Wind - Induced Vibration Control of Long - Span Bridges by Multiple Tuned Mass Dampers

Yuh-Yi Lin, Chii-Ming Cheng and David Sun

Department of Civil Engineering
Tamkang University, Tamsui, Taiwan 251, R.O.C.

Abstract

An analytical model is presented to examine the performance of multiple tuned mass dampers (MTMD) for the long span bridges subjected to wind excitation. The reduction of dynamic response and the increase of the flutter velocity by the attachment of the MTMD to the bridge are discussed. Through a parametric analysis, the characteristics of MTMD are studied and the design parameters including mass, damping, bandwidth, and total number of TMDs are proposed. A comparison of effectiveness between a single TMD and MTMD is also presented in this paper. The results indicate that the MTMD, designed with the recommended parameters, is not only more effective but also more robust than the usual single TMD against wind-induced vibration. The superior robustness of the MTMD is especially significant in the torsional direction.

Key Words : Tuned mass damper, Buffeting response, Flutter; Long-span bridge

1. Introduction

The developments of vibration control theories have led to the wide use of tuned mass dampers on many engineering structures such as tall buildings, long-span bridges, and so on. Installing a TMD on a long-span bridge has been proven to be effective for suppressing wind-induced vibrations both analytically and experimentally [2-4]. In recent years, the design concept of a TMD is extended to multiple tuned mass dampers (MTMD) which are composed of several small oscillators attached to the main structure [1,5,8]. The main idea of this design is to distribute the natural frequencies of MTMD around the natural frequency of the suppressed mode of the structure for lessening the resonant effects. From the previous studies [1,5,8], we found that MTMD is less sensitive to the offset of the tuning frequency than a single TMD, and the mass of each TMD can be made smaller. The latter is specially important for large structures, because the massive size of the damper may cause difficulties with bridge construction and maintenance.

The theories of MTMD have been discussed extensively by some researchers [1,5,8], which have provided design formulas and recommendations in their papers. However, those formulas were primarily derived for general engineering structures and may not be directly used for flexible bridges subjected to wind excitation. For the involvement of aerodynamic damping and aerodynamic stiffness, the wind-induced response of the long-span bridge is somewhat more complex than that of general structures subjected to the harmonic loads. Furthermore, the assumption of treating wind loads as white noise, adopted in some papers [5], is not completely valid, because the contribution of the background part to the total response can not be ignored in most cases. Hence, further studies of wind-induced vibration control of flexible bridges by the MTMD are still needed.

In this paper an analytical model is presented to examine the performance of the MTMD used in the long-span bridge. The dynamic response reduction and the increase of the flutter velocity of the flexible bridge are discussed. A cable-stayed bridge subjected to buffeting is chosen as the target for evaluating the performance of the MTMD in this analysis. Then, the design parameters of the MTMD are proposed through this parametric study.

2. Formulations of Equations of Motion

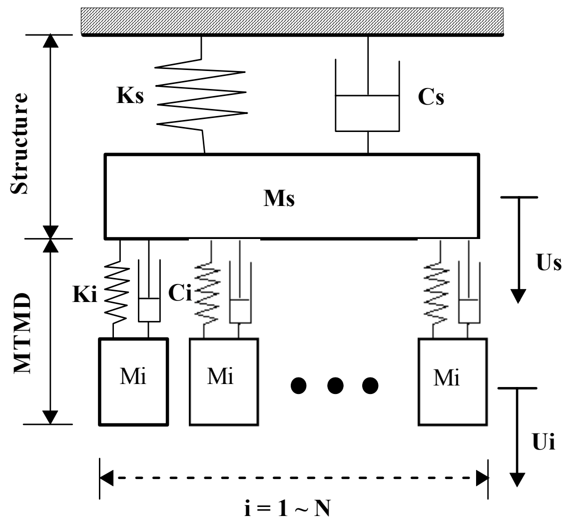


Fig. 1 Bridge - MTMD system

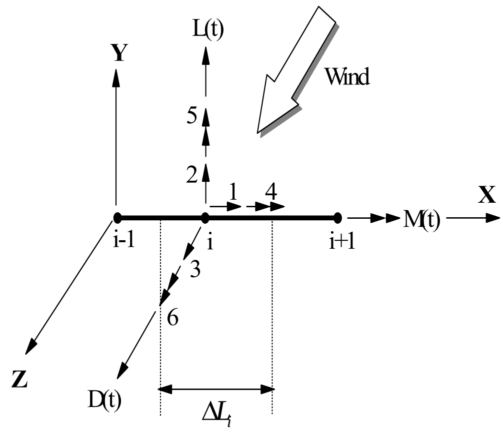


Fig.2 Finite element model of bridge deck subjected to wind loads

The vertical or torsional motion of the long-span bridge is generally dominated by the structure's first mode in that direction. Hence, it is possible to model the bridge as a single degree of freedom (SDOF) system and each TMD of the MTMD is also modeled as a SDOF system. Provided that there are N TMDs used in the structure, then the bridge-MTMD system, shown in Fig. 1, oscillates with N +1 degrees of freedom. The equations of motion of the structural model can be expressed in the generalized coordinate system as

$$M_s \ddot{u}_s + (C_s + \sum_{i=1}^{N} C_i)\dot{u}_s - \sum_{i=1}^{N} C_i \dot{u}_i + (K_s + \sum_{i=1}^{N} K_i) u_s - \sum_{i=1}^{N} K_i u_i = \mathbf{\Phi}^{\mathbf{T}} \mathbf{F}_{\mathbf{se}} + \mathbf{\Phi}^{\mathbf{T}} \mathbf{F}_{\mathbf{ex}} \tag{1}$$

$$M_i \ddot{u}_i - C_i \dot{u}_s + C_i \dot{u}_i - K_i u_s + K_i u_i = 0 \qquad (i = 1, N) \tag{2}$$

where u is the generalized displacement, M, C, and K are respectively the generalized mass, damping, and stiffness, $\mathbf{\Phi}$ is the matrix containing the first mode of the bridge, $\mathbf{F}_{\mathbf{se}}$ is the self-excited force matrix and $\mathbf{F}_{\mathbf{ex}}$ is the buffeting force matrix. The subscript s stands for the bridge and i for the ith TMD. Let ϕ (x_i) be the component of $\mathbf{\Phi}$ at the coordinate x_i where the ith TMD is located. Then, the expressions of M_s and M_i are known as

$$M_s = \mathbf{\Phi}^{\mathbf{T}} \mathbf{M} \mathbf{\Phi} \tag{3}$$

$$M_i = \phi^2(x_i) m_i \tag{4}$$

where **M** is the mass matrix of the bridge and m_i is the mass of the ith TMD. The other properties such as damping and stiffness are defined in a similar manner.

