision power. A simplified modification is suggested by the following aggregation operations on two separation measures of the group:

$$\overline{S_i^+} = \left(u^1 \oplus S_i^{1+}\right) \otimes \ldots \otimes \left(u^K \oplus S_i^{K+}\right), \quad \text{for alternative } i, \text{ and} \tag{10}$$

$$\overline{S_i^-} = \left(u^1 \oplus S_i^{1-}\right) \otimes \ldots \otimes \left(u^K \oplus S_i^{K-}\right), \quad \text{for alternative } i. \tag{11}$$

Here, u^k, $k = 1, \ldots, K$, is the decision power of DM k by his or her relative weight, and $\sum_{k=1}^{K} u^k = 1$, which can be obtained through a predetermined way (Ramanathan and Ganesh 1994). The greater the value of the weight is, the more power the decision maker has in the group. A DM with greater weight also means that he or she seems to possess more influence than others on the final decision of the group.

There are many choices in the operation of internal aggregation, e.g., weighted arithmetic mean, weighted geometric mean, or other modifications. If we take a weighted arithmetic mean of all individual measures, then the group measures, Eqs. (10) and (11), from PIS and NIS are:

$$\overline{S_i^+} = \left(\sum_{k=1}^{K} u^k S_i^{k+} \right) \Big/ K, \quad \text{for alternative } i, \text{ and} \tag{12}$$

$$\overline{S_i^-} = \left(\sum_{k=1}^{K} u^k S_i^{k-} \right) \Big/ K, \quad \text{for alternative } i, \tag{13}$$

where $i = 1, \ldots, m$; $k = 1, \ldots, K$. If the weighted geometric mean is considered, then the group measures from PIS and NIS are:

$$\overline{S_i^+} = \prod_{k=1}^{K} \left(S_i^{k+} \right)^{u^k}, \quad \text{for alternative } i, \text{ and} \tag{14}$$

$$\overline{S_i^-} = \prod_{k=1}^{K} \left(S_i^{k-} \right)^{u^k}, \quad \text{for alternative } i. \tag{15}$$

In the following the final ranking is determined by the group relative closeness, as a group preference, of the ith alternative A_i:

$$\overline{C_i^*} = \frac{\overline{S_i^-}}{\overline{S_i^+} + \overline{S_i^-}}, \quad i = 1, \ldots, m, \tag{16}$$

where $0 \leq \overline{C_i^*} \leq 1$. The larger the index value is, the better the rank of the alternative will be.

We note that all efforts attempt to obtain the composited separation measures, including ordinal information and group information aggregation. Thus, the proposed procedure is uniformly coherent with the traditional TOPSIS steps and keeps a straightforward procedure.

3.3 Proposed Procedure

Based on the developments of the above two sections, we modify the procedure of group TOPSIS of Shih et al. (2007) as follows.

Step 1 Construction of mixed-data decision matrix individually, Eq. (1).
(1.1) Acquisition of cardinal and ordinal performance information.
(1.2) Transformation of the ordinal performance to be the adjusted performance, Eqs. (3)–(7).

Step 2 Construction of normalized decision matrix individually.

Step 3 Determination of PIS/NIS individually.

Step 4 Assignment of weights to the criteria individually.
(4.1) Elicitation of the weights from the importance of the criteria.
(4.2) Processing the importance numerically directly from comparisons or ranks.
Step 5 Determination of decision power for all group members.
(5.1) Assignment of members' weights to all members by a supra decision maker or by participants under an agreement.
Step 6 Calculation of separation measure from PIS/NIS for the group, Eqs. (10)–(15).
Step 7 Calculation of relative closeness for the group, Eqs. (16).
Step 8 Ranking and selection of preferable alternative(s).
(8.1) Ranking of all alternatives.
(8.2) Examination of the preferable alternative(s) and screening them out if the alternative(s) are questionable.
(8.3) Selection of preferable alternative(s).

We can see that one major difference is Step 5 being added for processing the decision power among the group members. Some minor modifications are also shown at Steps 1, 4, and 8. In addition, an assumption is implicitly presented in the procedure that all group members are willing to obtain a compromised or consensus resolution. If the group members here do not agree with the resolution, then extra effort is needed to reach a consensus by some techniques of group decision making (Hwang and Lin 1987), e.g., the Delphi technique. Some decision support systems can also facilitate such a consensus, e.g., Shih et al. (2004).

4 Illustrative Examples

Two group decision examples are illustrated as follows. Example 1 demonstrates the example of fighter selection by various combinations of the parameters. Example 2 presents a case of personnel recruitment. We also show the original fighter selection problem (Hwang and Yoon 1981) with ordinal information on subjective criteria in "Appendix" for ease of tracing the proposed procedure.

Example 1 A fighter selection problem (Hwang and Yoon 1981)

Four alternatives are evaluated by six criteria for the selection. Table 2 shows the detailed information after adding one DM. The different evaluations between two DMs are on the last two criteria, which are subjective with different level ordinal performances. The first decision maker keeps the original five levels of measurement, and the second one only uses three levels of measurement. These measurements are handled by Eqs. (3)–(8). Here, we assume equal decision power for both DMs and employ the weighted arithmetical mean to aggregate the preferences of both DMs in which decision power $u^1 = u^2 = 0.5$ for Eqs. (12) and (13). The rank results are tabled by three sets of weights on the criteria—the given weights from the original example, equal weights, and the ordinal weights from ROC—with vector normalization on performance measures as shown in Table 3. In its three sub-tables, each has 18 combinations of ranks that are classified by different group aggregations, i.e., geometric and arithmetical means and Borda function, Minkowski's L_p metric parameters (p=1,

