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Market Collective Wisdom Discovery for Portfolio Investments

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Abstract

The goal of numerous investing strategies, as opposed to hedging strategies, is "to beat the market", i.e. to secure returns higher than those guaranteed by tracking market indices. In order to achieve this goal, one needs to identify key factors which drive markets and cause security prices to fluctuate.

We assume that distinctive key market factors exist, though it is not known how such factors correlate and aggregate, and eventually push a market from one quotation to another. In other words, we purport that at a given time there is the collective wisdom in a market which shapes the collective investment pattern for the future. We engage ourselves to reverse engineer that wisdom. Specifically, we attempt to reverse engineer it from market returns (which we interpret as collective market wisdom embodiment) with the use of the notions of *vectors of concessions and compromise half lines*, recently introduced into Multiple Criteria Decision Analysis. We illustrate our approach with preliminary calculations for selecting portfolios of international investment funds.

Keywords: Multiple criteria decision making, investment portfolio, knowledge discovery.

1. Introduction

A portfolio selection model was first proposed in the 1952 by Harry Markowitz [20]. The problem was formulated as a bi-criteria optimization problem, with the expected rate of return and risk as conflicting criteria. A survey on the Markowitz model and its modifications was given in [30] and the fifty-year retrospective of this model was presented by Mark Rubenstein in [25].

On the basis of the approach developed by Markowitz, a number of authors independently proposed the Capital Asset Pricing Model (CAPM) to estimate the relationship between the expected return and risk of individual assets [19, 22, 27, 32]. Since that time, the appropriateness of CAPM has been discussed in numerous publications. Only as late as in 2004 it was shown that the original CAPM cannot correctly capture correlation between risk and the expected return, which put the question mark on its practical applicability [9]. An extension of the original CAPM model, known as the Black CAPM, was proposed in [32] under assumption of the absence of risk-free assets. In [27], some sort of security analysis was used to improve portfolio selection in Black CAPM.

Despite many publications on the Markowitz model and CAPM (forming together what is known under the name of the Modern Portfolio Theory), there were numerous publications attempting to extend these original concepts. From these attempts the Post-Modern Portfolio Theory emerged, and this term was first used in [24]. The most distinctive element of the latter theory were new risk measures, such as the Sortino index [28, 29] or the Omega measure [16], proposed as possible replacements for the Sharpe index. Effectiveness of the Sortino index was confirmed in [5].

Since the publication of the original work by Markowitz, hundreds of papers related to portfolio selection were published. Papers [17, 23] are examples of still active research in this field. An alternative non-classical approach to portfolio selection made use of the Bayesian analysis [19], whereas in [22] an approach based on fuzzy probabilities was proposed. Some recent works propose to use metaheuristic algorithms for portfolio selection, such as immune algorithms [9] or genetic programming [3].

In this paper we deal specifically with portfolios composed of international investment funds (IIF). Investing in IIF relieves investors from explicit concerns about portfolio diversification. Recent approaches related to various aspects of international diversification can be found e.g. in [11, 37].

By a market we understand any IIFs trading system. However, the development we present here applies to any market tradable assets.

The main idea which underlines our considerations is our conviction that in a market there is some sort of collective wisdom in period $T-1$ which shapes the collective investment pattern for period T. That collective investment pattern is not directly observable but only a posteriori, as asset (here: IIF) market valuations.

To discover the market collective wisdom we assume that to evaluate IIFs the *market* (here understood as an aggregate of all investors) considers a set of selected indicators. We call that selection *key market indicators* (KMIs). On the base of assessments of KMIs values, the market "invests" and the result of such an investment are IIF market valuations, represented by rates of return. Our aim is to reverse engineer this process. Specifically, we aim at identifying a model which would relate key market indicators in period $T-1$ to IIF rates of return in period T. With such a model at hand and assuming stationary conditions on the market, we would be in a position to invest in period T in the IIF yielding high returns in $T+1$.

The outline of the paper is as follows. In Section 2 we describe the main development, whereas in Section 3 we present the results of numerical experiments. Section 4 concludes.

2. Market Collective Wisdom Discovery

2.1. Preliminaries and Notation

Market indices available to investors are meant to help analyzing IIFs ability to yield profit. To avoid redundancy, from a variety of indices available for investors, it is rational to focus on subsets of indices of low correlation. We call indices of such subsets *key market indicators* (KMIs). We attribute to KMIs the role of key market driving forces. The process of selecting KMIs is described in Appendix 1.

We assume that all the investors while making investment decisions, evaluate IIFs traded on the market by the same set of k KMIs. To simplify the presentation, we also assume that the higher is the KMI value, the more attractive is the IIFs from investor's point of view (we transform indicators of the type "the less, the better" to the former type by multiplying their values by -1).

IIFs with their evaluations by KMIs can be represented by *a decision table* $Y = \{y^i\}$, $i = 1, \dots, m$, where $y^i = (y_1^i, \dots, y_k^i)$ corresponds to the i-th IIF denoted IIF^i, and y_l^i is the value of its l-th KMI, $l = 1, \dots, k$. The k-dimensional space spanned by KMIs is called the *outcome space*.