The widely accepted forms of the self-excited forces, expressed by the flutter derivatives, were proposed by Scanlan and Tomko [6]. These forms are adopted here to represent the interaction between fluid and structure. Since only vertical and torsional responses are concerned, the drag force is ignored. The self-excited forces acting on deck node j in vertical direction F_{se}^{l} and in torsion direction F_{se}^{t} are in the following forms:

$$F_{se}^{l}(t) = \frac{1}{2}\rho^2 U (2B)(K)\left[H_1^*(K)\frac{\dot{y}(t)}{U} + H_2^*(K)\frac{B\dot{\alpha}(t)}{U} + KH_3^*(K)\alpha(t)\right]\Delta L_j \tag{5}$$

$$F_{se}^{t}(t) = \frac{1}{2}\rho U^2 (2B^2)(K)\left[A_1^*(K)\frac{\dot{y}(t)}{U} + A_2^*(K)\frac{B\dot{\alpha}(t)}{U} + KA_3^*(K)\alpha(t)\right]\Delta L_j \tag{6}$$

where ρ is air density, U is wind velocity, B is deck width, $K = \dfrac{B\omega}{U}$ is the reduced frequency, ΔL_j is the tributary length of the node j (shown in Fig. 2), y, α are the vertical and torsional displacements, respectively, H_l^*, A_l^* (l=1,3) are the flutter derivatives. In this study the bridge deck is assumed insensitive to the aerodynamic coupling, the coupling terms in Eqs. (5)-(6) are neglected. Thus, Eqs. (5)-(6) can be simplified as

$$F_{se}^{l}(t) = \frac{1}{2}\rho U^{2}(2B)(K)\left[H_{1}^{*}(K)\frac{\dot{y}(t)}{U}\right]\Delta L_{j} \quad (7)$$

$$F_{se}^{t}(t) = \frac{1}{2}\rho U^{2}(2B^{2})(K)\left[A_{2}^{*}(K)\frac{B\dot{\alpha}(t)}{U} + KA_{3}^{*}(K)\alpha(t)\right]\Delta L_{j} \quad (8)$$

Substituting Eq. (7) or (8) into (1) and making some manipulations, we can rewrite the equations of motion as follows:

$$\ddot{u}_s + (2\overline{\xi}_s\overline{\omega}_s + \sum_{i=1}^{N} 2\mu_i\xi_i\omega_i)\dot{u}_s - \sum_{i=1}^{N} 2\mu_i\xi_i\omega_i\dot{u}_i + (\overline{\omega}_s^2 + \sum_{i=1}^{N}\mu_i\omega_i^2)u_s - \sum_{i=1}^{N}\mu_i\omega_i^2 u_i = \frac{\mathbf{\Phi}^{\mathrm{T}}\mathbf{F}_{\mathbf{ex}}}{M_s} \quad (9)$$

$$\ddot{u}_i - 2\xi_i\omega_i\dot{u}_s + 2\xi_i\omega_i\dot{u}_i - \omega_i^2 u_s + \omega_i^2 u_i = 0 \qquad (i = 1, N) \quad (10)$$

where ξ_i, ω_i are the damping ratio and circular frequency of the ith TMD, respectively, μ_i is the generalized mass ratio of the ith TMD to the bridge, $\overline{\xi}_s$, $\overline{\omega}_s$ are the effective damping and the effective frequency of the bridge, respectively. The mathematical form of μ_i is shown in the following:

$$\mu_i = \frac{\phi^2(x_i)m_i}{M_s} \quad (11)$$

If the vertical response is considered, the mathematical expressions of $\overline{\omega}_s$ and $\overline{\xi}_s$ can be stated as

$$\overline{\omega}_s = \omega_s \quad (12)$$

$$\overline{\xi}_s = \xi_s - \frac{\rho B^2 \sum_j \phi^2(x_j)\Delta L_j}{2M_s} H_1^* \frac{\omega}{\omega_s} \quad (13)$$

If the torsional response is considered, the mathematical forms of $\overline{\omega}_s$ and $\overline{\xi}_s$ can be expressed by

$$\overline{\omega}_s^2 = \omega_s^2 - \omega^2\left(\frac{\rho B^4 \sum_j \phi^2(x_j)\Delta L_j}{M_s} A_3^*\right) \quad (14)$$

$$\overline{\xi}_s = \frac{1}{\overline{\omega}_s}\left(\xi_s\omega_s - \frac{\rho B^4 \sum_j \phi^2(x_j)\Delta L_j}{2M_s} A_2^*\omega\right) \quad (15)$$

To solve Eqs. (9) and (10), the complex forms of the generalized displacements and the external force are used

$$\frac{\mathbf{\Phi}^{\mathrm{T}}\mathbf{F}_{\mathbf{ex}}}{M_s} = F_0 e^{i\omega t} \quad (16)$$

$$u_s = A_s e^{i\omega t} \quad (17)$$

$$u_i = A_i e^{i\omega t} \quad (18)$$

The transfer functions of the bridge and the ith TMD are then derived by substituting Eqs. (16)-(18) into (9)-(10) and setting the generalized force to be unity. If we define

$$R_F = \frac{\omega}{\overline{\omega}_s} \quad (19)$$

$$R_i = \frac{\omega_i}{\overline{\omega}_s} \quad (20)$$

$$D_i = R_i^2 - R_F^2 \quad (21)$$

$$E_i = 2\xi_i R_i R_F \quad (22)$$

then, the transfer function of the bridge can be stated as:

$$H_s(\omega) = \frac{1 / (M_s\overline{\omega}_s^2)}{\mathrm{Re}(Z) + i\,\mathrm{Im}(Z)} \quad (23)$$

where Re(Z) and Im(Z) are defined by

$$\mathrm{Re}(Z) = 1 - R_F^2 - \sum_{i=1}^{N}\frac{\mu_i R_F^2(R_i^2 D_i + E_i^2)}{D_i^2 + E_i^2} \quad (24)$$

$$\mathrm{Im}(Z) = 2\overline{\xi}_s R_F + \sum_{i=1}^{N}\frac{2\mu_i\xi_i R_i R_F^5}{D_i^2 + E_i^2} \quad (25)$$

Also, the transfer function of the ith TMD are obtained as

$$H_i(\omega) = H_s(\omega)\frac{R_i^2 D_i + E_i^2 - i2\xi_i R_i R_F^3}{D_i^2 + E_i^2} \quad (26)$$

These results are identical to those derived by Yamaguchi & Harnpornchai [8] except that the damping ratio ξ_s and the frequency ω_s are replaced by the effective damping $\overline{\xi}_s$ and the effective frequency $\overline{\omega}_s$ in this study.