Table 2 Evaluation data of the fighter selection example

Alternatives	Criteria							
	Objective criteria				Subjective criteria			
					Decision maker #1		Decision maker #2	
	Maximum speed (Mach)	Ferry range (NM)	Maximum payload (pounds)	Purchasing cost ($\$ \times 10^6$)	Reliability (five levels)	Maneuverability (five levels)	Reliability (three levels)	Maneuverability (three levels)
	X_1	X_2	X_3	X_4	X_5	X_6	X_5	X_6
A_1	2.0	1500	20,000	5.5	Average	Very high	Average	High
A_2	2.5	2700	18,000	6.5	Low	Average	Low	Average
A_3	1.8	2000	21,000	4.5	High	High	High	High
A_4	2.2	1800	20,000	5.0	Average	Average	Average	Low
Criterion characteristics	Benefit	Benefit	Benefit	Cost	Benefit	Benefit	The same as the left two columns	
Weight								
Given	0.2	0.1	0.1	0.1	0.2	0.3	The same as the left two columns	
Equal	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667		
Ordinal	0.1944	0.0833	0.0833	0.0833	0.1944	0.3611		

The data are from Hwang and Yoon (1981) with the given weights above. However, we add the subjective information of the other decision maker (DM#2) for illustrating the case of group decision making
The subjective criteria are evaluated by five levels and three levels for DM#1 and DM#2, respectively
Equal weights are given as 0.1667 each for the six criteria
Ordinal weights of ROC are originally given as 0.1944, 0.0833, 0.0833, 0.0833, 0.1944, and 0.3611, respectively, by the ranks of the six criteria. They are modified by taking the same corner points with the ranks of ties. Thus, the weights of the criteria of maximum speed and reliability take the weights as $\left(0 + \frac{1}{3} + \frac{1}{3} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}\right)/6 = 0.1944$. The weights of the criteria of ferry range, maximum payload, and purchasing cost are all equal to 0.0833

Table 3 The comparison of the ranks with group decision under vector normalization for the fighter selection example with equal decision power

Group aggregation	GM	AM	Borda	GM	AM	Borda	GM	AM	Borda	GM	AM	Borda	GM	AM	Borda	GM	AM	Borda
Weighting	Before			After			Before			After			Before			After		
L_p metric	$p = 1$			$p = 1$			$p = 2$			$p = 2$			$p = \infty$			$p = \infty$		
(3a) *By given weights*																		
Alternatives																		
A_1	2	2	1	2	2	1	2	2	1	2	2	1	2	1	1	2	1	1
A_2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
A_3	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	2	1
A_4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
(3b) *By equal weights*																		
Alternatives																		
A_1	2	2	2	2	2	2	2	2	1	2	2	2	2	1	1	2	2	2
A_2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
A_3	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1	1
A_4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
(3c) *By ordinal weights*																		
Alternatives																		
A_1	2	2	1	2	2	1	2	1	1	2	2	1	1	1	1	1	1	1
A_2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
A_3	1	1	1	1	1	1	1	2	1	1	1	1	2	2	1	2	2	1
A_4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3

In group aggregation, GM and AM represent the geometric mean and the arithmetic mean of the separation measures of the two DMs, respectively. The Borda function is for external aggregation of the ranks of the two DMs

There are three metric parameters for the distance function: $p = 1$, $p = 2$, and $p = \infty$

There are two aspects on weighting the steps: before the PIS/NIS calculation, which is the traditional TOPSIS (Hwang and Yoon 1981), or after PIS/NIS calculation, whose weights are given to the calculation of separation measures (Shih et al. 2007). The results are tabled by three sets of the weights on the criteria

Table 4 Sensitivity analysis on the differentiated decision power of the two DMs in the fighter selection example

	Rank								
Alternatives									
A_1	1	2	2	2	2	2	2	2	2
A_2	4	4	4	4	4	4	4	4	4
A_3	2	1	1	1	1	1	1	1	1
A_4	3	3	3	3	3	3	3	3	3
Weights									
DM#1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1

The results are under vector normalization, ordinal weights on criteria, weighting after PIS/NIS, L_p metric with $p = 2$, and weighted arithmetic mean of the two DMs for differentiated decision power

2, and ∞), and by different weighting steps, whereby before the PIS/NIS calculation as traditional TOPSIS (Hwang and Yoon 1981) is marked as "before", and after the PIS/NIS calculation whose weights are given to the calculation of separation measures (Shih et al. 2007) is marked as "after".

We see that the ranks are rather stable in terms of the three weight sets under the aggregation by geometric and arithmetical means, but the results from the Borda function seem to show not much discrimination ability here, as 14 cases out of 18 have ties. This could be one advantage of the internal aggregation of TOPSIS instead of the external aggregation by the Borda function. In addition, aside from the case of $p = \infty$, the ranks are stable under the same combinations across the three weight sets.

To understand the effects of the weights on the criteria and other weights on decision power, we execute two sensitivity analyses. The former deals with the weight change on the criterion of maneuverability, whose weight is 0.3611 from ROC, with the range of $\pm 40\,\%$. Under the conditions of considering geometric mean aggregation, weighting the step after PIS/NIS calculation, and metric parameter $p = 2$, the ranks of alternatives are $A_3 \succ A_1 \succ A_4 \succ A_2$ and are unchanged within the range. The ranks from the Borda function are $A_3 \approx A_1 \succ A_4 \succ A_2$, within the range -10 to $40\,\%$, and their ranks switch to the previous ranks during the range -20 to $-40\,\%$. The latter inspects the effect of the weight changes on decision power in which DM#1 holds weights of 0.9 decreasing to 0.1. The basic condition of the combinations is to maintain the same as the former. The ranks of alternatives are $A_3 \succ A_1 \succ A_4 \succ A_2$, within the range of weights from 0.8 decreasing to 0.1, and the ranks of alternatives are $A_1 \succ A_3 \succ A_4 \succ A_2$ with the weights being 0.9 as shown in Table 4. With this small example, the presented results of the proposed model are stable.

Example 2 A personnel recruitment problem.