Given IIfs and the decision table, the information which is immediately derivable from it is the *ideal element* $\hat{y}$ of the outcome space, which is defined as

$$\hat{y}_l = \max_{i=1,\dots,m} y_l^i, \quad l = 1, \dots, k. \tag{2.1}$$

Clearly, all IIFs satisfy the condition $y_l^i \le \hat{y}_l$, $l = 1, \dots, k$. In the outcome space, any IIFs can be reached from $\hat{y}$ along a *compromise half line*

$$\theta = \{y \in \mathbb{R}^k \mid y = \hat{y} - t\tau, t \ge 0\}, \tag{2.2}$$

where τ is the *vector of concessions* [13]$-$[15]. Components of τ represent proportions of concessions which are made on KMIs values when replacing unattainable (in general) $\hat{y}$ by y^j.

Below, for technical reasons, instead of element $\hat{y}$ we make use of element y^* defined as $y_i^* = \hat{y}_i + \varepsilon$, $\varepsilon > 0$, $l = 1, \dots, k$. For small values of ε, the decision analysis considerations presented below produce the same results irrespective of use of $\hat{y}$ or y^*.

The function

$$\min_{i=1,\dots,m} \max_l \lambda_l (y_l^* - y_l^i), \tag{2.3}$$

where $\lambda_l > 0$, $l = 1, \dots, k$, called the Tchebycheff function, is widely used in Multiple Criteria Decision Making as an effective engine[1] for derivation of (weakly) efficient IIFs [8], [13]$-$[15], [21]. For the definition of efficiency see the next subsection. Moreover, the Tchebycheff function is also often used to provide rankings of variants [6, 34]. In addition to some interesting technical features exploited below, the most distinctive feature of the Tchebycheff function is that it is noncompensatory, i.e. the value of one component is not compensable by a value of another one.

[1]By an engine we mean a parametrized optimization problem. For each instance of parameters, solving the optimization problem selects an efficient IIF. As the set of IIfs considered for investments is never very large, efficient IIF can be identified by enumeration. However, in some portfolio models, as e.g. the Markowitz model, the set of portfolios is infinite and therefore one has to resort to optimization engines as (2.3). To ensure a uniform interpretation for any case, even in cases where derivation of efficient assets or efficient portfolios of assets becomes trivial, we also make use of (2.3) as the IIF selecting mechanism.

Below, we use of the Tchebycheff function to study the process of ranking IIFs by investors.

2.2. Regressing variants on compromise half lines

Let a compromise half line Θ, defined as in (2.2) be given. If λ_l are such that

$$\lambda_l = \frac{1}{\tau_l}, \qquad l = 1, \ldots, k, \tag{2.4}$$

then the contours of the Tchebycheff function (2.3) are as in Figure 1 [12, 13]. In particular, if $\tau_l = y_l^* - y_l^u$, $l = 1, \ldots, k$, for some y^u, then y^u is at the apex of a contour of the Tchebycheff function with λ_l, $l = 1, \ldots, k$, defined as in (2.4). Those y^i which are off the compromise half line are *regressed* by the Tchebycheff function (in the sense: *projected*) on the compromise half line. This can be interpreted as a specific *regression*, with the Tchebycheff function and the regression line defined by λ and y^*. The order on the compromise half line of regressed y^i corresponds to the ranking by the values of the Tchebycheff function.

We recall now the definition of the dominance relation. We put it in the form specific for the context of IIFs.

Dominance relation $\prec$ on Y is defined as follows: i-th IIF $\prec$ j-th IIF $\Leftrightarrow$ $y^i \ll y^j$, where $\ll$ denotes $y_l^i \leq y_l^j$, $l = 1, \ldots, k$, and $y_l^i < y_l^j$ for at least one l.

IIF^i for which there exists no other IIF^j such that $y^i \prec y^j$, i.e. no other IIF dominates it, is called *efficient*.

If y^u is efficient, the value of the Tchebycheff function for y^u, where $\lambda_l = \tau_l^{-1} = (y_l^* - \hat{y}_l)^{-1}$, $k = 1, \ldots, l$, is minimal over $\{y^1, \ldots, y^m\}$ (and equals to 1), see [8, 13, 21].

From the infinite number of possible regressions of that sort (there is the infinite number of λ), we are interested only in those ones which place an indicated IIF high in the rankings.

Lemma 1. *IIF^i gets the highest rank in rankings by the decreasing Tchebycheff function values when the compromise half line passes through y^i.*

For the proof see Appendix 2.

2.3. Market wisdom discovery and market wisdom driven investment

Markets are not egalitarian, in the sense that gains are not the same for every investor. Those who invest wisely receive higher returns.

In period T it is immediate to identify which investment in period $T-1$ yielded the highest return in T. We denote such IIF by y^H. With that IIF we construct the compromise half line such that y^H is on that line, namely

$$\theta^H = \{y \in \mathbb{R}^k \mid y = \hat{y} - t\tau,\ t \geq 0\}, \tag{2.5}$$

where $\tau_l = y_l^* - y_l^H$, $l = 1, \ldots, k$.