3. Buffeting Response

As the buffeting response is considered, the external force term F_{ex} in Eq. (1) is substituted by the vertical or torsional buffeting force that is well-known as [7]

$$F_b^l\left(x_j,t\right)=\frac{1}{2}\rho U^2 B\left[C_L\frac{2u}{U}+\left(\frac{dC_L}{d\alpha}+C_D\right)\frac{w}{U}\right]\Delta L_j \quad (27)$$

$$F_b^t\left(x_j,t\right)=\frac{1}{2}\rho U^2 B^2\left[C_M\frac{2u}{U}+\frac{dC_M}{d\alpha}\frac{w}{U}\right]\Delta L_j \quad (28)$$

where C_L, C_D, and C_M are respectively the lift, drag, and moment coefficients, u and w are the wind speed fluctuations in horizontal and vertical directions, respectively. It should be noted that the response calculation is based on the first structural mode in either vertical or torsional direction, only one mode is taken into account for the analysis.

Using the random theory and wind velocity spectra, we can obtain the co-spectrum of buffeting forces between deck node p and q, which is denoted by $S^C_{F_pF_q}$. The generalized force spectrum S_F is then obtained

$$S_F=\sum_p\sum_q \phi(x_p)\phi(x_q)S^C_{F_pF_q} \quad (29)$$

The displacement spectrum of the bridge at node j is in the following form:

$$S_d(x_j)=\phi^2(x_j)S_F\left|H_s(\omega)\right|^2 \quad (30)$$

Similarly, the displacement spectrum of the ith TMD is expressed by

$$S_{id}(x_i)=\phi^2(x_i)S_F\left|H_i(\omega)\right|^2 \quad (31)$$

Integrating Eqs.(30) and (31) with the frequency w, we can obtain the mean square of the response of the bridge at node j

$$\sigma_s^2(x_j)=\int_0^\infty S_d(x_j)d\omega \quad (32)$$

Also, the variance of the response of the ith TMD can be calculated from the following equation:

$$\sigma_i^2(x_i)=\int_0^\infty S_{id}(x_i)d\omega \quad (33)$$

4. Evaluation of Fluttervelocity

The main objective of using TMD or MTMD is to suppress excessive response induced by buffeting. In addition, an accompanying effect is the increase of the flutter velocity especially for the torsion-resistant dampers. When the torsional resistance of the bridge system is concerned, the use of MTMD not only can reduce the torsional response but increase the critical velocity. In general, the motion of the bridge is reasonably assumed to be both structurally and aerodynamically uncoupled, flutter in this case will be "single-degree-of-freedom flutter". Since this type of flutter is dominated by the first torsional mode, the flutter analysis based on the equations of motions shown in Eqs.(9)-(10) is plausible.

We consider Eqs. (9) and (10) and drop the external force term in Eq. (9), because the external force is not relevant to the flutter analysis. Then, by substituting Eqs. (17)-(18) into this system of equations, a complex eigen-value problem is yielded and can be stated in a matrix form

$$\left([G]-[\lambda]\right)\{A\}=0 \quad (34)$$

where [G] is a square matrix with rank of N+1, [λ]is a diagonal matrix with rank of N+1, {A} is an amplitude matrix. The definitions of these matrices are

$$[G]=i\omega\begin{bmatrix} 2\overline{\xi}_s\overline{\omega}_s+\sum_{i=1}^{N}2\mu_i\xi_i\omega_i & -2\mu_1\xi_1\omega_1 & -2\mu_2\xi_2\omega_2 & \cdots & -2\mu_N\xi_N\omega_N \\ -2\xi_1\omega_1 & 2\xi_1\omega_1 & 0 & \cdots & 0 \\ -2\xi_2\omega_2 & 0 & 2\xi_2\omega_2 & \cdots & 0 \\ . & . & . & \cdots & . \\ -2\xi_N\omega_N & 0 & 0 & \cdots & 2\xi_N\omega_N \end{bmatrix}+$$

$$
\begin{bmatrix}
\overline{\omega}_s^2 + \sum_{i=1}^{N} \mu_i \omega_i^2 & -\mu_1 \omega_1^2 & -\mu_2 \omega_2^2 & \cdots & -\mu_N \omega_N^2 \\
-\omega_1^2 & \omega_1^2 & 0 & \cdots & 0 \\
-\omega_2^2 & 0 & \omega_2^2 & \cdots & 0 \\
\cdot & \cdot & \cdot & \cdots & \cdot \\
-\omega_N^2 & 0 & 0 & \cdots & \omega_N^2
\end{bmatrix} \tag{35}
$$

$$[\lambda] = [\omega^2] \tag{36}$$

$$\{A\} = [A_s\ A_1\ A_2 \ldots A_N]^T \tag{37}$$

The square root of the eigenvalue obtained from Eq. (34) is the frequency in a complex form. The ratio of the imaginary part to the real part of this frequency is interpreted as the total damping of the structural system. The total damping is contributed by the bridge itself, MTMD, and aerodynamics. At some wind velocity, when the damping being zero, the structural response will approach infinity, that is, flutter will occur. The corresponding wind speed is called the "flutter velocity", and the real part of the frequency is called "flutter frequency". It is noted that Eq. (34) should be solved by iteration for each wind speed, because the matrix [G] consists of the unknown $\tilde{}$ The iterative calculation generally can yield a convergent solution by using an appropriate initial value and a reliable convergence criterion.

5. Parametric Analysis

The structure used in this study is a cable -stayed bridge. The bridge has a total span of 1460m and a width of 21.5m. Two 200-m-high towers are supported by cables. A finite element model, consisted of beam elements and cable elements, is used to calculate the natural frequencies of the structure. The geometry of this bridge is shown in Fig.3. Through the calculation, the natural frequencies of the first vertical mode and the first torsional mode are 0.141Hz and 0.354Hz, respectively. The flutter derivatives H_1^* and A_i^* (i = 2,3), modified from Scanlan and Tomko [6], are shown in Fig.4. The drag, lift, and torsion coefficients C_D , C_L , and C_M , used for buffeting calculations, are adopted from reference [9] and shown in Fig. 5. The roughness length of 1.2m is used. The mass and the damping ratio of each TMD are assumed the same for practical reasons. Since the TMDs are closely mounted on the bridge deck, the location of each TMD connected to the bridge deck can be theoretically assumed the same without any huge error. In this analysis the design parameters of the MTMD include the total mass ratio, the damping ratio, the number of TMDs, and the frequency bandwidth. To account for the mistuning problem, the effect of offset is also considered. The performance of a TMD will increase in proportion to the mass ratio. Generally mass ratios less than 1% or larger than 4 % would be too light or too heavy for practical purposes. Therefore, the range of total mass ratio of the TMDs is chosen from 1% to 4%. The damping ratio range of the TMDs is chosen from 1% to 7%. The total number of TMDs is related to the bandwidth of the MTMD, and the ranges of these parameters are studied from 1 to 21 and from 0.1 to 0.5, respectively.

(1) Performance of the MTMD for Suppressing Vertical Buffeting Response

Since the characteristics of MTMD in the vertical direction will generally not vary with the wind velocity, the following studies are investigated at a single wind speed only.