A semiconductor corporation is extending its business by selling its newly-developed light-emitting diode (LED) components to lighting and other manufacturers of LED products. The sales and marketing department of the corporation initializes a recruitment of two sales engineers to promote the new business. After a screening process on the applications, 15 candidates remain on the final list for an interview in the second stage. The interview evaluation includes four dimensions with the items they belong to in the corresponding parentheses: knowledge (management and specialty), skills (communication, expressiveness, problem-solving, and analytic), behavior (team-work and customer-oriented), and personal characteristics (vigor, confidence, compressive resistance, and integrity). Since these 12 items are difficult to evaluate by cardinal information, e.g., 0–100 scores, interval values are used with some limited levels for ease of evaluation. The human resources department (HR) wants to introduce a new evaluating system to fit the company's needs. For ease of manipulation, HR sets up ordinal data by five levels for the interview evaluation, and our proposed approach satisfies the firm's needs. Table 5 lists the evaluation sheet of the 15 candidates on the 12 items by two managers from the employing unit (EU) and HR, respectively. The weights of the evaluating items or criteria are tested by the given criteria weights from AHP, equal weights, and ROC weights.

For a group decision, it is not easy to define the decision power for both departments to reflect their expertise. The final decision relies heavily upon the evaluation of the EU, because the candidates will work for that unit later, but the opinion of HR is mainly as a reference for the decision makers. However, HR does keep veto power on the final decision if any preferable candidate has an unethical or unacceptable behavior in the past (at Step 8 of the proposed procedure). In our case, they agree on EU obtaining a weight of 0.8 and HR obtaining the rest after a long and tedious discussion, as mentioned at Step 5.

Table 6 presents a comparison of group ranks for the case from 18×3 combinations, i.e., three weight sets for criteria, weighted geometric or arithmetic means for differentiated decision power, L_p metric parameters ($p = 1, 2$, and ∞), and a weighting step before or after PIS/NIS calculation, and all combinations are under vector normalization and EU is assigned the decision power of 0.8. We also list the results of external aggregation by the Borda function for comparison, and the differentiated decision power is inapplicable to the function.

Based on the shown results, Candidate A_{11} should be the best one without much debate, but the second one shows a little bit of diversity. Candidate A_5 or A_{10} could be the second best, while Candidate A_5 is selected due to stable ranks among the different combinations. Furthermore, we see some unexpected results with metric parameter $p = \infty$ under aggregation by the geometric mean. This is caused by the zero value of separation measures, i.e., the alternatives are at PIS or NIS, and the geometric mean does not work well. In addition, the metric parameter $p = \infty$ also gives us slightly inconsistent results. This manipulation takes an extreme value from multiple dimensions, and the other decision information is almost neglected. It easily makes the candidates less discriminated under full ordinal information. We would suggest to avoid taking metric parameter $p = \infty$ as a choice in using TOPSIS evaluation. From the same table, we find that the results aggregated from the Borda function have many ties, which are not good for our evaluation.

To illustrate the effect of differentiated power on the final decision, we execute a sensitivity analysis on the unequal decision weights for both departments using Eqs. (12) and (13) and consider the weighted arithmetical mean for the aggregation in which EU takes a weight from 0.5 (equally shared decision power between EU and HR) to 0.95 (major decision by EU). The results in Table 7 are based on ordinal

Table 5 The interview evaluation of 15 candidates of the case company

Candidate	Knowledge		Skill				Behavior		Personal characteristics			
	Management	Specialty	Communication	Expressiveness	Problem solving	Analytic	Team-work	Customer-oriented	Vigor	Confidence	Compressive resistance	Integrity
	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
(5a) *From employing unit*												
A_1	Poor	Average	Average	Average	Poor	Average	Average	Average	Good	Good	Average	Average
A_2	Good	Good	Average	Average	Good	Average	Average	Good	Average	Good	Average	Good
A_3	Poor	Average	Average	Good	Good	Good	Average	Average	Good	Average	Average	Average
A_4	Average	Good	Average	Average	Average	Average	Good	Average	Good	Good	Average	Good
A_5	Good	Good	Good	Good	Good	Average	Good	Good	Good	Good	Good	Good
A_6	Good	Good	Good	Average	Good	Good	Average	Good	Average	Good	Average	Average
A_7	Good	Good	Good	Average	Good	Good	Average	Good	Average	Good	Average	Average
A_8	Average	Average	Good	Good	Average	Average	Good	Good	Average	Good	Good	Average
A_9	Average	Good	Average	Average	Poor	Average	Good	Average	Average	Good	Average	Good
A_{10}	Average	Average	Good	Good	Good	Average	Average	Average	Good	Good	Average	Average
A_{11}	Good	Good	Good	Average	Good	Good	Good	Good	Good	Good	Good	Good
A_{12}	Average	Average	Average	Average	Average	Average	Average	Good	Average	Average	Average	Average
A_{13}	Very poor	Average	Good	Good	Average	Average	Average	Good	Good	Good	Good	Average
A_{14}	Average	Average	Average	Good	Good	Good	Average	Average	Good	Good	Average	Average
A_{15}	Average	Average	Average	Average	Good	Average	Average	Average	Average	Good	Average	Average
Weight												
Given	0.0095	0.0571	0.0134	0.0102	0.0204	0.0464	0.0647	0.0582	0.1924	0.1401	0.0991	0.2885
Equal	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833
Ordinal	0.0069	0.0544	0.0229	0.0145	0.0321	0.0425	0.0850	0.0683	0.1753	0.1336	0.1058	0.2586