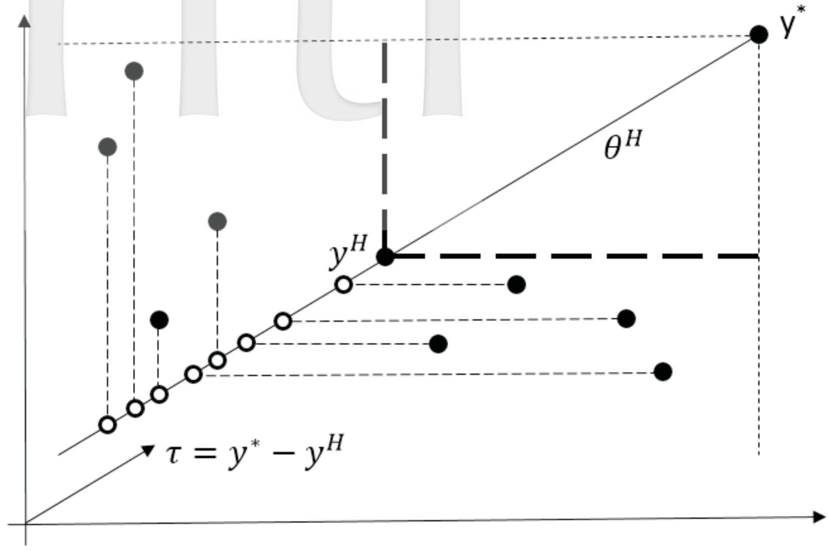

Figure 1: Compromise half line θ^H defined by IIF$^{\rm H}$ and regressions of IIFs on θ^H by the Tchebycheff function; the case of two KMIs.

It is rational to expect that in period $T-1$, IIF$^{\rm H}$ had attractive (high) values of KMIs. This does not necessarily mean that IIFH was efficient in period $T-1$, i.e there could be another IIF which in period $T-1$ had higher values of KMIs. Nonetheless, values of KMIs for y^H were in $T-1$ attractive enough for the market wisdom to make IIF$^{\rm H}$ the best rate of return yielder. In other words, in period $T-1$ the market decreed a specific aggregation of KMI values of y^H to made it the winner in period T. We hypothesise that the aggregation takes the form of the Tchebycheff function with λ as defined by τ discovered by the construction of θ^H in (2.5).

Under the already assumed stationarity conditions on the market, we used the discovered aggregation to invest into a portfolio constructed of n funds in period T, in order to profit in period $T+1$. We call the resulting portfolio the Market Wise (MW) portfolio. The procedure of selecting this portfolio is formalized as Algorithm 1.

Algorithm 1. Selecting MW portfolio for investing in period $T+1$

(1) **for** all l, $l = 1, \ldots, k$, **do**
  (a) calculate $\hat{y}_l = \max_i y_l^i$.

(2) **for** all l, $l = 1, \ldots, k$, **do**
  (a) $y_l^* := \hat{y}_l + \varepsilon$, $l = 1, \ldots, k$.

(3) Select IIF$^{\rm H}$, i.e. the IIF with the maximal rate of return in investing period T.

(4) **for** all l, $l = 1, \ldots, k$, **do**

(a) calculate $\lambda_l^H = \frac{1}{y_l^* - y_l^H}$.

(5) **for** for all i, $i = 1, \dots, m$, **do**

(a) calculate $\max_l \lambda_l^H (y_l^* - y_l^i)$ (the Tchebycheff function value).

(6) Select first n IIFs from all IIFs sorted in the increasing order of the Tchebycheff function value.

3. Numerical Experiments

We conducted numerical tests of performance of MW portfolios on the data for the set of funds presented in Appendix 1. Each portfolio was built for 1-month investment in the investing periods from May 2014 till April 2015 (12 months). We compared performance of MW portfolios against two other type portfolios. The first ones, denoted TOP, consisted of n first IIFs ranked in the decreasing order by their rates of return. The second ones, denoted M, consisted of all m IIFs.

In all three portfolios, MW, TOP and M, the share of IIFs selected for the portfolio was the same. The rates of return of portfolios were calculated as the simple averages of the rates of returns of individual IIFs. In all cases and for the whole investment period, the set of KMI was the same.

We experimented with portfolios for $n = 5$, $n = 10$ and $n = 15$. In all cases the short term portfolio return measure has been monthly rate of return (the collective market wisdom embodiment). The following two sets of KMIs have been selected.

For the experiment 1:

- rate of return 3M,
- rate of return 6M,
- rate of return 1Y,
- Sharpe index.

For the experiment 2:

- rate of return 3M,
- rate of return 6M,
- rate of return 1Y,
- rate of return *Till Now*,
- Sharpe index.

The rationale for such KMIs selections is given in Appendix 1.

Experiment 1. The results are presented graphically in Figure 2 and in tabular form in Table 1. The best values in the table for each month are set in bold.