(a) Effect of damping ratio

When 1% total mass ratio is used, the relationship of the damping ratio and response reduction ratio for various numbers of TMDs is shown in Fig.6. For a single TMD, the performance increases with the damping ratio and reaches the maximum at 5% damping ratio. This result is coincident with that in previous studies [3]. For the MTMD the result is quite different. The response reduction ratio sharply increases to its optimum at a low damping ratio and then slowly decreases as the damping ratio increases. The comparison of the results indicates that the performance of the MTMD is somewhat better than that of the single TMD. This conclusion can be expected because the frequency range of the displacement spectrum at the resonant part is wide band, which results in the superiority of the MTMD. We can also conclude that with larger number of TMDs, the smaller is the optimum damping ratio. However, this tendency will not be obvious when the number of TMDs is more than 9. For design purposes, the damping ratio should be selected on the safe side, that is, the preferred value is the one that is larger than the optimum. By inspection from Fig. 6, we can suggest that 2% damping is appropriate for the MTMD with 1% total mass ratio.

(b) Effect of bandwidth

The bandwidth is one of the important design parameters of MTMD. It designates the range of the distributed frequencies of TMDs and is defined here as the ratio of the difference of the maximum and the minimum frequencies of the TMDs to the structural frequency. Fig. 7 shows the response