Table 5 continued

Candidate	Knowledge		Skill				Behavior		Personal characteristics			
	Management X_1	Specialty X_2	Communication X_3	Expressiveness X_4	Problem solving X_5	analytic X_6	Team-work X_7	Customer-oriented X_8	Vigor X_9	Confidence X_{10}	Compressive resistance X_{11}	Integrity X_{12}
(5b) *From human resources department*												
A_1	Average	Average	Average	Average	Average	Poor	Average	Average	Average	Good	Average	Average
A_2	Average	Average	Average	Good	Average	Good	Average	Good	Good	Good	Average	Average
A_3	Poor	Average	Average	Good	Average	Good	Average	Average	Good	Good	Average	Average
A_4	Poor	Average	Average	Average	Average	Average	Good	Average	Average	Average	Average	Good
A_5	Average	Average	Good	Good	Good	Average	Good	Good	Good	Average	Good	Good
A_6	Good	Good	Good	Good	Good	Good	Average	Average	Good	Good	Average	Average
A_7	Average	Average	Good	Good	Good	Good	Average	Average	Good	Good	Average	Average
A_8	Average	Average	Good	Good	Good	Average	Good	Good	Good	Good	Good	Good
A_9	Average	Average	Good	Good	Average	Average	Good	Average	Average	Average	Average	Good
A_{10}	Average	Good	Good	Good	Good	Good	Good	Good	Good	Good	Good	Good
A_{11}	Good	Good	Good	Good	Good	Good	Good	Good	Average	Good	Good	Good
A_{12}	Average	Average	Good	Average	Average	Average	Good	Good	Good	Average	Average	Average
A_{13}	Very poor	Average	Good	Good	Average	Average	Average	Good	Good	Good	Average	Average
A_{14}	Average	Average	Average	Good	Good	Good	Average	Average	Good	Good	Average	Average
A_{15}	Average	Good	Average	Good	Average	Good	Average	Average	Average	Good	Average	Average

There are 12 evaluating items or criteria belonging to four categories
There are three sets of weights: given weights from AHP, equal weights, and ordinal ROC weights

Table 6 The results of group ranks under different combinations for the human recruitment case with differentiated decision power

Group aggregation	GM	AM	Borda	GM	AM	Borda	GM	AM	Borda	GM	AM	Borda	GM	AM	Borda	GM	AM	Borda
weighting	Before			After			Before			After			Before			After		
L_p metric	$p = 1$			$p = 1$			$p = 2$			$p = 2$			$p = \infty$			$p = \infty$		
(6a) *By given weights*																		
Candidates																		
A_1	13	13	13	13	13	13	13	11	13	13	13	14	11	11	11	n/a	13	14
A_2	7	5	7	7	5	7	7	5	7	7	5	7	7	5	7		8	8
A_3	12	12	12	12	12	12	10	10	12	12	12	12	8	8	8		9	9
A_4	5	3	4	5	3	4	4	3	3	5	3	3	4	3	3		5	4
A_5	3	2	2	3	2	2	3	2	1	3	2	2	3	2	1		3	1
A_6	9	10	8	9	10	8	11	12	10	9	10	9	12	12	12		9	9
A_7	11	11	11	11	11	11	12	13	11	11	11	11	12	12	12		9	9
A_8	4	6	3	4	6	3	5	7	6	4	6	3	6	7	6		2	2
A_9	6	4	6	6	4	6	6	4	3	6	4	3	5	4	5		5	4
A_{10}	2	8	4	2	7	4	2	6	3	2	7	6	2	6	4		4	7
A_{11}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2		1	3
A_{12}	15	15	14	15	15	14	15	15	14	15	15	13	15	15	14		15	13
A_{13}	8	7	8	8	8	8	8	8	8	8	8	8	8	8	8		7	6
A_{14}	10	9	10	10	9	10	9	9	9	10	9	10	8	8	8		9	9
A_{15}	14	14	14	14	14	14	14	14	14	14	14	14	14	14	15		13	14

Table 6 continued

Group aggregation weighting	GM	AM	Borda	GM	AM	Borda	GM	AM	Borda	GM	AM	Borda	GM	AM	Borda	GM	AM	Borda
	Before			After			Before			After			Before			After		
L_p metric	$p=1$			$p=1$			$p=2$			$p=2$			$p=\infty$			$p=\infty$		
(6b) *By equal weights*																		
Candidates																		
A_1	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	n/a	15	15
A_2	8	8	7	8	8	7	8	8	7	8	8	7	9	9	9		9	9
A_3	12	12	12	12	12	12	12	12	13	12	12	13	13	12	13		12	13
A_4	11	10	11	11	10	11	11	9	11	11	9	11	7	7	7		6	7
A_5	2	2	2	2	2	2	2	2	2	2	2	2	3	2	2		2	2
A_6	5	4	4	5	4	4	5	4	4	5	4	4	5	6	4		5	4
A_7	6	5	5	6	5	5	6	6	5	6	6	5	9	9	9		9	9
A_8	4	3	2	4	3	2	4	3	2	4	3	2	3	2	2		2	2
A_9	10	11	10	10	11	10	10	11	11	10	11	11	7	7	7		6	7
A_{10}	3	6	6	3	6	6	3	5	6	3	5	6	1	5	5		8	5
A_{11}	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1		1	1
A_{12}	14	14	12	14	14	12	14	14	14	14	14	14	14	14	14		14	14
A_{13}	7	7	9	7	7	9	7	7	9	7	7	9	6	4	6		4	5
A_{14}	9	9	8	9	9	8	9	10	8	9	10	8	9	9	9		9	9
A_{15}	13	13	12	13	13	12	13	13	10	13	13	10	12	13	12		13	12
(6c) *By ordinal weights*																		
Candidates																		
A_1	14	13	13	14	13	13	13	11	13	15	13	14	11	11	11	n/a	13	14
A_2	7	6	7	7	6	7	7	5	7	7	6	7	7	5	7		8	8