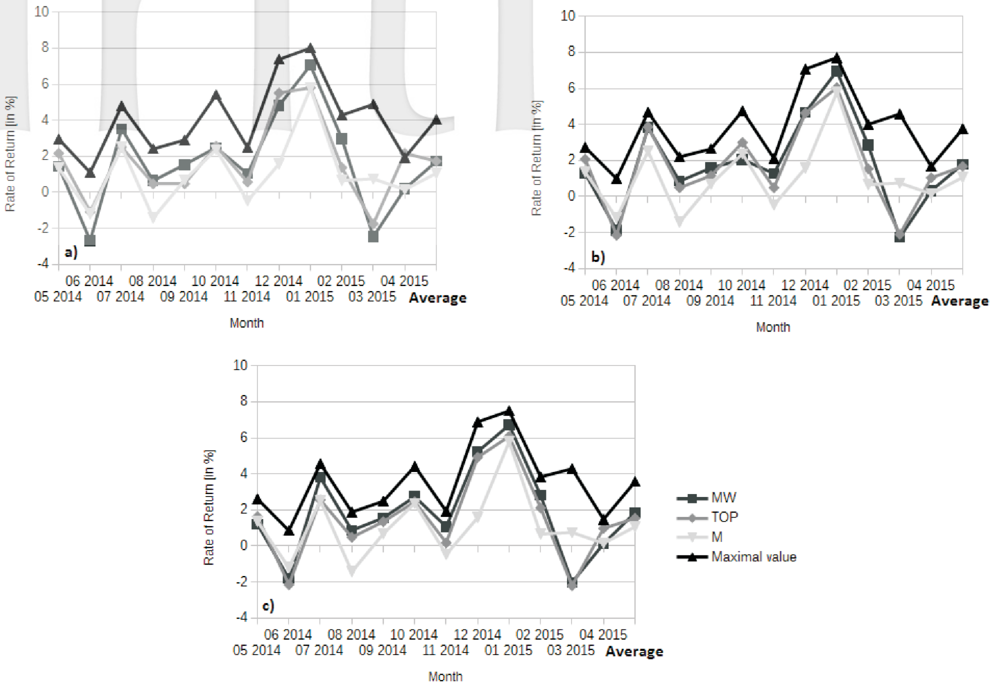

Figure 2: Portfolio yields: (a) $n = 5$, (b) $n = 10$, (c) $n = 15$.

For $n = 5$ and $n = 10$, MW portfolio outperformed TOP and M portfolios in 6 out of 12 months; and for $n = 15$, better results were achieved in the case of 8 months. Moreover, in two out of three cases MW portfolios outperformed TOP and M portfolios in rates of return averaged over the whole period of 12 months, with the following average rates of return for MW: 1.80 for $n = 10$ and 1.86 for $n = 15$. Results obtained for MW $n = 5$ where only slightly worse than TOP portfolio and were equal to 1.72.

As a reference, for each months of the investing period we selected n IIFs which yielded the highest rate of return. Those values are represented in Figure 2 as a "maximal value" and may be described as the maximal possible rate of return from n IIFs to achieve in a given month. The space between the interpolated reference rates of return and the rates of return of MW, TOP and M portfolios shows the "playground", i.e. the margin where possible improvements in portfolio selection could be found if tested in the same manner as we did.

Portfolios TOP clearly performed as worst. This can be attributed to the high variability of the sole KMI used to construct them, namely the rate of return. The high variability of the rate of return for IIFs and the investing period is illustrated in Figure 3 which shows monthly rates of return for 5 randomly selected funds. In Figure 3 no trend can be observed. This clearly shows that the market wisdom builds on more than one KMI aggregated by some aggregating structures. One of such possible aggregating structure is that one proposed in this work.

Table 1: Experiment 1 rate of return in every month (in percent).

	5 funds			10 funds			15 funds		
	MW	TOP	M	MW	TOP	M	MW	TOP	M
05 2014	1.43	**2.17**	1.36	1.29	**2.08**	1.36	1.21	**1.64**	1.36
06 2014	-2.68	**-1.08**	-1.2	-1.87	-2.15	**-1.2**	-1.81	-2.15	**-1.2**
07 2014	**3.53**	2.52	2.52	3.83	**3.87**	2.52	**3.8**	2.56	2.52
08 2014	**0.65**	0.47	-1.43	**0.85**	0.49	-1.43	**0.87**	0.48	-1.43
09 2014	**1.54**	0.47	0.68	**1.59**	1.16	0.68	**1.56**	1.35	0.68
10 2014	2.49	**2.57**	2.38	2.06	**3.0**	2.38	**2.74**	2.46	2.38
11 2014	**1.03**	0.57	-0.47	**1.28**	0.49	-0.47	**1.07**	0.18	-0.47
12 2014	4.82	**5.52**	1.58	**4.68**	4.64	1.58	**5.24**	4.93	1.58
01 2015	**7.06**	5.81	5.79	**6.96**	6.05	5.79	**6.72**	6.09	5.79
02 2015	**2.97**	1.38	0.66	**2.85**	1.54	0.66	**2.82**	2.12	0.66
03 2015	-2.46	-1.75	**0.75**	-2.27	-2.1	**0.75**	-2.02	-2.21	**0.75**
04 2015	0.2	**2.17**	0.15	0.32	**1.04**	0.15	0.12	**0.98**	0.15
Average	*1.72*	***1.74***	*1.06*	*1.8*	*1.68*	*1.06*	***1.86***	*1.54*	*1.06*