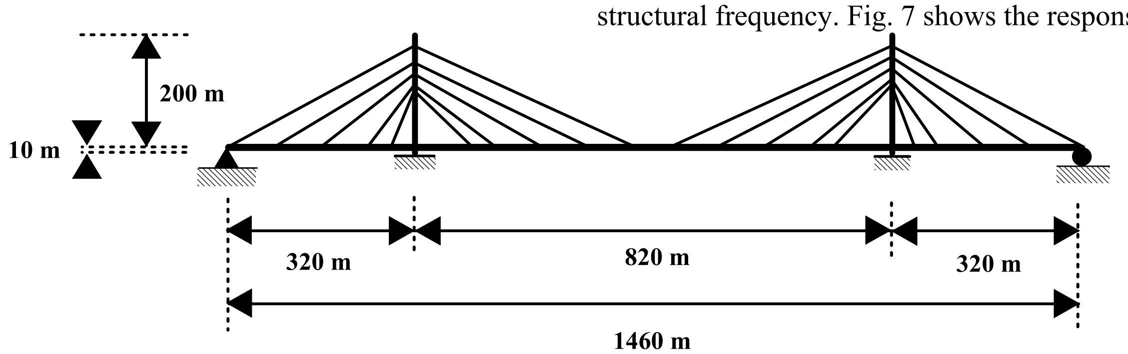


Fig 3 Geometry of the cable-stayed bridge

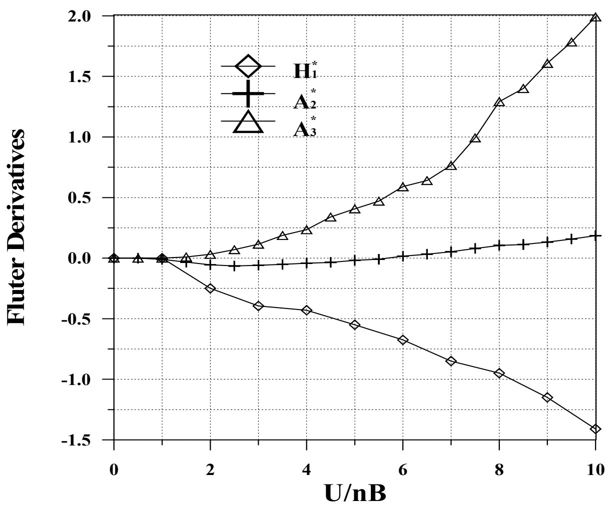


Fig. 4 Flutter derivatives

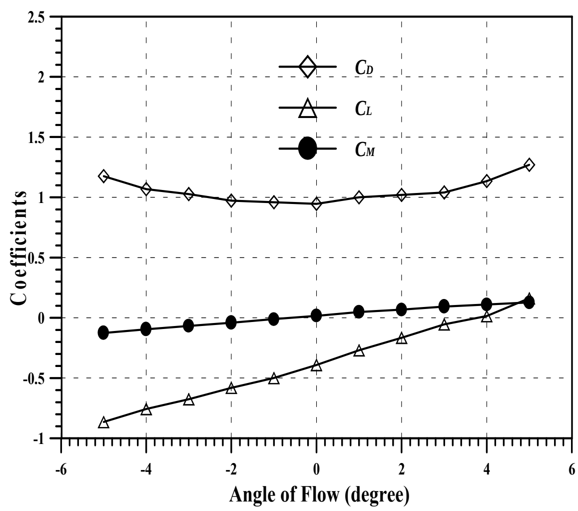


Fig. 5 Lift, drag, and torsional coefficients

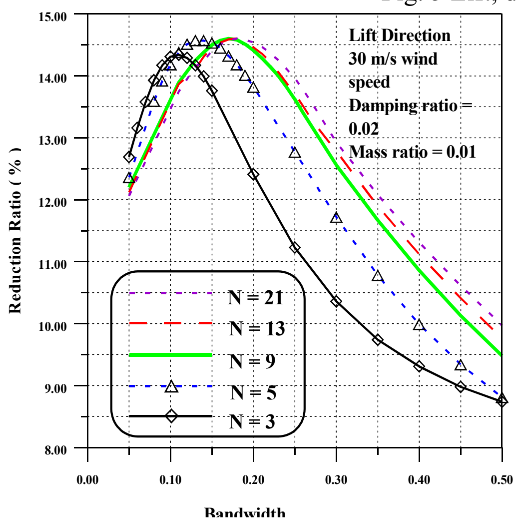


Fig. 7 Response reduction ratio versus bandwidth for different number of TMDs

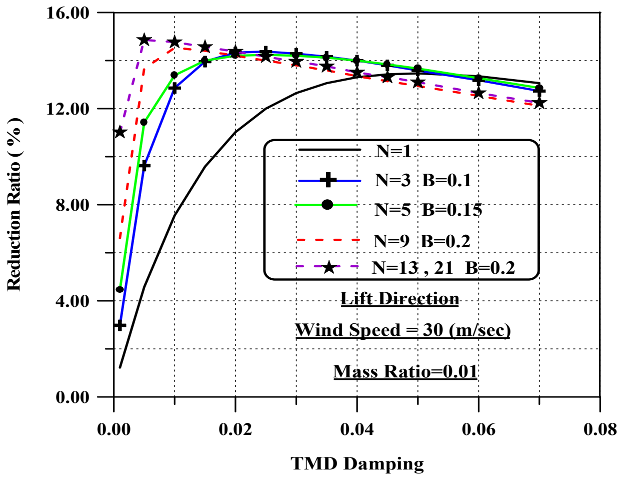


Fig. 6 Response Reduction Ratio versus TMD damping for different numbers of TMDs

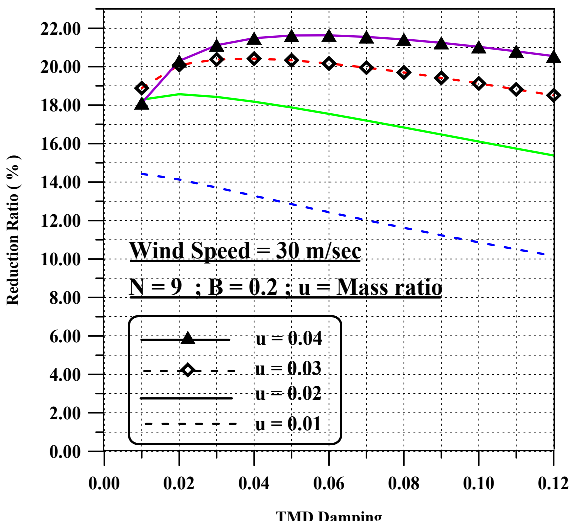


Fig. 8 Response reduction ratio versus TMD damping for different mass ratios

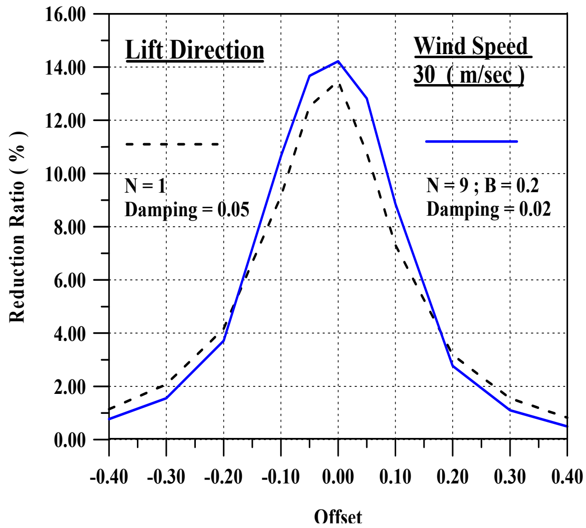


Fig. 9 Comparison of robustness between 9 TMDs and a single TMD

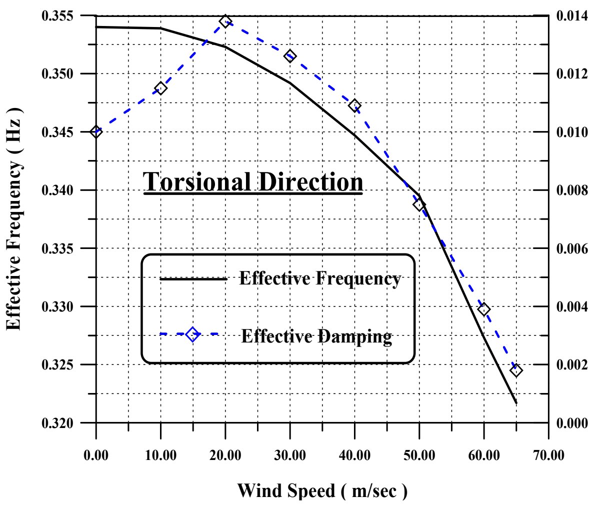
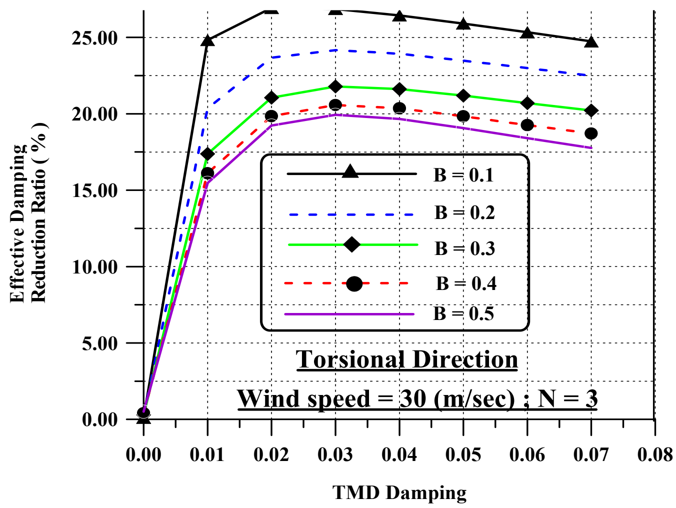
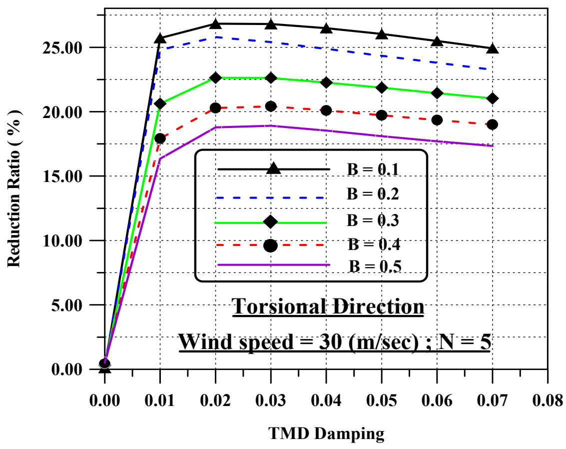


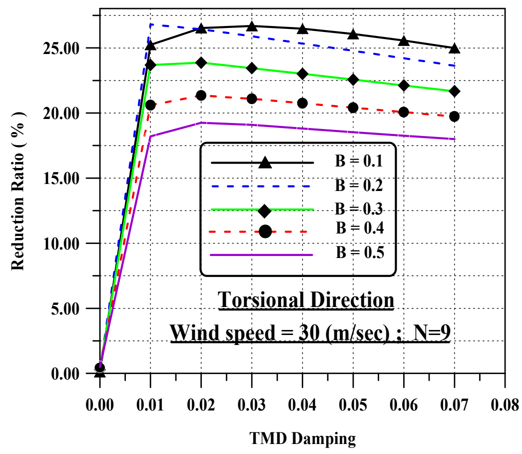
Fig.10 Effective frequency and damping versus wind speed for the typical bridge