Table 6 continued

Group aggregation weighting	GM	AM	Borda	GM	AM	Borda	GM	AM	Borda	GM	AM	Borda	GM	AM	Borda	GM	AM	Borda
	Before			After			Before			After			Before			After		
L_p metric	$p = 1$			$p = 1$			$p = 2$			$p = 2$			$p = \infty$			$p = \infty$		
A_3	12	12	12	12	12	12	10	10	12	12	12	12	8	8	8		9	9
A_4	5	3	4	5	3	4	5	3	3	5	3	5	4	3	3		4	4
A_5	3	2	2	3	2	2	3	2	1	3	2	2	3	2	1		2	1
A_6	9	10	8	9	10	8	11	12	9	9	9	8	12	12	12		9	9
A_7	11	11	11	11	11	11	12	13	11	11	11	10	12	12	12		9	9
A_8	4	4	3	4	4	3	4	6	3	4	4	3	6	7	6		3	2
A_9	6	5	4	6	5	4	6	4	3	6	5	3	5	4	5		4	4
A_{10}	2	8	4	2	8	4	2	7	6	2	8	6	1	6	4		7	7
A_{11}	1	1	1	1	1	1	1	1	1	1	1	1	2	1	2		1	3
A_{12}	15	15	13	15	15	13	15	15	14	14	15	12	15	15	14		15	13
A_{13}	8	7	8	8	7	8	8	8	8	8	7	8	8	8	8		6	6
A_{14}	10	9	10	10	9	10	9	9	9	10	10	10	8	8	8		9	9
A_{15}	13	14	15	13	14	15	14	13	14	13	14	14	14	15	15		13	14

All results are under vector normalization

In group aggregation, GM and AM represent the weighted geometric and arithmetic means, respectively, for the separation measures of the two DMs, while EU keeps the weight of 0.8. The Borda function is for external aggregation of group TOPSIS without differentiated decision power

There are three metric parameters for the distance function: $p = 1$, $p = 2$, and $p = \infty$

There are two aspects on weighting the steps: before the PIS/NIS calculation, which is the traditional TOPSIS (Hwang and Yoon 1981), or after PIS/NIS calculation, whose weights are given to the calculation of separation measures (Shih et al. 2007). The results are tabled by three sets of the weights on criteria

"n/a" means no ranks of alternatives are provided by that combination

Table 7 Sensitivity analysis on the differentiated decision power of the two DMs (EU and HR)

	Rank									
Candidates										
A_1	13	13	13	13	13	13	13	13	13	15
A_2	5	5	6	6	7	7	7	7	7	7
A_3	12	12	12	12	12	12	12	12	12	12
A_4	3	3	3	3	3	3	4	5	5	5
A_5	2	2	2	2	2	2	2	2	2	2
A_6	10	10	10	9	9	9	9	9	9	9
A_7	11	11	11	11	11	11	11	11	11	11
A_8	6	6	5	4	4	4	3	3	4	4
A_9	4	4	4	5	5	6	6	6	6	6
A_{10}	8	8	8	8	6	5	5	4	3	3
A_{11}	1	1	1	1	1	1	1	1	1	1
A_{12}	15	15	15	15	15	15	15	15	14	13
A_{13}	7	7	7	7	8	8	8	8	8	8
A_{14}	9	9	9	10	10	10	10	10	10	10
A_{15}	14	14	14	14	14	14	14	14	15	14
Weights										
EU	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50

The results are under vector normalization, ROC weights on the criteria, weighting after PIS/NIS, L_p metric with $p = 2$, and weighted arithmetic mean of EU and HR for differentiated decision power

weighting on the criteria, L_p metric $p = 2$, and vector normalization. Although there are 15 candidates, the ranks of the leading two candidates are unchanged. The case demonstrates that our proposed approach is stable for the problem. In the end, the case company is satisfied with the proposed model for making better judgments without using cardinal scores.

5 Discussions

Based on the analyses of the two examples, we now offer some discussions on ordinal information, score conversion, group aggregation, and decision power, with some limitations.

5.1 Ordinal Information

The ordinal information provided for decision making has no need for assumptions on the equality of the ordinal intervals or levels and the units of measurement (Coombs 1950). The processed information can include some defined categories in a partial order set for the performance measure. We think the proposed model is rather flexible as shown in Example 1 with ordinal data of three levels and five levels. The settings of different levels do not also generate distinguishable rank results in the example.

One might argue why not just directly transfer the ordinal information into cardinal information as in Hwang and Yoon (1981). In fact, such an assignment would incur a subjective measure in which the values and their intervals might not be represented very well. It seems that they force partial information to be complete before the TOPSIS process. This transformation could be distorted due to a scaling effect or inappropriate assignment. In addition, such a subjective measure should be serious in a group decision making environment due to the multiple heterogeneous preferences. Different attitudes of DMs, e.g., conservative or liberal, will make the subjective measure more complicated. It is worth it to make direct use of ordinal information at the cost of extra computation.

The concept of approximating weights based on ranks is an attempt to divert the difficulties of weight elicitation in MCDM problems. The basis in using such weights is that DMs can differentiate among rank positions by reflecting their preferences (Ahn and Choi 2012), and the ordinal weights are applicable after the establishment of these weak preference relations on the criteria. For various ordinal weights, Barron and Barrett (1996) examined four approximate rank-order weights and found that ROC weights are the most accurate. Roberts and Goodwin (2002) classified weight elicitation methods as three categories—direct rating, point allocation, and ranking—and pointed out that ROC weights are appropriate to use as a substitute for point allocation methods, but the ROC weights do not provide the best approximation to the original weights in direct rating methods. On this point, they proposed rank order distribution (ROD) weights for a better approximation, and ROD weights will be close to the rank sum weights as the numbers of criteria increase. Ahn (2011) also confirmed that ROC weights result in the highest performance in MCDM evaluation. Though there are some debates on which one is the best, we divert the debates by executing extra sensitivity analyses on ROC weights to show the stability on rankings of our proposed model.