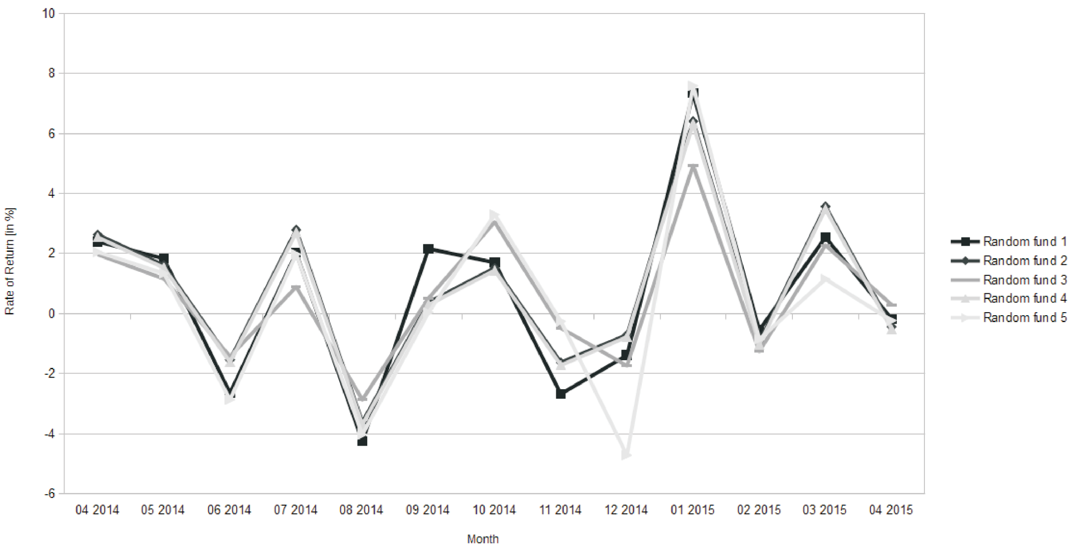

Figure 3: Experiment 1 — rate of return over 1 month for five randomly selected funds.

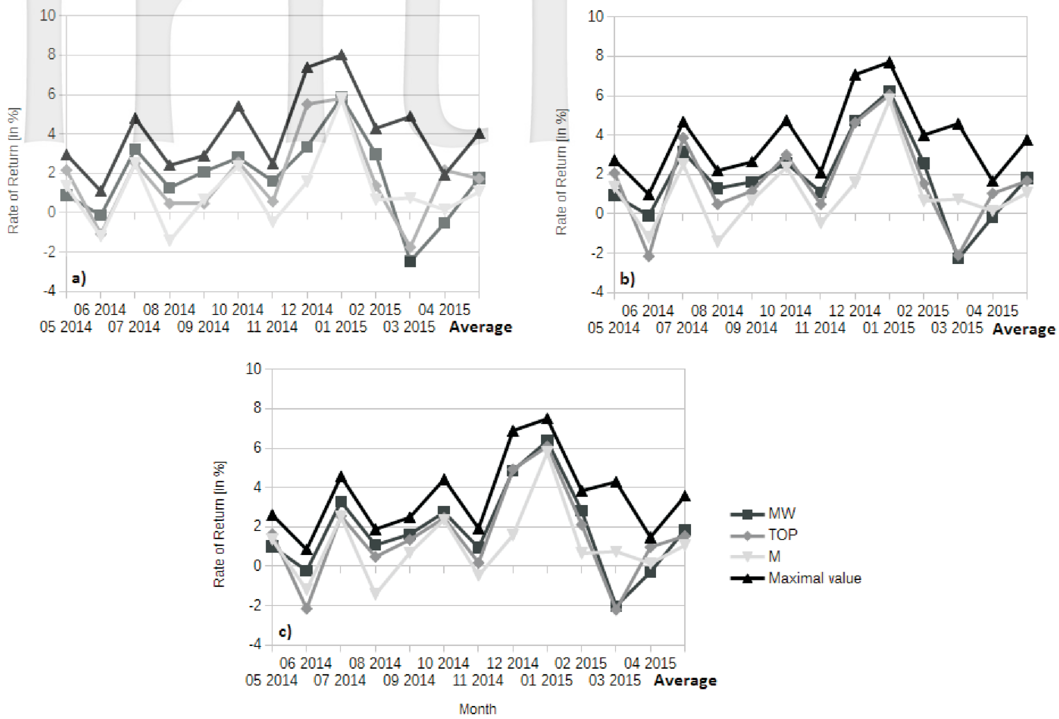

Figure 4: Portfolio yields: (a) $n = 5$, (b) $n = 10$, (c) $n = 15$.

Experiment 2. The results are presented graphically in Figure 4 and in tabular form in Table 2. The best values in the table for each month are set in bold.

For $n = 5$, MW portfolio outperformed TOP and M portfolios in 8 out of 12 months; and for $n = 10$, better results were achieved with MW in the case of 7 months. For $n = 15$, it was also 8 out of 12. Moreover, MW portfolios outperformed TOP and M portfolios in rates of return averaged over the whole period of 12 months, with the following average rate of return for MW: 1.75 for $n = 5$, 1.81 for $n = 10$ and 1.85 for $n = 15$. Though rates of return for MW portfolios for the second set of KMI have higher values than TOP and M approach in larger number of months, in some cases (like July 2014 or January 2015) the advantage of the proposed MW is not as spectacular as it was for the same months in Experiment 1.

4. Concluding Remarks

Portfolios built with the market wisdom discovery approach, as presented above, yielded definitely better average rates of return than the other type portfolios. However to proof the general viability of the results they need to be confirmed for other investing periods and for different markets.

Effectiveness of the presented approach could be certainly improved with the more in-depth analysis of alternative KMIs selection. Sortino index, mentioned earlier in the

Table 2: Experiment 2 rate of return in every month (in percent).