(a) n=3

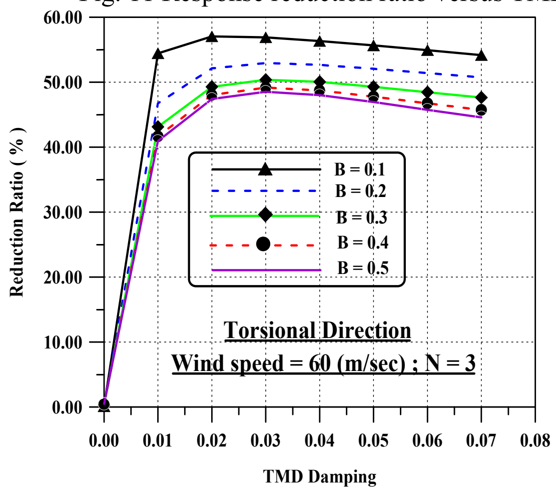


(b) n=5

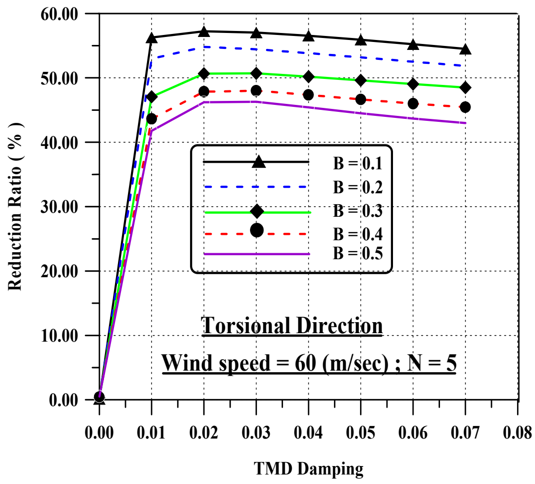


(c) n=9

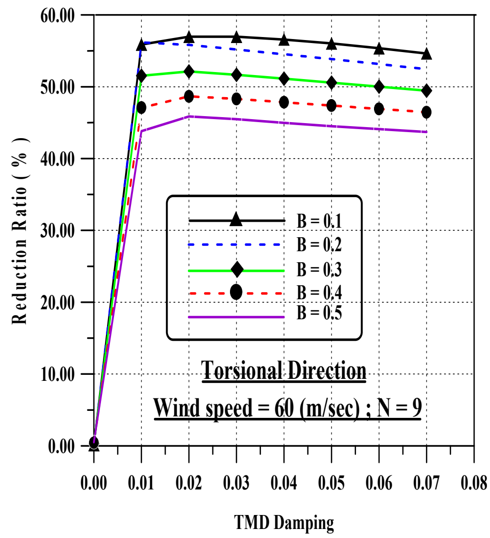
Fig. 11 Response reduction ratio versus TMD damping at 30 m/s wind speed (a) n=3 (b) n=5 (c) n=9



(a) n=3



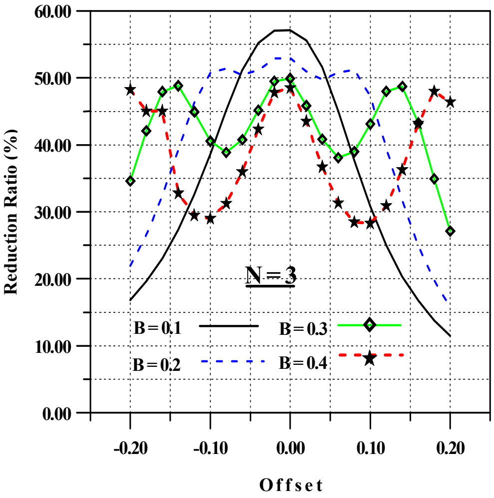
(b) n=5



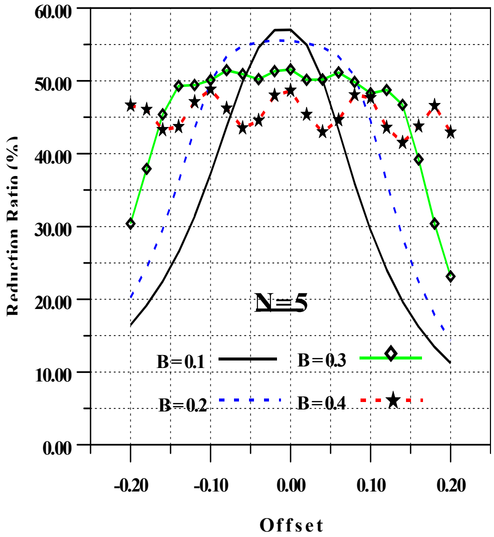
(c) n=9

Fig. 12 Response reduction ratio versus TMD damping at 60 m/s wind speed (a) n=3 (b) n=5 (c) n=9

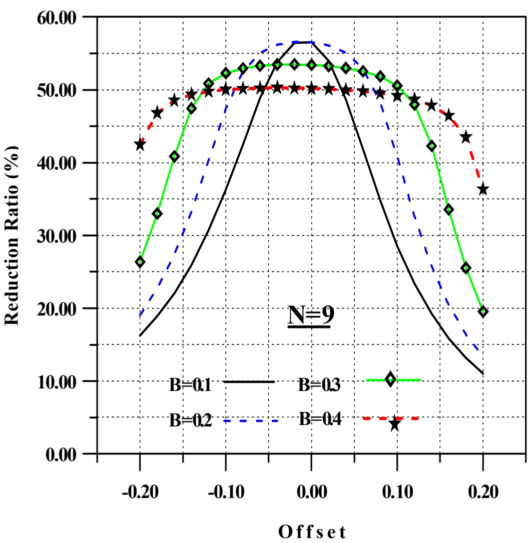
(a) n=3



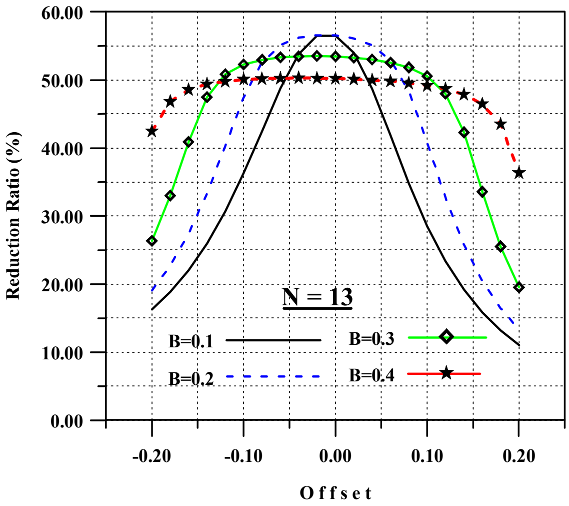
(a) n=3



(b) n=5



(c) n=9



(d) n=13

Fig. 13 Response reduction ratio versus offset at 60 m/s wind speed (a) n=3 (b) n=5 (c) n=9 (d) n=13