5.2 Score Conversion

In Sect. 3, Eqs. (7) and (8) are derived for converting an ordinal performance measure into a cardinal performance measure based on the frequency of dominating and dominated cases of any qualitative criterion by pairwise comparison among alternatives. It is common to count the frequency of the superior and inferior situations as EVAMIX. EVAMIX utilizes two weighting steps: the first one posts criterion weights into dominating and dominated cases for obtaining dominance scores; and the second one uses the weights of qualitative and quantitative criteria to combine the corresponding standardized dominance scores into the total standardized dominance scores for the final ranking. On the other hand, the core of TOPSIS is to count PIS/NIS by a distance function. We try to keep the core in the TOPSIS extension so that the conversion process on an ordinal measure is rather simple. Following the TOPSIS procedure, Eq. (8) can be thought of a vector normalization process. After the converted data of qualitative criteria have the same style as that of the quantitative criteria with vector normalization, PIS/NIS becomes obtainable for further use.

5.3 Group Aggregation

When using MCDM techniques in group decision making, there will be an extra consideration on how to aggregate the group preference. It is assumed that the group can achieve an agreement or work in a cooperative environment so that a consensus can be reached (Bezerra et al. 2014). This study takes advantage of Baucells and Sarin's work (2003) for the development, and some common operators can be employed. The operator of arithmetic mean has a sense of an equal distance to all members in the group, whereas the operator of geometric mean considers an equal ratio to all members (Medhi 2006). The latter is a better fit to normalized results and is especially good for AHP (Forman and Peniwati 1998). After checking the results of both examples, we perceive that rank ties frequently occur by external aggregation of the Borda function. It seems that the discrimination ability of the external aggregation in TOPSIS is questionable if we want to have a total differentiated rank. This can be considered as one advantage for internal aggregation of TOPSIS in our proposed model. Though the geometric mean is rather common for the aggregation in AHP (Ramanathan and Ganesh 1994), it appears to not work well for the case with full ordinal performance measures, because some measures are easy to reach the points of PIS or NIS. Thus, the generated zero value in the denominator undermines the relative closeness.

There are various issues on aggregating individual preferences from a technical aspect. We do not provide an operation that is involved in the preference variations among a large number of group members, as it is difficult to reach an agreement. Please check the contents of Huang and Li (2012) for details.

5.4 Decision Power

There indeed exists differentiated decision power in the real world, such as business and political organizations, yet formulating the power with differentiation is a big issue. The power is usually implicitly expressed in a group with many types (Griffin and Moorhead 2013). Since there is no explicit clue from the area of organizational behavior, we refer to Ramanathan and Ganesh's work (1994) and extend it to the group utility function of Baucells and Sarin (2003) to aggregate individuals' utility functions with differentiated decision power. Other elegant ways to formulate the decision power could be left for future study. In addition, it is possible that there exist conflicts or regrets regarding decision power, meaning a consensus cannot be reached. Extra work is needed for consensus facilitation.

Ramanathan and Ganesh (1994) also indicated two approaches for assigning weights of the decision power. In our case study, the participatory approach to determine weights is feasible by interpersonal pairwise comparison or discussions among the group members. The features of the job statement for the candidate position can be thought of a basis for predetermining the weights of the power among different units. It is worth noting that social choice functions, e.g., the Borda function, are usually not involved in differentiated decision power.

For the personnel recruitment problem, HR keeps a veto power on the final decision if the target candidate has any unethical or unacceptable behavior in the past. If his or her bad record cannot be found in the screening process, then HR will demonstrate its veto power later, even during the probation period or later on. This situation cannot be covered in our model. Moreover, for the selection of senior managers the reference check plays a vital role on the final decision. That means that our model can be quite applicable to junior positions, whose decisions heavily rely on candidates' performances.

6 Concluding Remarks

This study has proposed a decision support model to manage the mixed-data model with differentiated decision power for group TOPSIS. We integrate ordinal and cardinal information with differentiated weights into the traditional TOPSIS steps based on DMs' distance-based utilities. The suggested model maintains a straightforward procedure like the original TOPSIS does. The illustrated examples demonstrate that our approach is useful in a cooperative group decision making environment. From the viewpoint of group aggregation, the proposed model with internal aggregation has an edge over the Borda function for external aggregation in group TOPSIS, achieving better discrimination ability. Sensitivity analysis also demonstrates the robustness of the model on alternative ranks.

After testing 54 combinations of different parameters on three weight sets on the criteria, group aggregation, two different weighting steps, and three L_p metric parameters for group TOPSIS, the fighter selection example performs quite stable with two sets of rankings, but the personnel recruitment case with $p = \infty$ for the L_p metric gives slight different rank results when dealing with full ordinal performance information. In addition, the recruitment case with full ordinal data shows us that aggregating group preferences by geometric mean seems inadequate, because it has little ability to process a zero value under metric $p = \infty$. Though there exists a slight distortion on using vector normalization of TOPSIS (Kaliszewski et al. 2014), we do not notice its effect on the final ranks.

The core concept of the proposed decision model with mixed data and differentiated decision power can be extended to other cardinal MCDM techniques in a group decision environment. In this study we have not dealt with stochastic dominance in group TOPSIS or with incomplete decision information. We also do not take into account the heterogeneous preferences for a large number of DMs as well as conflicts or regrets regarding decision power or weights on the criteria. These types of problems could be left for future study.

Acknowledgments The author wish to thank the editor and the anonymous reviewers for their constructive comments that have improved the quality of this paper. The author is also thankful to Li-Teng Chiang at Lite-On Semiconductor Corporation for his help in the development of the illustrative case, and appreciate David L. Olson, Dmitry Podkopaev, Ignacy Kaliszewski, and Hai-Ming Chen for their valuable help to the study. Special thanks are given to Minister of Science and Technology, Taiwan, R.O.C. for the financial support under grant numbers: NSC 101-2221-E-032-019 and NSC 101-2911-I-032-501.

Appendix: Comparison Results of the Fighter Selection Problem with Ordinal Information

Example 3 A fighter selection problem (Hwang and Yoon 1981).