	5 funds			10 funds			15 funds		
	MW	TOP	M	MW	TOP	M	MW	TOP	M
05 2014	0.9	**2.17**	1.36	0.95	**2.08**	1.36	1.01	**1.64**	1.36
06 2014	**-0.13**	-1.08	-1.2	**-0.08**	-2.15	-1.2	**-0.22**	-2.15	-1.2
07 2014	**3.23**	2.52	2.52	3.14	**3.87**	2.52	**3.29**	2.56	2.52
08 2014	**1.27**	0.47	-1.43	**1.3**	0.49	-1.43	**1.08**	0.48	-1.43
09 2014	**2.09**	0.47	0.68	**1.63**	1.16	0.68	**1.63**	1.35	0.68
10 2014	**2.8**	2.57	2.38	2.59	**3.00**	2.38	**2.75**	2.46	2.38
11 2014	**1.61**	0.57	-0.47	**1.11**	0.49	-0.47	**0.94**	0.18	-0.47
12 2014	3.36	**5.52**	1.58	**4.72**	4.64	1.58	4.86	**4.93**	1.58
01 2015	**5.87**	5.81	5.79	**6.24**	6.05	5.79	**6.37**	6.09	5.79
02 2015	**2.97**	1.38	0.66	**2.57**	1.54	0.66	**2.82**	2.12	0.66
03 2015	-2.46	-1.75	**0.75**	-2.27	-2.1	**0.75**	-2.02	-2.21	**0.75**
04 2015	-0.51	**2.17**	0.15	-0.15	**1.04**	0.15	-0.29	**0.98**	0.15
Average	***1.75***	*1.74*	*1.06*	***1.81***	*1.68*	*1.06*	***1.85***	*1.54*	*1.06*

text, is a natural candidate. It is worth noting that since numerical experiments covered only the period of 12 months, long-term risk indicators like "standard deviation" or β indicator were not applicable. Nevertheless, for 1-month investments we were able to identify high-yield portfolios without resorting to risk indicators. An interesting alternative worth verification is flexible window indicators, calculated similarly to *Till Now* return. Another promising direction of research is to use, instead of the first IIF in the ranking, a number of s first IIFs in the ranking to construct a regressing compromise half line θ (cf. Figure 5, for the case $s = 2$). This can bring the effect of immunizing the proposed market discovery mechanism against some noisy, short term behavior of the first IIFs in the ranking, making the method more robust.

Appendix 1.

Data on 92 IIFs have been collected from SITCA website (http://www.sitca.org.tw/). Time of analysis spanned for 13 months: from April 2014 till April 2015 with 1-month time windows. All IIFs are characterized by the following market indicators:

- rate of return over 1 month (RR 1M),
- rate of return over 3 and 6 months (RR 3M and RR 6M),
- rate of return over 1, 2, 3, 5, 10 years (RR 1Y, RR 2Y, RR 3Y, RR 5Y and RR 10Y),

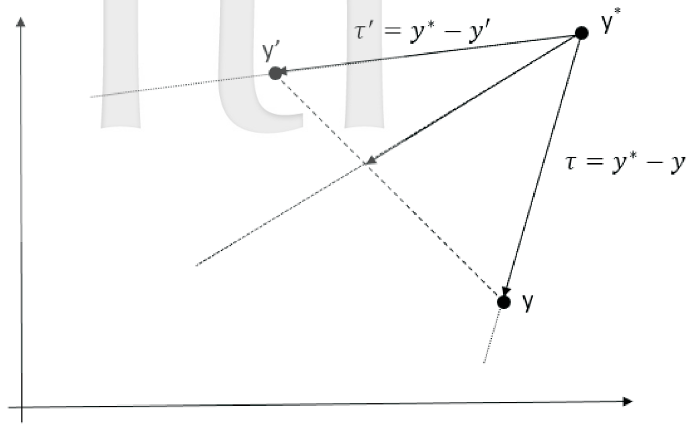

Figure 5: Constructing a compromise half line for market wisdom discovery with two IIFs.

- rate of return *Till Now*, calculated over the period from the beginning of the calendar year until present,
- 3 year standard deviation,
- 3 year β index,
- Sharpe index,
- Treynor index.

As data cover 13-month period, we have neglected long term market indicators: RR $3Y$, RR $5Y$ and RR $10Y$. For the same reason, 3-year standard deviation and 3-year β index have not been considered as KMIs.

In order to keep the number of KMIs limited, pairs of market indicators with high correlation (calculated over all 13 months in the data set) have been regarded as complementary and only one indicator from such a pair has been selected. From the highly correlated pair Sharpe index — Treynor index, the former has been selected. Market indicator *Till Now*, because of its construction, has not been included into the correlation analysis.

Eventually we have been left with the following market indicators:

- rate of return 3M,
- rate of return 6M,
- rate of return 1Y,
- rate of return *Till Now*,
- Sharpe index.

	Rate.of.Return.1M	Rate.of.Return.3M	Rate.of.Return.6M	Rate.of.Return.1Y	Std.Dev	Beta	Sharpe	Treynor
Rate.of.Return.1M	1	0.66	0.53	0.44	-0.13	0.03	0.3	0.14
Rate.of.Return.3M	0.66	1	0.87	0.72	-0.07	0.05	0.5	0.28
Rate.of.Return.6M	0.53	0.87	1	0.82	-0.03	0.08	0.55	0.35
Rate.of.Return.1Y	0.44	0.72	0.82	1	0.24	0.23	0.33	0.22
Std.Dev	-0.13	-0.07	-0.03	0.24	1	0.55	-0.48	-0.48
Beta	0.03	0.05	0.08	0.23	0.55	1	-0.11	-0.29
Sharpe	0.3	0.5	0.55	0.33	-0.48	-0.11	1	0.83
Treynor	0.14	0.28	0.35	0.22	-0.48	-0.29	0.83	1

Figure 6: Correlation between market indicators in the data set.

From this set we have formed two sets of KMI. The first one, for Experiment 1, included all the above market indicators as KMIs except *Till Now* (4 KMIs), and the second one, for Experiment 2, consisted of all 5 indicators as KMIs.