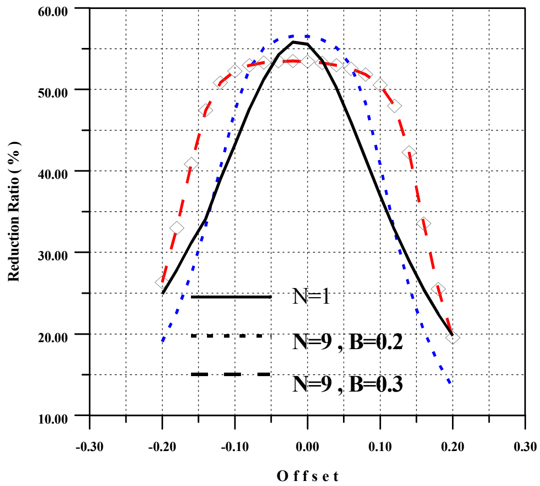


Fig. 14 Comparison of robustness between 9 TMDs and a single TMD in torsional direction

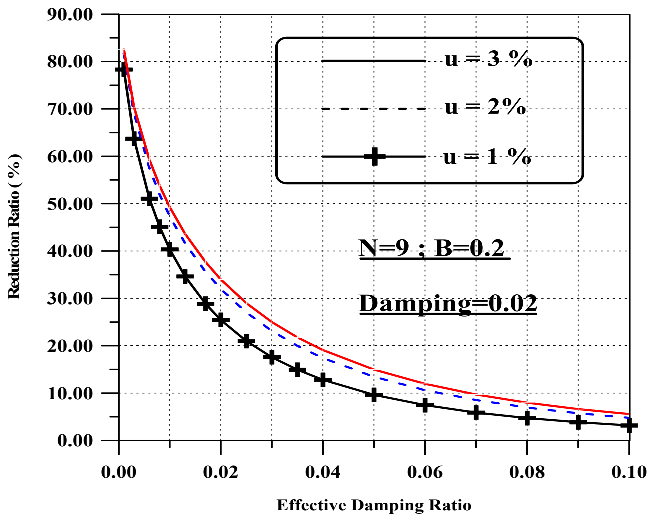


Fig. 15 Response reduction ratio versus effective structural damping for different mass ratios

Table 1 Suggested bandwidth for different number of TMDs

no. of TMDs	lower bound ($B^n_{\min}$)	Suggested value (B^n)	upper bound ($B^n_{\max}$)
3	0.07	0.1	0.15
5	0.09	0.15	0.20
9	0.11	0.2	0.24
13	0.11	0.2	0.25
21	0.12	0.2	0.25

Table 2 Flutter velocity of the bridge

Flutter velocity w/o TMDs = 68.33 (m/sec) N = no. of TMDs ; B = bandwidth ; D=damping ratio μ = total generalized mass ratio ; ( ) = increase of the flutter velocity in percent			
	μ = 1 %	μ = 2 %	μ = 3 %
N = 1			
D =0.050	77.46 (+13.4%)	82.02 (+20.0%)	86.16 (+26.1%)
D =0.071		87.57 (+28.2%)	
D =0.087			97.74 (+43.0%)
N = 9 ; B=0.2			
D =0.02	81.41 (+19.1%)	81.80 (+19.7%)	81.77 (+19.7%)
D =0.03	84.20 (+23.2%)	89.19 (+30.5%)	90.37 (+32.3%)
D =0.05	86.76 (+27.0%)	97.27 (+42.4%)	101.98 (+49.2%)

reduction ratio versus bandwidth for different number of TMDs. We can observe that the optimum bandwidth increases with the number of TMDs but will converge to a value of about 0.18 as the number is equal to or larger than 9. A large bandwidth means that some of the frequencies are far away from the structural frequency and will lower the effectiveness of the MTMD. On the other hand, a small bandwidth implies the characteristics of the MTMD are similar to those of a single TMD and will lose the benefits of MTMD. Therefore, the bandwidth should be properly selected in the MTMD's design to ensure better performance. For design purposes we take 95% of the optimum reduction ratio as the design target, and the corresponding larger and smaller bandwidths are the upper and lower bounds, respectively. The suggested values are shown in Table 1.

(c) Effect of the number of TMDs and mass ratio

An odd number of TMDs is often used in the MTMD system, the central frequency is tuned around the structural frequency and the others are equally spaced on both sides of the central one. From the results in Fig. 7, the optimum number is around 9. The use of the larger number of TMDs does not increase the performance significantly. There also may be difficulties to tune the frequencies precisely for small spacing of the frequencies.

Earlier discussions of design parameters of the MTMD are based upon 1% total mass ratio. It is known that the performance of MTMD increases with the mass ratio and the corresponding optimum damping ratio also increases. To study this effect, we use 9 TMDs and bandwidth of 0.2 for the analysis. The relationship between response reduction ratio and damping ratio for different mass ratios is shown in Fig. 8. The results indicate that 2% damping ratio can yield a good performance when the mass ratio falls between 1 % and 3%, but a higher damping ratio (about 5%) is more suitable for a mass ratio of 4%. Another finding is that the increase of response reduction ratio is significant when the mass ratio is raised from 1% to 2 or 3%, but it is not obvious when 4% mass ratio is used. For design purposes, the appropriate value of the upper bound of total mass ratio is about 3%.

(d) Robustness

Fig. 9 shows the comparison of the robustness between the MTMD and the single TMD. The definition of the offset used here is the ratio of the difference between the tuned and peak frequencies to the peak frequency. The results show that the MTMD is better than a single TMD but the difference is not significant. The reason is that the response is relevant to the enclosed area bounded by the displacement spectrum and frequency, and the change of this area due to mistuning is not sensitive.

Furthermore, the structural frequency in the vertical direction will not change with the wind speed and the mistuning problem is not so serious. However, this effect will become more important in the torsional direction and will be discussed later.

(2) Performance of the MTMD for Suppressing Torsional Buffeting Response

There are some structural characteristics in the torsional direction that are different from those in the vertical direction. First, the effective damping in the torsional direction decreases with wind speed and may induce flutter. Second, the effective frequency also changes with wind speed and the tuning frequencies of MTMD become more important. These relationships between wind speed, the effective damping, and the effective frequency for a typical bridge are shown in Fig. 10.

(a) Effect of damping ratio and bandwidth

With 1% mass ratio, the relationships between performance and damping ratio corresponding to 30m/s and 60m/s wind velocities are shown in Figs. 11-12, respectively. We can see that a 2% damping ratio for the MTMD can yield a good performance in all cases. This suggested damping ratio is the same as that in the vertical direction. Therefore, we can conclude that the optimum damping ratio is independent with the effective frequency of the structure.

The results in Figs. 11-12 also show that the choice of bandwidth is nearly independent of wind velocity (or effective damping ratio), and a smaller bandwidth often results in better performance. However, a smaller bandwidth may cause a tuning problem for the MTMD. The reason is that the frequency interval of the MTMD may be too small to be tuned precisely. For this concern it seems that the best choice is to use 3 TMDs with a bandwidth of 0.1 and a damping ratio of 2%. The other choice is to use 9 TMDs with a bandwidth of 0.2 and a damping ratio of 2% that also produce a good performance. However, the choice of the bandwidth is also dependent on the robustness that may be the dominating factor and will be discussed in the next section.