Table 8 Comparison of the ranks for the fighter selection example from different combinations under vector normalization

TOPSIS	Traditional	Proposed	Traditional	Proposed	Traditional	Proposed	Traditional	Proposed	Traditional	Proposed	Traditional	Proposed	EVAMIX
Weighting	Before		After		Before		After		Before		After		
Metric	$p = 1$		$p = 1$		$p = 2$		$p = 2$		$p = \infty$		$p = \infty$		
(8a) *By given weights*													
Alternatives													
A_1	2	1	1	1	1	1	2	1	1	1	1	1	4
A_2	4	4	3	4	4	4	3	4	4	4	4	4	2
A_3	1	2	2	2	2	2	1	2	2	2	2	2	1
A_4	3	3	4	3	3	3	4	3	3	3	3	3	3
(8b) *By equal weights*													
Alternatives													
A_1	2	2	2	2	2	2	2	2	2	2	2	2	4
A_2	4	4	3	4	4	4	3	4	4	4	4	4	2
A_3	1	1	1	1	1	1	1	1	1	1	1	1	1
A_4	3	3	4	3	3	3	4	3	3	3	3	3	3
(8c) *By ordinal weights*													
Alternatives													
A_1	1	1	1	1	1	1	1	1	1	1	1	1	2
A_2	4	4	4	4	4	4	4	4	4	4	4	4	3
A_3	2	2	2	2	2	2	2	2	2	2	2	2	1
A_4	3	3	3	3	3	3	3	3	3	3	3	3	4

There are comparisons between the traditional TOPSIS procedure (Hwang and Yoon 1981) and the proposed TOPSIS with ordinal information.
There are two weighting steps: before the PIS/NIS calculation (marked "before") and after PIS/NIS calculation (marked "after")
There are three metric parameters: $p = 1$, $p = 2$, and $p = \infty$
The column marked by green background is the ranks by the traditional TOPSIS procedure of Hwang and Yoon (1981)

Four alternatives are evaluated by six criteria for the selection problem and the decision information is illustrated in Table 2 with partial information. Please delete the left two columns on the subjective criteria of DM #2 for this example with a single DM. The analysis is based on four combinations. The first one is related to traditional TOPSIS steps, converting ordinal performances by pre-determined 0–10 scales, and to our proposed steps, comparing ordinal information by different levels using Eqs. (4)–(8). The second one deals with two weighting processes that are on the normalized decision matrices (marked "before" whose weights are used before PIS/NIS calculation as in traditional TOPSIS) or the separation measures (marked "after" whose weights are combined with the separation measures, after PIS/NIS calculation, as detailed in Step 5a of Shih et al. (2007). The third one includes three common Minkowski's L_p metric parameters, i.e., $p = 1$, $p = 2$, and $p = \infty$. The fourth one concerns three weight sets: the given weights, equal weights, and ordinal weights. Table 8 lists the rank results, which are separated by three sub-tables with different weight sets. All information in the table is derived by vector normalization on performance measures from Table 2. The last column of Table 8 gives the rank results by EVAMIX for the purpose of comparison.

According to the results, we first observe that the ranks of the four alternatives by our approach for ordinal information are rather stable compared to traditional TOPSIS. We believe that the proposed pairwise comparison provides better discriminating ability than traditional TOPSIS. Second, the weighting step on the normalized decision matrices or on the separation measures does not have much effect on the proposed approach; however, it has a slight effect on the traditional approach. Third, the difference L_p metric parameters also have a slight effect on the traditional approach, but not much impact on the proposed approach. Fourth, the ordinal weight set provides the most stable ranks, confirming the results of Barron and Barrett (1996). Compared to the ranks from EVAMIX, TOPSIS provides more similar ranks under the given weight set.

References

Alfares HK, Duffuaa SO (2009) Assigning cardinal weights in multi-criteria decision making based on ordinal ranking. J Multi-criteria Decis Anal 15:125–133

Ahn BS (2011) Compatible weighting method with rank order centroid: maximum entropy ordered weighted averaging approach. Eur J Oper Res 212:552–559

Ahn BS, Choi SH (2012) Aggregation of ordinal data using ordered weighted averaging operator weights. Ann Oper Res 201:1–16

Arrow KJ (1963) Social choice and individual values. Wiley, New York

Bana e Costa C, Chagas MP (2004) A career choice problem: an example of how to use MACBETH to build a quantitative value model based on qualitative value judgments. Eur J Oper Res 153:323–331

Barron FH, Barrett BE (1996) Decision quality using ranked attribute weights. Manag Sci 42(11):1515–1523

Barzilai J, Lootsma FA (1997) Power relations and group aggregation in the multiplicative AHP and SMART. J Multi-criteria Decis Anal 6:155–165

Baucells M, Sarin RK (2003) Group decisions with multiple criteria. Manag Sci 49(8):1105–1118

Behzadian M, Otaghsara SK, Yazdani M, Ignatius J (2012) A state-of-the-art survey of TOPSIS. Expert Syst Appl 39:13051–13069

Belenson SM, Kapur KC (1973) An algorithm for solving multicriterion linear programming problems with examples. Oper Res Q 24(1):65–77