Appendix 2

Proof of Lemma 1. We consider ranks of IIF^i with respect to:

(a) IIFs which dominate IIF^i,

(b) IIFs which are dominated by IIF^i,

(c) IIFs which are neither dominated by IIF^i nor dominate it.

In the case of (a) it is immediate to show that irrespective of λ, all IIFs which dominate IIF^i get higher rank. Indeed, let $\mathrm{IIF}^i \prec \mathrm{IIF}^j$. Then, by the definition of dominance $y_l^i \leq y_l^j$, $l = 1,\ldots,k$, and $y_l^i < y_l^j$ for at least one l. Since $\lambda_l \geq 0$, $l = 1,\ldots,k$, $\lambda_l(y_l^* - y_l^i) \geq \lambda_l(y_l^* - y_l^j)$, $l = 1,\ldots,k$, and $\lambda_l(y_l^* - y_l^i) > \lambda_l(y_l^* - y_l^j)$, $l = 1,\ldots,k$ for at least one l. Finally, $\max_{l=1,\ldots,k} \lambda_l(y_l^* - y_l^i) > \max_{l=1,\ldots,k} \lambda_l(y_l^* - y_l^j)$, what proofs case (a).

In the case of (b), by a similar argument as in case (a), irrespective of λ, all IIFs which are dominated by IIF^i get lower rank.

In the case of (c) let IIF^i and IIF^j be such that neither $\mathrm{IIF}^i \prec \mathrm{IIF}^j$ nor $\mathrm{IIF}^j \prec \mathrm{IIF}^j$. Hence there exists l such that $y_l^i < y_l^j$. By this, for λ such that the compromise half line passes through y^i we have $\lambda_l(y_l^* - y_l^i) > \lambda_l(y_l^* - y_l^j) = \tau_l^{-1}(y_l^* - y_l^j) = \frac{1}{y_l^* - y_l^j}(y_l^* - y_l^j) = 1$. In consequence, $\max_{l=1,\ldots,k} \lambda_l(y_l^* - y_l^i) > \max_{l=1,\ldots,k} \lambda_l(y_l^* - y_l^j)$, what proofs case (c).

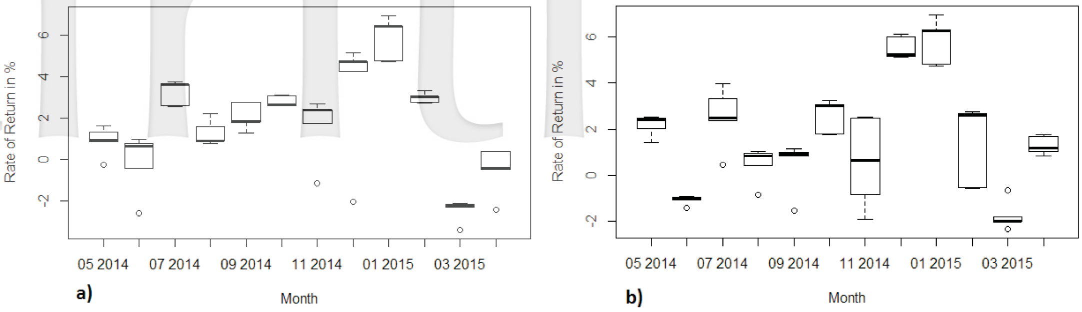

Figure 7: Boxplots for rates of return value, $n = 5$; (a) MW portfolios; (b) TOP portfolios.

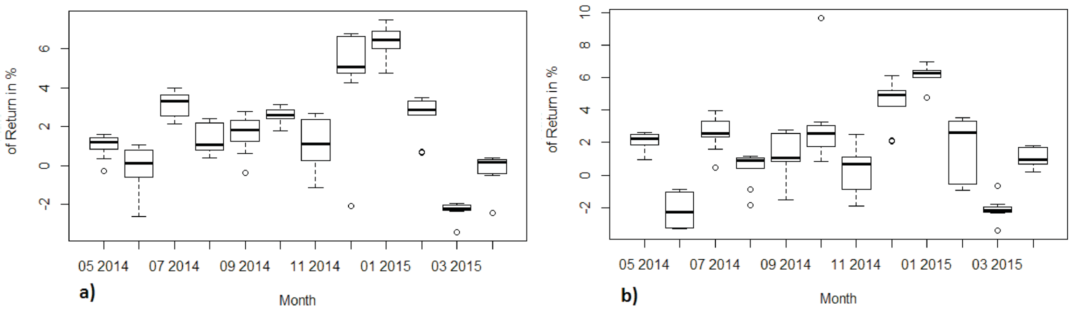

Figure 8: Boxplots for rates of return value, $n = 10$; (a) MW portfolios; (b) TOP portfolios.

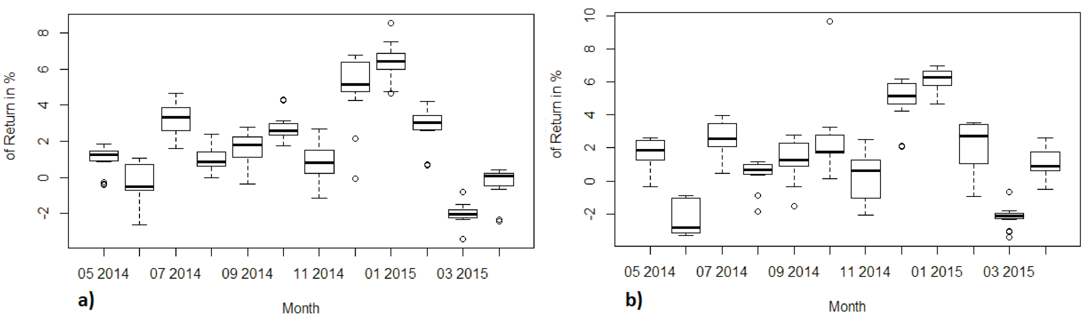

Figure 9: Boxplots for rates of return value, $n = 15$; (a) MW portfolios; (b) TOP portfolios.