(b) Robustness

The effective frequency of the bridge subjected to wind excitation will be changed by the aerodynamic stiffness. For this reason it is not possible to exactly tune the TMD frequency to the frequency of the peak response for each wind speed and there will be some offset to the peak value. Furthermore, the natural frequency discrepancies between the real structure and the prototype are, in practice, inevitable. Therefore, the offset should be taken into account for determining the design parameters to ensure the MTMD performance. Because the total number of TMDs and bandwidth will affect the robustness, these factors are investigated in the following analysis.

To simplify the study, 1% mass ratio and 2% damping ratio are used in this analysis. At 60 m/s wind speed, the relationships of response reduction ratio and offset for 3, 5, 9, and 13 TMDs are shown in Fig. 13. For 3 TMDs, the curve of bandwidth of 0.1 is a bell-like shape; the response reduction ratio reaches the peak (57%) at zero offset and reduces rapidly with the increase of offset. In this case, robustness is similar to that of a single TMD and the allowable offset is small. As a bandwidth of 0.2 is used, the response reduction ratio fluctuates with offset and produces three peaks. The allowable offset is larger but the maximum response reduction ratio drops to 53%. As the bandwidth is increased to 0.3 and 0.4, the peaks are more obvious and the allowable offset is larger but the maximum response reduction ratio is smaller. This explains that the robustness increases with the bandwidth but the performance decreases with it. To satisfy both robustness and performance requirements, a bandwidth of 0.2 seems to be a best value for 3 TMDs. For 5 TMDs, the relationship between offset and response reduction ratio is similar to that of 3 TMDs. In this case, the curve of bandwidth of 0.2 is more flat and the response reduction ratio is about 55% which is slightly larger than that of 3 TMDs. We then can conclude that for a given bandwidth more TMDs are more robust and produce better performance. This conclusion can be verified further for 9 or 13 TMDs in which the performance of a bandwidth of 0.2 is even better. However, the comparison of the results between 9 and 13 TMDs indicates that the maximum performance is achieved as 9 TMDs are used. The performance of 13 TMDs is almost the same as that of 9 TMDs. In the case of 9 or 13 TMDs, another finding is that the allowable offset is almost a half of the bandwidth. For design procedures, the allowable offset should be determined first and then the bandwidth. Due to the change of the effective frequency with wind velocity, the allowable offset $\triangle$ S should be controlled by the following:

$$\Delta \quad S = \frac{n_s - n_f}{n_s} \tag{38}$$

where n_s is the structural frequency, n_f is the flutter frequency. After the offset is evaluated from the above equation, the required bandwidth is twice of the offset.

From the comparison of the results, shown in Fig. 14, we can find that the MTMD with 9 TMDs is superior to the usual single TMD.

(c) Effect of the number of TMDs and mass ratio

The effect of the number of TMDs on the performance of the MTMD can be clearly explained in Fig. 13. The results indicate that the required minimum number of TMDs to obtain best performance in the torsional direction is 9, which is the same as that in the vertical direction.

The performance of the MTMD definitely increases with mass ratio but it does not gain much at the low effective damping ratio as shown in Fig. 15. For design purposes, a 1% mass ratio can obtain a good performance and is recommended in the torsional direction. Also, Fig. 15 can be a useful tool to predict the response reduction ratio when the effective damping ratio of the bridge is known.

(d) Increase of flutter velocity

The critical velocity of the typical bridge without dampers is 68.33 m/s. A single TMD and 9 TMDs with various combinations of damping ratios and mass ratios are analyzed to study the increase of the flutter velocity by the addition of the tuned mass dampers. The frequency of the central TMD is tuned to the natural frequency of the first torsional mode. The results, illustrated in Table 2, indicate that the increase of flutter velocity is about 13-43% for a single TMD and 19-49% for the MTMD. Generally the flutter velocity increases with damping and mass ratios. For a fixed damping ratio, a higher mass ratio yields a higher flutter velocity in the case of a single TMD. However, in the case of MTMD with a fixed damping ratio, the increase of flutter velocity due to the increase of mass ratios is not obvious. In this case, a higher damping ratio should be used for a higher mass ratio to efficiently raise the flutter velocity. Comparison of a single TMD and the MTMD, both designed with the same mass, shows that the MTMD is more effective than a single TMD for increasing the structure's stability.

It should be noted that the results shown in Table 2 are based on the assumption that the frequency of the central damper is tuned to the natural frequency of the first torsional mode. If the increase of torsional stability is the major concern, this frequency should be tuned less. A suitable value for achieving this purpose is the flutter frequency without using TMD.

6. Design Recommendation of MTMD

A summary of the parametric analysis on the MTMD in the vertical and torsional directions can be stated as follows:

(1)For the vertical MTMD, the suggested frequency of the central damper is the frequency of the first vertical mode of the structure. For the torsional MTMD, the suggested tuning frequency of the central damper is the average between the first torsional mode frequency and the flutter frequency of the bridge without using TMD. If the increase of torsional stability is the major concern, this frequency can be simply tuned to the flutter frequency without using TMD.

()For both the vertical and torsional MTMD, the suggested number of dampers is 9.

For the vertical MTMD, the total mass is suggested to be 2% or more to ensure the vertical performance and the corresponding damping ratio is 2%. For the torsional MTMD, the mass is suggested to be 1% which is sufficient for obtaining good performance and the corresponding damping ratio is also 2%.

For the vertical MTMD, the suggested bandwidth is shown in Table 1. For the torsional MTMD, the suggested bandwidth is the larger one between the values obtained from Eq. (38) and Table 1.

7. Concluding Remarks

A parametric analysis of the MTMD used for suppressing aerodynamic response of long-span bridges is presented. Through this analysis the suggested design parameters including damping, mass, number, and bandwidth of the MTMD are proposed. The results show that the MTMD is more effective than an optimized single TMD for suppressing buffeting response and increasing the critical flutter speed. With a proper design, the use of MTMD can be advantageous to improve the aerodynamic behavior of a long-span bridge. The characteristics of the MTMD used in the vertical and torsional directions are almost the same except the tuning frequency and bandwidth that are affected by the aerodynamic stiffness. The bridge and the aerodynamic coefficients used here are chosen arbitrarily, due to the fact that these structural properties and aerodynamic effects have been normalized. Therefore, the values recommended in this paper are still useful for other bridges. It should be mentioned that the relative displacement between TMD and the structure is not included in this study; if this displacement is not allowable, the suggested damping ratio of TMD should be increased.

Acknowledgements

The writers gratefully acknowledge the financial support of part of this work by the National Science Council (R. O. C.) under the grant NSC 87 -2211 -E -032 -017.

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Manuscript Received: Apr. 28, 2000
Revision Received: May. 19, 2000
And Accepted: May, 19, 2000