Bernasconi M, Choirat C, Seri R (2014) Empirical properties of group preference aggregation methods employed in AHP: theory and evidence. Eur J Oper Res 232:584–592
Bezerra F, Melo P, Costa JP (2014) Visual and interactive comparative analysis of individual opinions: a group decision support tool. Group Decis Negot 23:101–125
Coombs CH (1950) Psychological scaling without a unit of measurement. Psychol Rev 57(3):145–158
Eckenrode RT (1965) Weighting multiple criteria. Manag Sci 12(3):180–192
Edwards W (1977) How to use multiattribute utility measurement for social decision making. IEEE Trans Syst Man Cybern 7(5):326–340
Edwards W, Barron FH (1994) Numerical vs cardinal measurements in multiattribute decision making: how exact is enough? Organ Behav Hum Decis Process 60:306–325
Forman E, Peniwati K (1998) Aggregating individual judgments and priorities with the analytic hierarchy process. Eur J Oper Res 108:165–169
Garcia F, Guijarro F, Moya I (2010) A goal programming approach to estimating performance weights for ranking firms. Comput Oper Res 37(9):1597–1609
Goh CH, Tung YCA, Cheng CH (1996) A revised weighted sum decision model for robot selection. Comput Ind Eng 30(2):193–199
Griffin RW, Moorhead G (2013) Organ Behav, 11th edn. South-Western, Mason
Guitouni A, Martel J (2006) Tentative guidelines to help choosing an appropriate MCDA method. Eur J Oper Res 109:501–521
Hinloopen E, Nijkamp P, Rietveld P (1983) The regime method: a new multicriteria technique. In: Hansen P (ed) Essay and surveys on multiple criteria decision making. LNEMS 209. Springer, Berlin, pp 146–155
Horsky D, Rao MR (1984) Estimation of attribute weights from preference comparison. Manag Sci 30(7):801–822
Hung CC, Chen LH (2010) A fuzzy TOPSIS decision making model with entropy weight under intuitionistic fuzzy environment. Lect Notes Elect Eng 52:17–26
Huang YS, Li WH (2012) A study on aggregation of TOPSIS ideal solutions for group decision-making. Group Decis Negot 21(4):461–473
Hwang CL, Lin MJ (1987) Group decision making. Springer, Berlin
Hwang CL, Yoon K (1981) Multiple attribute decision making. Springer, Berlin
Jones DF, Mardle SJ (2004) A distance-metric methodology for the derivation of weights from a pairwise comparison matrix. J Oper Res Soc 55:869–875
Kaliszewski IS, Podkopaev D, Shih HS (2014) TOPSIS demistifed. Research report: RB/13/2014, Systems Research Institute, Polish Academy of Sciences, Warsaw, Poland
Kirkwood CW, Sarin RK (1985) Ranking with partial information: a method and a application. Oper Res 33(1):38–48
Korhonen P (1997) Comments on Barzilai and Lootsma. J Multi-criteria Decis Anal 6:167–168
Lahdelma R, Miettinen K, Salminen P (2003) Ordinal criteria in stochastic multicriteria acceptability analysis (SMAA). Eur J Oper Res 147:117–127
Langroudi MZA, Emrouznejad A, Mustafa A, Ignatius J (2013) Type-2 TOPSIS: a group decision problem when ideal values are not extreme endpoints. Group Decis Negot 22(5):851–866
Larichev OI (1992) Cognitive validity in design of decision-aiding techniques. J Multi-criteria Decis Anal 1(3):127–138
Larichev OI, Olson DL, Moshkovich HM, Mechitov AJ (1995) Numerical vs cardinal measurements in multiattribute decision making: how exact is enough? Organ Behav Hum Decis Process 64(1):9–21
Medhi J (2006) Statistical methods: an introductory text, 2nd edn. New Age International, New Delhi
Olson DL, Dorai VK (1992) Implementation of the centroid method of Solymosi and Dombi. Eur J Oper Res 60:117–129
Parkan C, Wu ML (1997) On the equivalence of operational performance measurement and multiple attribute decision making. Int J Prod Res 35(11):2963–2988
Ramanathan R, Ganesh LS (1994) Group preference aggregation methods employing in AHP: an evaluation and an intrinsic process for deriving members' weightages. Eur J Oper Res 79:249–265
Rao RV (2006) Machinability evaluation of work materials using a combined multiple attribute decision-making method. Int J Adv Manufact Technol 28:221–227
Roberts R, Goodwin P (2002) Weight approximations in multi-attribute decision models. J Multi-criteria Anal 11:291–303
Saaty TL (1980) The analytic hierarchy process. McGraw Hill, New York

Shih HS, Shyur HJ, Lee ES (2007) An extension of TOPSIS for group decision making. Math Comput Model 45:801–813

Shih HS, Wang CH, Lee ES (2004) A multi-attribute GDSS for aiding problem-solving. Math Comput Model 39(11–12):1397–1412

Shyur HJ, Shih HS (2006) A hybrid MCDM model for strategic vendor selection. Math Comput Model 44(7–8):749–761

Torfi F, Farahani RZ, Rezapour S (2010) Fuzzy AHP to determine the relative weights of evaluation criteria and Fuzzy TOPSIS to rank the alternatives. Appl Soft Comput 10(2):520–528

Tsaur RC (2011) Decision risk analysis for an interval TOPSIS method. Appl Math Comput 218:4295–4304

Tsui CW, Wen UP (2014) A hybrid multiple criteria group decision-making approach for green supplier selection in the TFT-LCD industry. Mathematical Problems in Engineering Volume 2014, Article ID 709872, 13 pp

Van Delft A, Nijkamp P (1977) Multi-criteria analysis and regional decision-making. Martinus Nijhoff, Hague

Van den Honert RC (2001) Decision power in group decision making: a note on the allocation of group members' weights in the multiplicative AHP and SMART. Group Decis Negot 10:275–286

Voogd H (1982) Multicriteria evaluation with mixed qualitative and quantitative data. Environ Plan B 9:221–236

Voogd H (1983) Multicriteria evaluation for urban and regional planning. Pion, London

Widerén M (1994) Voting power in the EC decision making and the consequences of two different enlargements. Eur Econ Rev 38:1153–1170

Xu X, Lamond BF (2001) A multiple criteria ranking procedure based on distance between partial preorders. Eur J Oper Res 133:69–80

Xu Y, Wang H, Sun H, Yu D (2014) A distance-based aggregation approach for group decision making with interval preference orderings. Comput Ind Eng 72:178–186

Yue Z (2013) An avoiding information loss approach to group decision making. Appl Math Model 37:112–126

Zeleny M (1974) A concept of compromise solutions and the method of the displaced ideal. Comput Oper Res 1:479–496