Appendix 3

For both experiments we have calculated statistics for rates of return of IIFs selected for portfolios, such as those ones provided by boxplots, namely quartiles, maximal and minimal values, and medians. We focused on MW and TOP portfolios.

Experiment 1. Figure 7 presents boxplots for MW and TOP portfolios in the case $n = 5$. It is easy to see that for almost every month, the median value is higher for the MW portfolio. Moreover, the maximal heights of the boxes are smaller for MW portfolios

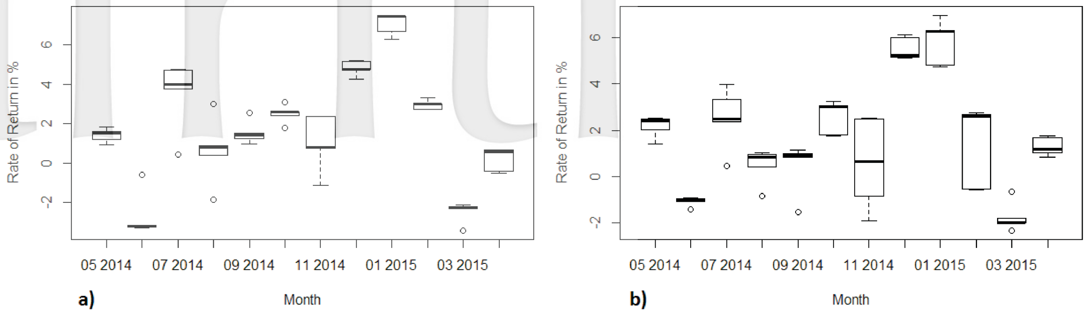

Figure 10: Boxplots for rates of return value, $n = 5$; (a) MW portfolios; (b) TOP portfolios.

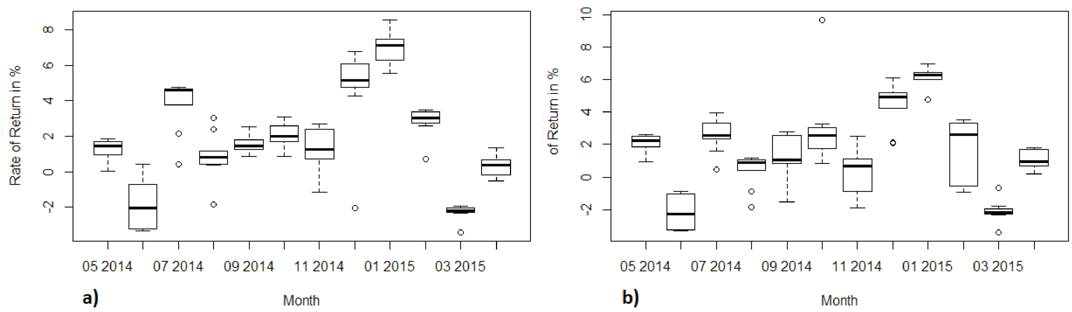

Figure 11: Boxplots for rates of return value, $n = 10$; (a) MW portfolios; (b) TOP portfolios.

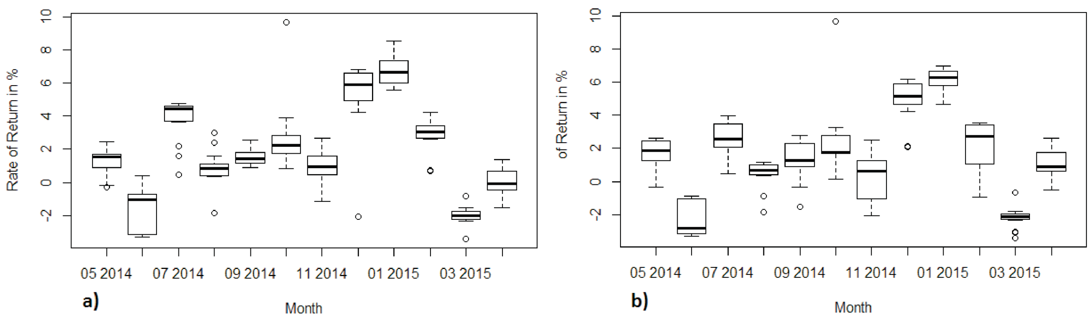

Figure 12: Boxplots for rates of return value, $n = 15$; (a) MW portfolios; (b) TOP portfolios.

than for TOP portfolios. As MW portfolios in average outperformed TOP portfolios, this may be a hint that portfolios with IFFs of high variability do not perform well.

Figures 8 and 9 present analogous boxplots for $n = 10$ and $n = 15$.

Experiment 2. Figures 10 −12 presents boxplots for MW and TOP portfolios in the cases $n = 5$, $n = 10$ and $n = 15$ respectively. One can make analogous observation as in for Experiment 1: median values are while maximal heights of the the boxes are smaller for MW portfolios comparing to TOP portfolios, which can indicate that portfolios with IFFs of high variability do not perform well.

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